To calculate the drag we need to know p on $r=a$

- flow is steady, inviscid + irrotational ($+ g = 0$)

so Bernoulli Fn. = const. everywhere

so $\frac{1}{2}\rho_0 \underset{\sim}{u}\cdot\underset{\sim}{u} + p = \frac{1}{2}\rho_0 U^2 + p_\infty$

so $p = p_\infty + \frac{1}{2}\rho_0 U^2 - \frac{1}{2}\rho_0 \underset{\sim}{u}\cdot\underset{\sim}{u}$

on $r=a$ $\quad \underset{\sim}{u} = (0, u_\theta)$ $\quad$ since $u_R = 0$

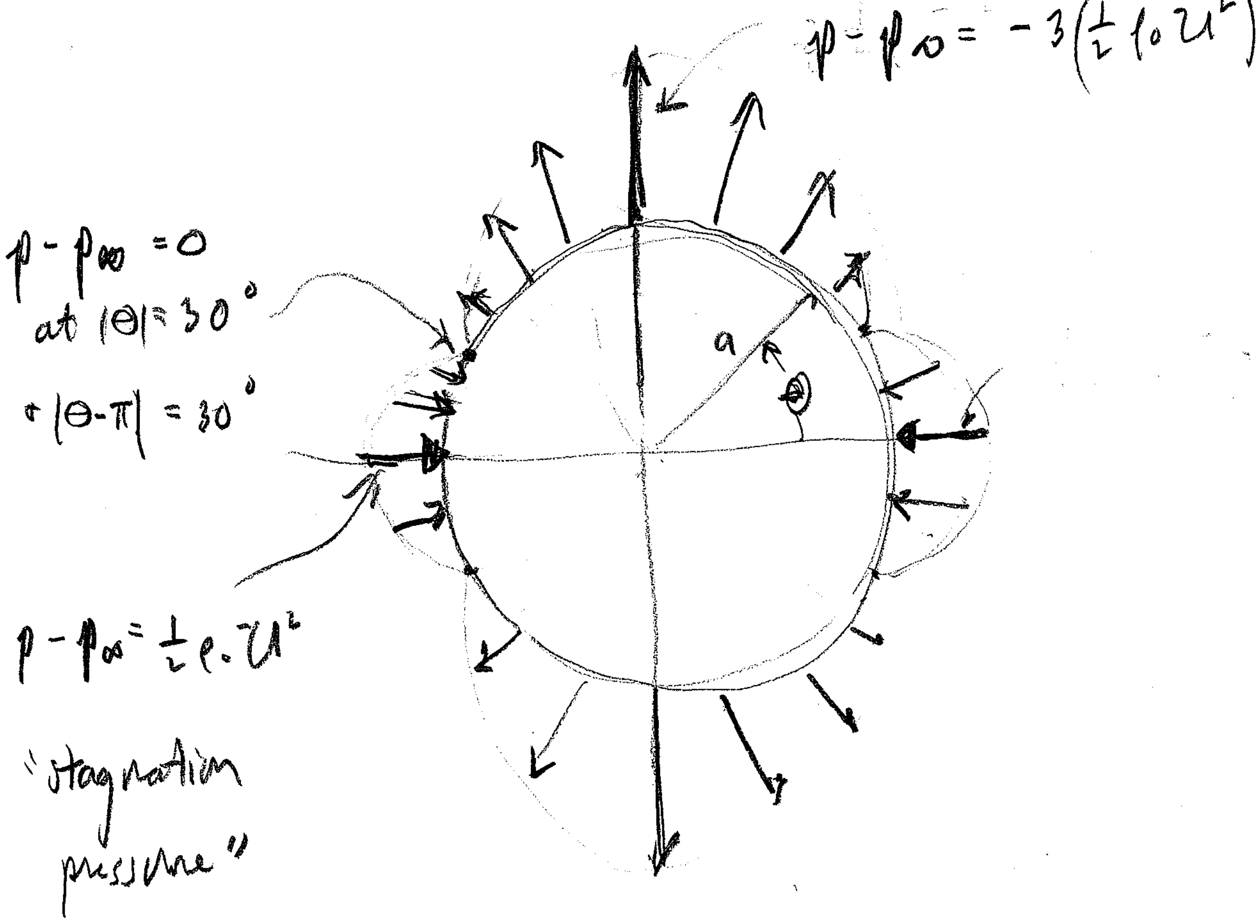
and $(u_R, u_\theta) = \nabla\varphi = \left(\frac{\partial\varphi}{\partial r}, \frac{1}{r}\frac{\partial\varphi}{\partial\theta}\right)$

$\Rightarrow u_\theta = \frac{1}{r}\frac{\partial\varphi}{\partial\theta} = -U\left(1 + \frac{a^2}{r^2}\right)\sin\theta = -2U\sin\theta$ on $r=a$

and $\frac{1}{2}\rho_0 U^2 - \frac{1}{2}\rho_0 \underset{\sim}{u}\cdot\underset{\sim}{u}\Big|_{r=a} = \frac{1}{2}\rho_0\left(U^2 - u_\theta^2\right)$

so $$p = p_\infty + \frac{1}{2}\rho_0\left[U^2 - 4U^2\sin^2\theta\right]$$

$$p = p_\infty + \frac{1}{2}\rho_0 U^2\left[1 - 4\sin^2\theta\right]$$

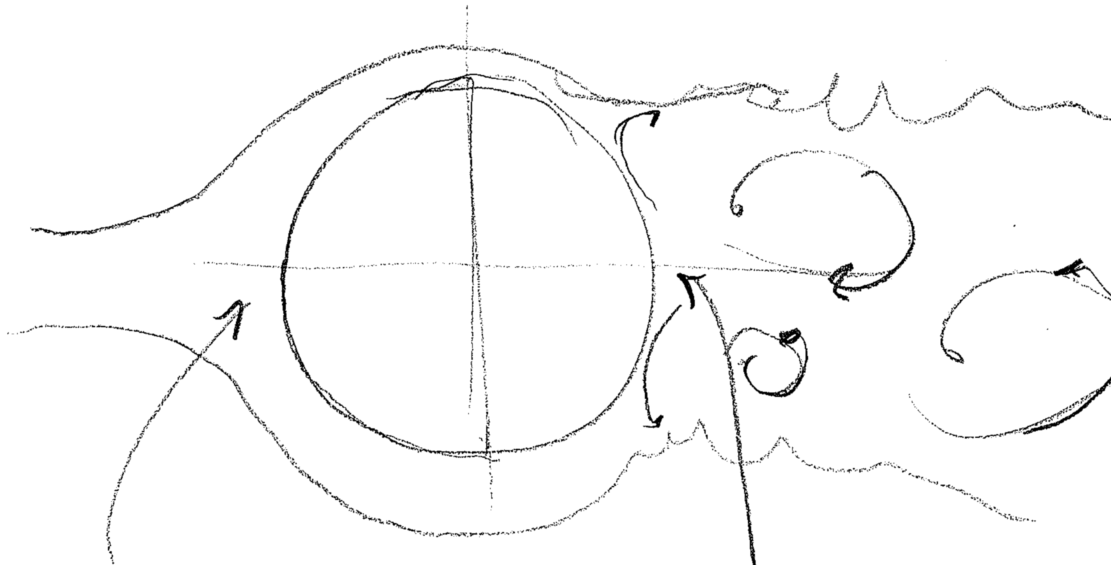


Pressure forces are symmetric

so drag = 0

For flow past a real cylinder there is "separation"

$U \Rightarrow$



$p \simeq p_\infty + \frac{1}{2}\rho_0 U^2$ on the front face

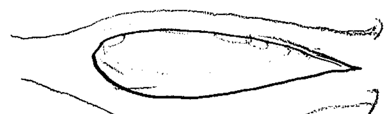
$p \simeq p_\infty$ on the back face

$\text{Drag} \sim \frac{1}{2}\rho_0 U^2 \cdot A$ (A = <u>frontal</u> area)

More generally:

$$\text{Drag} = \frac{1}{2}\rho_0 U^2 \, C_D \, A$$

a "drag coefficient"



$C_D = O(1)$ for the cylinder $C_D \ll 1$ for streamlined shapes