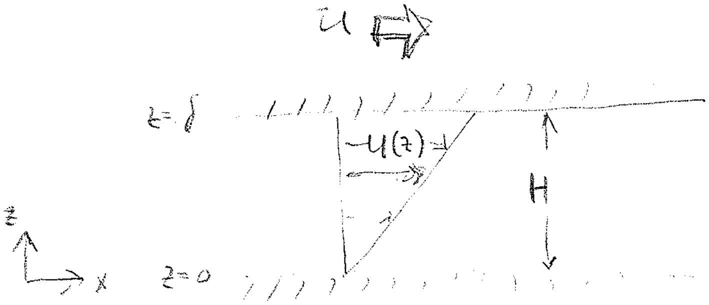
VISCOSITY

III.1

10/23/2009 ①



Two plates w/ fluid between

Top plate moving at speed U

("Couette Flow")

For real fluids we observe:

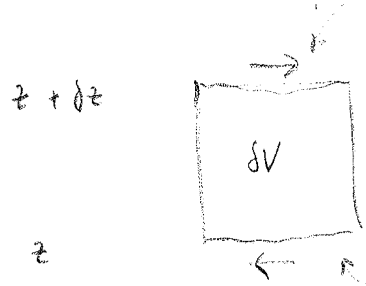
$u(0) = 0$
$u(H) = U$ } "No slip condition"

$\frac{\text{Force required to keep upper plate in motion}}{\text{unit horizontal area}} = \mu \frac{U}{H} = \mu \overbrace{\frac{\partial u}{\partial z}}^{\text{shear}} =$ "shear stress"

μ = "dynamic viscosity" $\left[\frac{kg}{m\,s}\right]$ This defines a "Newtonian Fluid"

Considering just a fluid parcel (with just $\partial u/\partial z$)

$x\text{-force} = \mu \frac{\partial u}{\partial z}\Big|_{z+\delta z} dy\,dx$ (pulls parcel to right)



$x\text{-force} = -\mu \frac{\partial u}{\partial z}\Big|_{z} dy\,dx$ (pulls parcel to left)

$$\Rightarrow \frac{\text{Viscous Force}^x}{\text{unit vol}} = \lim_{\delta V \to 0} \left\{ \frac{1}{dx\,dy\,dz} \left(\mu \frac{\partial u}{\partial z}\Big|_{z+\delta z} - \mu \frac{\partial u}{\partial z}\Big|_{z} \right) dx\,dy \right\} = \frac{\partial}{\partial z} \mu \frac{\partial u}{\partial z}$$

$$= \mu \frac{\partial^2 u}{\partial z^2}$$

due to $\mu\,\partial u/\partial z$ only

- in general we may neglect spatial variation of μ
- the relation stress $= \mu \times$ shear with $\mu =$ const. defines a "Newtonian fluid"
- Non-Newtonian fluids : μ may be a function of shear

    shear thinning fluids : ball point pen ink, ketchup

    " thickening " : Silly Putty

- can have eg. $\mu = \mu$(temperature) + still be "Newtonian"

Generalizing the viscous force on a fluid parcel

$$\frac{\text{Visc. Force}^x}{\text{unit vol}} = \mu\left(u_{xx} + u_{yy} + u_{zz}\right) = \mu \nabla^2 u = \mu \nabla \cdot (\nabla u)$$

(similar expressions y + z)