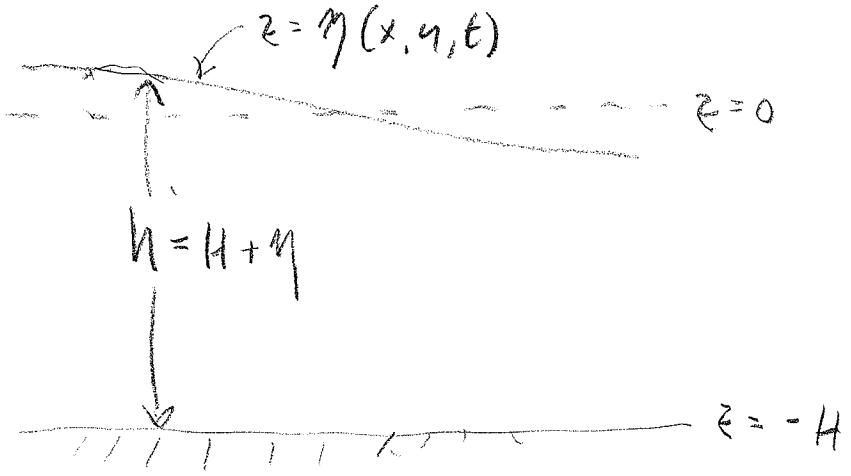
II.2

Shallow Water Equations

assume $\rho = \text{CONST.}$, $H \ll L$

flow with a free surface



Mass $\nabla \cdot \underset{\sim}{u} = 0$

x mom $\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$

y mom $\frac{Dv}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y}$

z mom $\frac{\partial p}{\partial z} = -\rho g$

} 4 equations in u, v, w, p

Note $\int_z^\eta \boxed{z \text{ mom}}\, dz$ gives

$$p_{ATM} - p(z) = -\rho g(\eta - z)$$

$$\Rightarrow \frac{\partial p}{\partial x} = -g\frac{\partial \eta}{\partial x} \quad \text{and} \quad \frac{\partial p}{\partial y} = -g\frac{\partial \eta}{\partial y}$$

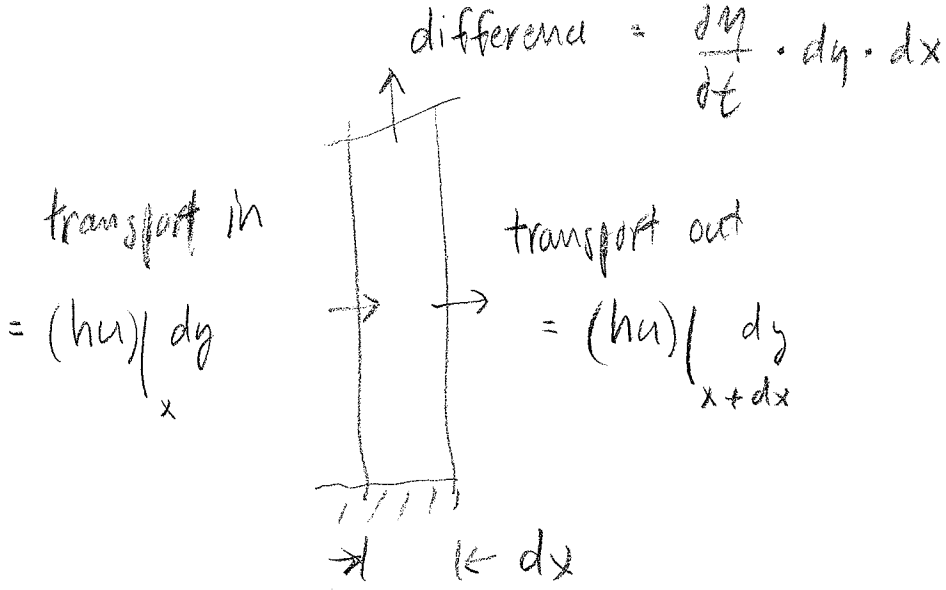
Also: because p_x & p_y are independent of z

then no shear develops: if $\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0$ initially

then will always be zero!

$$\Rightarrow \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \cancel{w\frac{\partial u}{\partial z}}^{\,0} \qquad \left(\text{same for } \frac{Dv}{Dt}\right)$$

To find an equation for η, consider flow with horizontal convergence



$$\therefore \quad \frac{\partial \eta}{\partial t} \, dx \, dy = \left[(hu)|_x - (hu)|_{x+dx} \right] dy$$

in the limit $dx \to 0$

$$\frac{\partial \eta}{\partial t} = - \frac{\partial}{\partial x}(hu)$$

then including v:

$$\boxed{\eta_t + (hu)_x + (hv)_y = 0}$$

this is really [mass] and may be derived directly from $\nabla \cdot \underset{\sim}{u} = 0$

Summarizing:

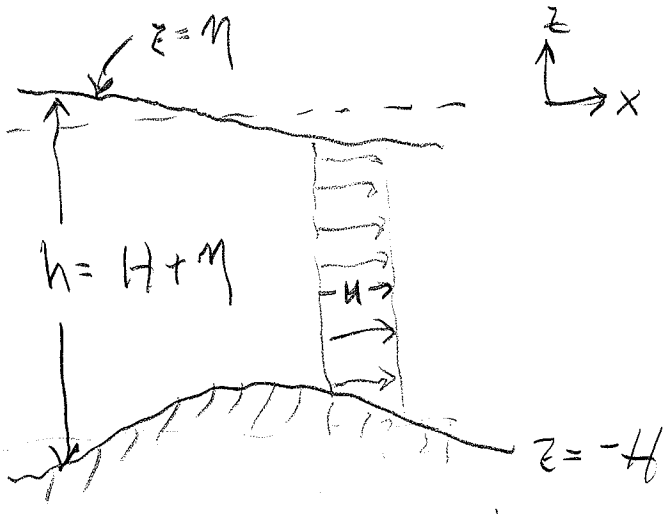
Our "Shallow Water" ($H/L \ll 1$) Equations are:

x mom $\quad u_t + uu_x + vu_y = -g\eta_x$

y mom $\quad v_t + uv_x + vv_y = -g\eta_y$

mass $\quad \eta_t + (hu)_x + (hv)_y = 0$

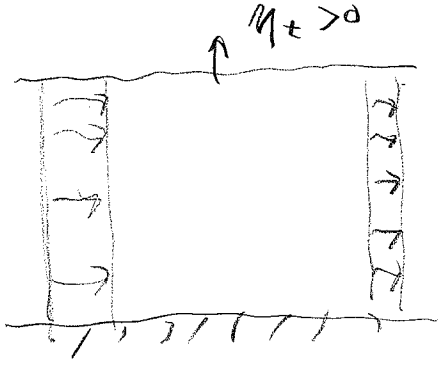
3 equations in u, v, η



OK if $H = H_0(x,y)$

Q: What does w look like?

e.g.



$\frac{\partial u}{\partial x} < 0$