$\boxed{V.1}$ 11/16/2009 (1)

Potential Flow under some conditions the velocity may be given by the gradient of a "potential"

$$\underset{\sim}{u} = \nabla\varphi \qquad \varphi = \varphi(\underset{\sim}{x}, t) \quad \text{scalar}$$

- Requires that flow be $\boxed{\text{irrotational}}$ ← the condition!

since $\nabla \times [\nabla(\;)] = 0$ for any vector field $(\;)$

then $\underset{\sim}{\omega} = \nabla \times \underset{\sim}{u} = \nabla \times [\nabla\varphi] = 0$

- If the flow is also $\boxed{\text{incompressible}}$

then $\nabla \cdot \underset{\sim}{u} = 0$ becomes $\nabla \cdot \nabla\varphi = \boxed{\nabla^2\varphi = 0}$ (††)

"Laplaces equation" linear

Boundary Conditions e.g.

- no flow through a boundary

$\underset{\sim}{u} \cdot \hat{n} = 0$ becomes $\dfrac{\partial\varphi}{\partial n} = 0$

- $\underset{\sim}{u} \to (U, 0, 0)$ at ∞

becomes $\dfrac{\partial\varphi}{\partial x} \to U$ at ∞

formally "the Neumann problem"

* Then solutions to $\nabla^2\varphi = 0$ are unique, Also because (††) is linear we may add solutions, and their sum must satisfy the desired b.c.!

Motivation

incompressible, irrotational, inviscid flow governed by

mass $\nabla \cdot \underset{\sim}{u} = 0$

$\boxed{\underset{\sim}{x} \text{ mom}}$ $\dfrac{\partial \underset{\sim}{u}}{\partial t} + \underbrace{\nabla \tfrac{1}{2} \underset{\sim}{u} \cdot \underset{\sim}{u} = -\tfrac{1}{\rho} \nabla p - \nabla (gz)}$

gives Bernoulli function for steady flow

4- equations in u, v, w + p, non linear

* Use of $\nabla^2 \varphi = 0$ reduces problem to linear 2nd order ODE in φ

+ Then p may be determined from $\boxed{\underset{\sim}{x} \text{ mom}}$

Note this works for unsteady flow too! since the solution to $\nabla^2 \varphi = 0$ only depends on the instantaneous value of the boundary conditions.

Examples 2-D (x,y)

- Note that for 2-D flow we may *also* define a "streamfunction" ψ

$$(u, v) = (\psi_y, -\psi_x)$$

which identically satisfies $u_x + v_y = 0$

and note that if $\hat{k}\cdot\underset{\sim}{\omega} = v_x - u_y = 0$

then $\nabla^2 \psi = 0$ — Laplace's equation again.

+ now ψ = const. on boundaries with no normal flow

e.g. 2D flow through a contraction