VII.1

12/10/2009 (1)

COMPRESSIBILITY ⟨KC 1.8-10⟩

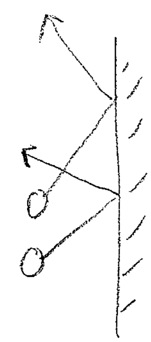
• Dry air can be treated as an ideal gas, with equation of state $\rho = \rho(p, T)$ derived from

$$p = \rho R \underset{K}{T} \qquad R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$$

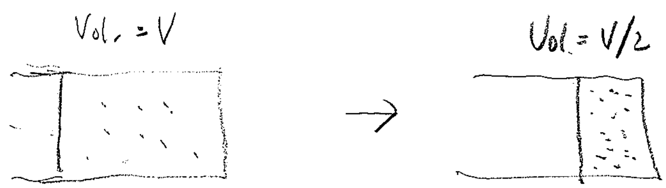
physical interpretation: the pressure on a wall depends on

(i) how <u>many</u> molecules hit it per unit time and what their <u>mass</u> is $\Rightarrow p \propto \rho$

(ii) the average <u>speed</u> of the molecules $\Rightarrow p \propto T$

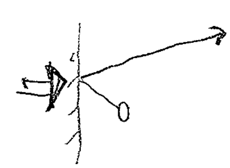


Consider <u>compressing</u> the air in an insulated piston



• Volume decreased $\Rightarrow \rho$ increased $\quad \rho \to 2\rho$

• T also increased due to KE added to molecules by collision with moving wall



$\therefore$ p more than doubles

Consideration of ideal gas thermodynamics for changes which are

- adiabatic $\rightarrow$ no external loss or gain of heat, and
- reversible (eg. no viscous dissipation)

then the system is "isentropic", and we may show

$$\boxed{\frac{p}{\rho^{\gamma}} = \text{const.}}$$ where $\gamma = 1.4$ for dry air

or $p = (\text{const.})\, \rho^{1.4}$ and still $p = \rho R T$

Now the effect of increasing T is incorporated into the exponent on ρ: $\gamma > 1$

<KC 16.1-2>

Sound waves rely on compressibility for their restoring force, instead of gravity

Assume small variations of p & ρ

$$p = p_0 + p'(x,t) \qquad [p'] \ll p_0$$

$$\rho = \rho_0 + \rho'(x,t) \qquad [\rho'] \ll \rho_0$$

also $[u] \ll c$ the wave speed

Linearize the equations:

$\boxed{\text{x mom}}$

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \Rightarrow \frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} = -\frac{1}{\rho_0}\left(\frac{\partial p}{\partial \rho}\right)\frac{\partial \rho}{\partial x} \qquad (1)$$

$\boxed{\text{mass}}$

$$\frac{D\rho}{Dt} + \rho \nabla\cdot\underline{u} \Rightarrow \frac{\partial \rho}{\partial t} + \rho_0 u_x = 0 \qquad (2)$$

then forming $\frac{\partial (1)}{\partial t} + \frac{\partial (2)}{\partial x} \Rightarrow \boxed{u_{tt} - \left(\frac{\partial p}{\partial \rho}\right) u_{xx} = 0}$ A wave eqn with phase speed $c_p = \left(\frac{\partial p}{\partial \rho}\right)^{\frac{1}{2}}$ (non-dispersive)

Then using $\frac{p}{\rho^\gamma} = \frac{p_0}{\rho_0^\gamma} \Rightarrow p = \frac{p_0 \rho^\gamma}{\rho_0^\gamma} \Rightarrow \frac{\partial p}{\partial \rho} = \frac{\gamma p_0}{\rho_0^\gamma}\rho^{\gamma-1} \cong \frac{\gamma p_0}{\rho_0^\gamma}\rho_0^{\gamma-1} = \gamma\frac{p_0}{\rho_0}$

& using $p = \rho R T \Rightarrow \boxed{\frac{\partial p}{\partial \rho} = c^2 = \gamma R T}$ $c \cong 340 \text{ m s}^{-1}$ for air at 20°C

$c \cong 1400 \text{ m s}^{-1}$ for water

Appendix

$\boxed{\text{Basic properties of our favorite fluids}}$

Air

N_2 78.1%, O_2 21.0%, ...

$\rho = \rho(T, p, q)$ ← Eqn. of state

specific humidity = $\dfrac{\text{mass of water vapor}}{\text{mass of moist air}}$

$\rho \simeq 1.2$ kg m^{-3} sea level

$\rho \to 0$ high up

Air often approximated as an "ideal" or perfect gas, with

$p = \rho R T$ ← deg. K

gas const. = 287 $\dfrac{J}{kg\,K}$ for dry air

Sea Water

H_2O, NaCl ~ 35 ‰ (ppt)

→ $\rho = \rho(S, T, p)$

salinity ≅ $\dfrac{\text{mass of salt}}{\text{mass of seawater}} \times 1000$

$\rho = 1000$ kg m^{-3} fresh

1025 ocean surface

1055 abyss

change mostly due to compressibility