$\boxed{\text{FLUID ENERGY continued ...}}$ $\boxed{\text{III.5}}$ 4/6/2009 ①

< see III.3 for $KE_m \to KE_v$ >

We can also form an equation for Potential Energy

Recall $PE_M = \frac{PE}{\text{unit mass}} = gz$

and note that

$$\frac{D}{Dt}(gz) = \cancel{\frac{\partial}{\partial t}(gz)}^{\,0} + u\cancel{\frac{\partial}{\partial x}}^{\,0}(gz) + v\cancel{\frac{\partial}{\partial y}}^{\,0}(gz) + w\frac{\partial}{\partial z}(gz) = gw$$

so

$$\boxed{PE_M} \quad \frac{D(gz)}{Dt} = wg$$

and we can also form an equation for $PE_v = \rho g z$

$$\rho\left[\frac{\partial}{\partial t}(gz) + \underset{\sim}{u}\cdot\nabla(gz)\right] + gz\overbrace{\left[\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\underset{\sim}{u})\right]}^{=0 \text{ by } \boxed{\text{MASS}}} = \rho g w$$

$$\Rightarrow \boxed{PE_v} \quad \boxed{\frac{\partial}{\partial t}(\rho g z) = -\nabla\cdot\left[\underset{\sim}{u}(\rho g z)\right] + \rho g w}$$

¿What do KE + PE look like for shallow water waves?

$$\underset{\sim}{u} \cong \left[u_0 \cos(kx - \omega t),\ 0,\ 0 \right]$$

$$\eta = \eta_0 \cos(kx - \omega t)$$

progressive wave

The $\dfrac{\text{KE}}{\text{unit horizontal area}} = \displaystyle\int_{-H}^{\eta} \tfrac{1}{2}\rho(\underset{\sim}{u}\cdot\underset{\sim}{u})\,dz \cong \int_{-H}^{0} \tfrac{1}{2}\rho u^2\,dz$

$$= \boxed{\tfrac{1}{2}\rho u^2 H = KE_A^{sw}}$$

And $\dfrac{\text{PE}}{\text{unit horizontal area}} = \displaystyle\int_{-H}^{\eta} \rho g z'\,dz' = \tfrac{1}{2}\rho g(\eta^2 - H^2) = PE_A^{sw}$

then defining the zero-PE background state as

$PE_{A0}^{sw} \equiv -\frac{1}{2}\rho g H^2$. Then we can define the Available Potential Energy

$$\boxed{APE_A^{sw} \equiv PE_A^{sw} - PE_0^{sw} = \tfrac{1}{2}\rho g \eta^2}$$