Optimization Methods for Minimum Power Multicasting in Wireless Networks with Sectored Antennas

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***Abstract*— We consider the problem of minimum power multicasting in wireless networks with sectored antennas. For omnidirectional wireless broadcast to a node, a transmission from node i to node j will also reach all nodes which are closer to i than j. Depending on the network geometry, this strategy can be highly power efficient. In ideal sectored antenna systems, however, this phenomenon is sector specific; $i.e.$, only those nodes which are located in the same sector as j will receive the transmission implicitly. Though this might seem an apparent disadvantage, the higher gains associated with directional antennas (as opposed to omni-directional antennas) allow for reduced transmission powers without sacrificing the signal-to-interference ratio at the receiver. In this paper, we first develop a mixed integer linear programming model for optimal solution of the minimum power multicast problem with sectored antennas. Subsequently, we discuss a biologically inspired algorithm for solving the problem to near-optimality at a very reasonable computation time. Experiments on randomly generated 10, 20 and 30-node networks indicate that near-optimal solutions can be obtained using the proposed algorithm.**

I. Introduction

The minimum power broadcast/multicast (MPB/MPM) problem in wireless networks was first investigated by Wieselthier *et al* [1] in the context of omni-directional antennas. They showed that the inherently broadcast nature of the wireless medium can be exploited effectively to obtain power efficient multicast trees. Specifically, they showed that, in certain cases (depending on the network geometry), it may be beneficial for a transmitting node i to transmit to the farthest node within its radio range (say j), thereby covering other nodes which are geometrically closer to i than j. They termed this property the "wireless multicast advantage". Subsequently, it was shown in [2] that the MPB/MPM problem is NP-complete, necessitating the need for good and fast heuristics.

While there has been considerable recent research activity on solving the MPB/MPM problem with omni-directional antennas, there has been relatively little research ([3], [4]) directed towards solving the problem with directional antennas, or more specifically, sectored antennas. Unlike omni-directional antennas, in sectored antenna systems, the wireless multicast advantage property is sector specific; $i.e.$, only those nodes which are located in the same sector as j will receive the transmission from i implicitly. Though this might seem an apparent disadvantage, the higher gains associated with directional antennas (as opposed to omni-directional antennas) allow for reduced transmitter powers without sacrificing the signal-to-interference ratio at the receiver.

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In this paper, we first develop a mixed integer linear programming (MILP) model for optimal solution of the minimum power multicast problem with sectored antennas. Subsequently, we discuss a biologically inspired algorithm, based on *swarm intelligence* [5], for solving the problem to near-optimality at a very reasonable computational time. Swarm intelligence appears in biological swarms of certain insect species. The main principle behind swarm intelligence interactions is stigmergy, or communication through the environment. An example is *pheromone* laying on trails followed by ants. Pheromone is a potent form of hormone that can be sensed by ants as they travel along trails. It attracts ants and causes them to follow trails which have high pheromone concentrations. This causes an auto-catalytic reaction, $i.e.$, ants attracted by a pheromone trail will deposit even more pheromone on the same trail, causing even more ants to be attracted. In essence, therefore, swarm intelligence paradigms use positive reinforcement as a search strategy.

The rest of the paper is organized as follows. In Section II, we describe the network model and outline our assumptions. In Section III, we formally define the problem and in Section IV, we develop the MILP model for solving the problem optimally. Section V explains the computational efficient swarm intelligent algorithm. Simulation results are discussed in Section VI.

II. Network Model

We assume a fixed N-node network with a specified source node which has to broadcast(multicast) a message to all(some) other nodes in the network. We assume static broadcasting(multicasting); $i.e.$, the same tree is used for the entire broadcast(multicast) duration. Any node can be used as a relay node to reach other nodes in the network.

All nodes are assumed to have S-sector antennas. The number of sectors, S, is related to the beamwidth, θ (in degrees), as follows:

$$S = 360/\theta \tag{1}$$

We make several simplifying assumptions on the antenna properties. These are listed below:

- Each sector is assumed to span the angular region $[(s-1)360/\theta, (s)360/\theta]$ in the 2-D plane, where $1 \leq s \leq S$ is the sector number.
- We ignore sidelobe effects and assume that when sector s is switched on, 100% of the radiated power is confined within that sector, providing an uniform gain within the angular region spanned by the sector.
- We consider antennas with 100% efficiency. That is, we ignore any antenna power losses.

Several operating modes are possible in networks with directional antennas. These are: (a) *Directional transmit, directional receive* (DTxDRx) (b) *Directional transmit, omni-directional receive* (DTxODRx) (c) *Omni-directional transmit, directional receive* (ODTxDRx) and (d) *Omni-directional transmit, omni-directional receive* (ODTxODRx). In this paper, we adopt the directional transmit, omni-directional receive (DTxODRx) operating mode.

Following our simplifying assumptions, the transmitter power at i necessary to support the link $(i \rightarrow j)$, $\mathbf{P}_{ij}$, can be written to be proportional (accounting for link/antenna gains and other factors) to d_{ij}^{α}/S, where d_{ij} is the Euclidean distance between nodes i and j. If (x_i, y_i) are the coordinates of node i and α (typically in the range $2 \leq \alpha \leq 4$) is the channel loss exponent, d_{ij} is given by:

$$d_{ij} \quad = \left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{1/2} \tag{2}$$

Without any loss of generality, we set the proportionality constant to be equal to 1 and therefore:

$$\mathbf{P}_{ij} = d_{ij}^{\alpha}/S \tag{3}$$

Finally, we assume that power expenditures due to signal reception and processing are negligible compared to signal transmission and hence the *cost* of a multicast tree is determined solely by the choice of transmitter powers.

III. Problem Statement

Let Y be a vector of node transmission powers, the element Y_i representing the total transmission power cost of node i. For an S-sector antenna, Y_i can be written as:

$$Y_i = \sum_{s=1}^{S} Y_{i,s} \tag{4}$$

where $Y_{i,s}$ is the transmission power cost corresponding to sector s of node i. We assume that each node has a constraint on the maximum transmitter power it can use per sector, denoted by $Y_{i,s}^{max}$. That is:

$$0 \leq Y_{i,s} \leq Y_{i,s}^{max} \; \forall i \in \mathcal{N} \tag{5}$$

where $\mathcal{N}$ is the set of all nodes in the network and $|\mathcal{N}| = N$.

Also, let $\mathcal{E}$ the set of all directed edges[1] and $\mathcal{D}$ the set of destination nodes, $\mathcal{D} \subseteq \{\mathcal{N} \setminus source\}$. Let the cardinality of these sets be E and D respectively; *i.e.*, , $E = |\mathcal{E}|$ and $D = |\mathcal{D}|$. Using the transmitter power constraint, the set of all edges, $\mathcal{E}$, is given by:

$$\mathcal{E} = \{(i \rightarrow j) : (i,j) \in \mathcal{N}, i \neq j, \mathbf{P}_{ij} \leq Y_{i,s}^{max}\} \tag{6}$$

The third condition in the right hand side of (6) defines the set of nodes which are reachable by a direct transmission from any transmitting node depending on its maximum sector power constraint.

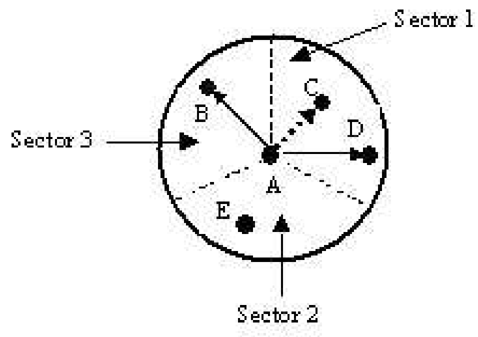

Fig. 1. Multicast illustration with a 3-sector antenna. Assume that the destination nodes are B, C and D. Since C and D are located in the same sector, node A can simply transmit to D, covering node C implicitly. Whether this is the best strategy depends on whether $\mathbf{P}_{AC} + \mathbf{P}_{AC} > \mathbf{P}_{AD}$. If the inequality is satisfied, a direct transmission to D is preferable over the multihop transmission $A \rightarrow C, C \rightarrow D$. We refer to the transmission $A \rightarrow C$ as an *implicit transmission* and $A \rightarrow D$ as an *actual transmission*. In this example, the transmitting node, A, needs to use two sectors (sectors 1 and 3) to reach all the destination nodes. The total transmission cost of node A is: $Y_A = Y_{A,1} + Y_{A,3}$, where $Y_{A,1} = max(\mathbf{P}_{AC}, \mathbf{P}_{AD}) = \mathbf{P}_{AD}$ and $Y_{A,3} = \mathbf{P}_{AB}$. Since sector 2 is not used, $Y_{A,2} = 0$.

Note that a transmission from node i to node j would also be received by all nodes which are located within the same sector as j and are geometrically closer to i than j. For example, in Figure 1, nodes C and D are located in the same sector w.r.t node A and therefore the transmission $A \rightarrow D$ would also be received by node C. Since no additional cost is required to reach node C, we refer to the transmission $A \rightarrow C$ as an *implicit transmission* and $A \rightarrow D$ as an *actual transmission*.

Let $\{X_{ij} : (i \rightarrow j) \in \mathcal{E}\}$ be a set of binary variables such that $X_{ij} = 1$ if the transmission $i \rightarrow j$ is used in the optimum tree and 0 otherwise. Also, let $ne(i,s)$ be the set of neighbors of node i which are within radio range of i and are located within the same sector, s, w.r.t node i. For example, in Figure 1, $ne(A,1) = \{C, D\}$, $ne(A,2) = \{E\}$ and $ne(A,3) = \{B\}$. Following our discussion in the previous paragraph, we can write:

$$Y_{i,s} = \max_j\{X_{ij}\mathbf{P}_{ij} : j \in ne(i,s)\} \tag{7}$$

where $X_{ij} = 1$ if node j is reached from node i (actually or implicitly) and 0 otherwise.

For minimum power multicasting with sectored antennas, our objective function is:

$$minimize\left(\sum_{i=1}^{N} Y_i\right) = minimize\left(\sum_{i=1}^{N}\sum_{s=1}^{S} Y_{i,s}\right) \tag{8}$$

[1] In this paper, we assume that all edges are directed. The notation $(i \rightarrow j)$ will be used to denote a directed edge from node i to j. The notation (i,j) will be used to refer to the node pair.

where $Y_{i,s}$ is as defined in (7). The objective function, (8), has to be solved subject to the following constraints: (a) all destination nodes must be reached, either actually or implicitly, (b) the source node must reach at least one other node, (c) the tree must be *connected*; *i.e.,* there must be directed paths from the source to all destination nodes, possibly involving other intermediate nodes and (d) the tree must not have any *cycles*.

In the next section, we develop a mixed integer programming (MILP) model for optimal solution of the above optimization problem.

IV. MILP MODEL

Let $\{F_{ij} : \forall (i \rightarrow j) \in \mathcal{E}\}$ be a set of flow variables (F_{ij} represents the flow from node i to node j), with $\mathcal{E}$ defined as in (6). The general multicast problem can be interpreted as a single-origin multiple-destination uncapacitated flow problem, with the source having D units of supply (no demand) and the destination nodes having one unit of demand each (see eqns. (9) to (11) below). For other nodes, the net in-flow must equal the net out-flow, since they serve only as relay nodes (see eqn. 12 below).

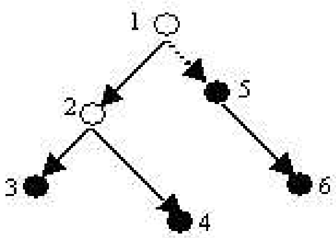

Fig. 2. Example multicast tree. Node 1 is the source. The shaded nodes are the destination nodes and the dashed line represents implicit transmissions. Assume that all nodes are equipped with a 3-sector antenna. Suppose that (a) nodes 2 and 5 are located in sector 3 w.r.t node 1 (b) node 3 is located in sector 1 w.r.t node 2 (c) node 4 is located in sector 3 w.r.t node 2 and (c) node 6 is located in sector 1 w.r.t node 5. The cost of the multicast tree is therefore: $Y_{1,3} + Y_{2,1} + Y_{2,3} + Y_{5,1} = \mathbf{P}_{12} + \mathbf{P}_{23} + \mathbf{P}_{24} + \mathbf{P}_{56}$.

At a conceptual level, the flow model can be viewed as a token allocation scheme where the source node generates as many tokens as there are destination nodes and distributes them along the "most efficient" tree such that each destination node gets to keep one token each. For example, for the multicast tree in Figure 2, the flow variables are: $F_{12} = F_{15} = 2$, $F_{23} = F_{24} = 1$, $F_{56} = 1$. All other flow variables are 0.

The supply and demand constraints discussed above can be expressed as the following *flow conservation equations* (see for example [7]):

$$\sum_{j=1}^{N} F_{ij} = D;\ i = source,\ (i \rightarrow j) \in \mathcal{E} \tag{9}$$

$$\sum_{j=1}^{N} F_{ji} = 0;\ i = source,\ (i \rightarrow j) \in \mathcal{E} \tag{10}$$

$$\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} = 1;\ \forall i \in \mathcal{D}, (i \rightarrow j) \in \mathcal{E} \tag{11}$$

$$\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} = 0;\ \forall i \in \mathcal{R}, (i \rightarrow j) \in \mathcal{E} \tag{12}$$

where $\mathcal{R} \stackrel{\triangle}{=} \mathcal{N} \setminus \{\mathcal{D} \cup source\}$ is the set of all nodes other than the source or destinations. Note that (10) effectively constrains all flows directed into the source node to 0.

Having set up the flow equations, we now need to link the flow variables to the power variables, $\{Y_{i,s} : i = 1, \cdots N; s = 1, \cdots S\}$. This is done in two stages. In the first stage (eqn. 13), we couple the flow variables and the indicator variables $\{X_{ij}\}$ and in the next stage (eqn. 14), we link the $\{X_{ij}\}$ variables to the power variables. Recall from Section III that $X_{ij} = 1$ if the edge $(i \rightarrow j)$ appears in the optimum solution (either as an actual transmission or as an implicit transmission) and 0 otherwise. The set of constraints which couple the flow variables and the X_{ij} variables is:

$$D \cdot X_{ij} - F_{ij} \geq 0;\ \forall (i \rightarrow j) \in \mathcal{E} \tag{13}$$

Note that (13) ensures that "$X_{ij} = 1$ if $F_{ij} > 0$". For the multicast tree in Figure 2, the status of the X_{ij} variables are $X_{12} = X_{15} = X_{23} = X_{24} = X_{56} = 1$, the rest being equal to 0. The coefficient of X_{ij} in (13) is due to the fact that the maximum flow out of any node on a single link is equal to the number of destination nodes. Equation (13), however, leaves open the possibility of X_{ij} being equal to 1 for $F_{ij} = 0$. We show later that this possibility can be discounted since it would unnecessarily increase the cost of the optimum solution. It should also be noted that the smallest integer value of X_{ij} which satisfies (13) for any nonzero flow out of node i (*i.e.*, if $\sum_j X_{ij} \geq 1$) is 1. Consequently, we can simply define the X_{ij}'s to be integers, instead of explicitly declaring them to be binary variables.

Next, we write down constraints linking the X_{ij} variables and the power variables. As discussed in Section III (see eqn. 7), the cost of spanning in multiple nodes, located within the same sector, from node i is simply the cost incurred in reaching the farthest node. This condition is expressed as:

$$Y_{i,s} - \mathbf{P}_{ij} X_{ij} \geq 0;\ \forall i \in \mathcal{N}, \forall j \in ne(i,s) \tag{14}$$

It is now clear that, if there is no flow out of node i (*i.e.*, $\sum_j F_{ij} = 0$), setting $X_{ij} = 1$ would result in a positive value for Y_i and thereby unnecessarily increase the cost of the optimal solution.

Next, consider the case when there are multiple flows out of node i, *i.e.*, $\sum_j F_{ij} > 1$. Suppose $j^* \in ne(i,s)$ is the node such that $\hat{Y}_{i,s} = \mathbf{P}_{ij^*} X_{ij^*} = \max_j (\mathbf{P}_{ij} X_{ij} : j \in ne(i,s))$ is part of the optimal solution. In this case, setting $X_{ij} = 1$, $j \neq j^*$, would not affect the cost of the optimal solution if $\mathbf{P}_{ij} X_{ij} \leq \mathbf{P}_{ij^*} X_{ij^*}$. If, however, $\mathbf{P}_{ij} X_{ij} > \mathbf{P}_{ij^*} X_{ij^*}$, this solution cannot be optimal since it can easily be improved by setting $X_{ij} = 0$.

The final set of constraints express the integrality of the X_{ij} variables and non-negativity of the F_{ij} and $Y_{i,s}$ variables.

$$X_{ij} \geq 0,\ \text{integer};\ \ \forall (i \rightarrow j) \in \mathcal{E} \tag{15}$$

$$F_{ij} \geq 0;\ \ \forall (i \rightarrow j) \in \mathcal{E} \tag{16}$$

$$0 \leq Y_{i,s} \leq Y_{i,s}^{max};\ \ \forall i \in \mathcal{N}, s = 1, 2, \cdots S \tag{17}$$

To summarize, solving the objective function (8), subject to constraints (9) to (17) solves the minimum power multicast problem in wireless networks with sectored antennas. The number of integer variables in the MILP model is equal to E while the number of continuous variables is equal to $E + SN$. The number of constraints is approximately on the order of $E + N(1 + S)$.

Since integer programming is known to be NP-complete, we are currently using the model to benchmark the performance of heuristic algorithms on small and medium size networks. Research is under way to develop fast approximation algorithms based on the model. In the following section, we describe an *ant colony system* (ACS) approach to solving the MILP optimization problem. ACS algorithms, first proposed by Dorigo and Gambardella [6] for solving the celebrated travelling salesman problem, have their roots in the foraging behavior of ants and, in essence, are positive reinforcement intelligent search strategies.

V. ACS Optimization Approach

We start out by establishing the following notation.

A. Notation

t = time index
t^{max} = maximum time index
N_A = number of Type-A ants
N_B = number of Type-B ants
τ^{min} = minimum pheromone level
τ^{max} = maximum pheromone level
$\tau_{ij}(t)$ = pheromone level on edge ($i \rightarrow j$) at time t, $\tau^{min} \leq \tau_{ij}(t) \leq \tau^{max}$
η_{ij} = local visibility of node j from node $i \triangleq 1/\mathbf{P}_{ij}$
β_A = tunable parameter to control η_{ij} for Type-A ants, $0 < \beta_A \leq 1$
β_B = tunable parameter to control η_{ij} for Type-B ants, $0 < \beta_B < \beta_A \leq 1$
$T_m(t)$ = tree developed by ant m at time t
$C_m(t)$ = the cost of $T_m(t)$
ρ = pheromone decay coefficient, $\rho \in (0, 1]$
q = uniformly distributed random variable over the interval [0,1]
q_0 = tunable parameter, $q_0 \in [0, 1]$

At time $t = 0$, the pheromone level on all edges is initialized to τ^{min}; *i.e.*,

$$\tau_{ij}(0) = \tau^{min} : \forall (i \rightarrow j) \in \mathcal{E} \tag{18}$$

B. Tree building by an ant

Tree building[2] is an iterative process which starts with a transmission from the source and continues till all the intended destination nodes are reached. The iteration converges in *at most* $N - 1$ iterations (*i.e.*, $k \leq N - 1$). It should be noted that, because of the inherently broadcast nature of the wireless medium, the number of iterations can range from as few as 1 (this will be the case if all destination nodes are located within the same sector and are within direct radio range of the source) to $N - 1$ (*e.g.*, if exactly one new node is reached during each iteration and the last edge chosen reaches a destination node). Before explaining the tree-building process, we define the following additional parameters:

k = transmission step number
$\mathbf{NR}^k$ = set of new nodes reached at transmission step k ($\mathbf{NR}^0$ = [$source$])
$\mathbf{NR}^{0:k}$ = set of all nodes reached till transmission step k $\triangleq \bigcup_{x=0}^{k} \mathbf{NR}^x$
$\mathbf{NNR}^k$ = set of all nodes *not reached* till transmission step k $\triangleq \mathcal{N} \setminus \mathbf{NR}^{0:k}$

Node i is said to be newly reached at step k if $i \in \mathbf{NR}^k$ but $i \notin \mathbf{NR}^{0:k-1}$.

In general, at any transmission step k, an ant m can travel *from* any node which has been reached till step $k - 1$, *to* any node which has not yet been reached till step $k - 1$. The set of possible edges to choose from, $edge\_list_m^k$, is therefore given by:

$$edge\_list_m^k = \{(i \rightarrow j) : \ i \in \mathbf{NR}^{0:k-1}, j \in \mathbf{NNR}^{k-1}, (i \rightarrow j) \in \mathcal{E}\} \tag{19}$$

where $\mathcal{E}$ is as defined in (6). The decision rule governing which edge is chosen at step k of the tree building process is *pseudo-random-proportional*, as described in Figure 3. Starting with $k = 0$, this decision rule is executed till all the intended destination nodes are reached. It can be easily seen from Figure 3 that the worst case complexity of the tree-building procedure is on the order of $O(N^2)$.

It can be seen from Figure 3 that edges are chosen either deterministically or probabilistically. The extent to which probabilistic decisions are made is controlled by the tunable parameter q_0. Probabilistic edge selection is used for efficient exploration of the search space. In our simulations, we varied q_0 with t so that decision making is predominantly probabilistic during the initial stages of the algorithm and mostly deterministic during the latter stages. This is discussed in Section VI.

The factors which determine the desirability of choosing an edge (i, j) at iteration k and time t are:

• local visibility of node j from i, scaled exponentially by the parameter β_A or β_B, depending on the type of ant. Higher the local visibility, higher the desirability of choosing that edge. The degree of desirability can be varied by properly selecting β_A and β_B, as explained subsequently.

• pheromone level, $\tau_{ij}(t)$, on the edge at time t. Since edges which are part of better solutions are positively reinforced[3], presence of a high pheromone level on an edge is used to boost the desirability of choosing that edge. A very high pheromone level on any edge, therefore, makes it much more probable for that edge to be included in the final tree.

We now explain how the degree of desirability of choosing an edge can be controlled by varying the parameters β_A and β_B. Consider an arbitrary 4-node network. Suppose we have

[2] In keeping with swarm intelligence terminology, we will refer to the tree-building agents as ants.

[3] At any time t, the pheromone level on the edge (i, j), $\tau_{ij}(t)$, reflects the cumulative knowledge acquired by the ants till time $t - 1$ on the desirability of moving to node j from node i.

1. Prepare a list of candidate edges to choose from, $edge\_list_m^k$.
2. Randomly choose a transmitting node from the set of possible transmitters in $edge\_list_m^k$. Let the transmitting node be f^k.
3. Let $\mathbf{A}^{k,m} = \{a_{ij} : i = f_k, (i \to j) \in edge\_list_m^k\}$ be the decision matrix based on which ant m makes its decision for selecting an edge at step k. The probabilities $\{a_{ij}\}$ are computed as follows:

$$a_{ij} = \begin{cases} \frac{[\tau_{ij}(t)][\eta_{ij}]^{\beta_A}}{\sum_x [\tau_{ix}(t)][\eta_{ix}]^{\beta_A}}, & i = f^k, \quad \text{Type-}A \text{ ants} \\ \frac{[\tau_{ij}(t)][\eta_{ij}]^{\beta_B}}{\sum_x [\tau_{ix}(t)][\eta_{ix}]^{\beta_B}}, & i = f^k, \quad \text{Type-}B \text{ ants} \end{cases} \tag{20}$$

where $(f^k \to x) \in edge\_list_m^k$.
4. Sample q from a uniform distribution over [0,1].
5. **if**($q < q_0$) /* $Deterministic\ decision\ making$ */
Choose the strongest edge, (f^k, t^k), from $\mathbf{A}^{k,m}$.

$$(f^k, t^k) = \text{argmax}_{i,j}\{a_{ij}\} \tag{21}$$

else /* *Explore. Probabilistic decision making.* */
Choose an edge (f^k, t^k) from $\mathbf{A}^{k,m}$ probabilistically, $e.g.$, using a roulette-wheel type mechanism.
end if
6. Set $\mathbf{NR}^k = \{t^k\}$. If other nodes, located within the same sector as t^k, are reached implicitly (see Figure 1 for illustration), include them in $\mathbf{NR}^k$.
7. Update the sets:

$$\mathbf{NR}^{0:k} \leftarrow \mathbf{NR}^{0:k-1} \bigcup \mathbf{NR}^k \tag{22}$$

$$\mathbf{NNR}^k \leftarrow \mathcal{N} \setminus \mathbf{NR}^{0:k} \tag{23}$$

Fig. 3. *Pseudo-random-proportional* edge selection criterion at any iteration k of the tree building process by ant m. Note from eqn. (20) that the edge-selection criterion is dependent on the time instant t through the pheromone level parameters $\tau_{ij}(t)$. Multicast trees are built by repeated application of this edge-selection criterion which terminates when all destination nodes are reached.

1. Set $t = 0$.
2. Set $\tau_{ij}(0) = \tau^{min} : \forall (i \to j) \in \mathcal{E}$.
3. Let T^{best} be the tree grown by the global best ant and Y^{best} its cost.
4. Let $T^{best}(t)$ be the best tree grown by any ant during iteration t and $Y^{best}(t)$ its cost.
5. **while**($t < t^{max}$)
for($m = 1 : N_A + N_B$) /* ant number */
/* *Tree building depends on whether the ant is Type A or B* */
• Build the tree $T_m(t)$; /* *See Figure 3* */
/* $prune(T)$ *is a function which takes a tree* T*, prunes it, and returns the updated tree* */
• $T_m(t) \leftarrow prune(T_m(t))$;
• Compute the cost $C_m(t)$ of $T_m(t)$; /* *See Figure 2* */
/* *Local pheromone update* */
• $\tau_{ij}(t) \leftarrow \rho\tau_{ij}(0) + (1-\rho)\tau_{ij}(t), \ \forall (i \to j) \in T_m(t)$;
end for
if($t == 0$)
$T^{best} \leftarrow T^{best}(t), \ Y^{best} \leftarrow Y^{best}(t)$;
else
if($Y^{best}(t) < Y^{best}$)
$T^{best} \leftarrow T^{best}(t), \ Y^{best} \leftarrow Y^{best}(t)$;
end if
end if
/* *Global pheromone update* */
• $\tau_{ij}(t+1) \leftarrow \rho/Y^{best} + (1-\rho)\tau_{ij}(t), \ \forall (i \to j) \in T^{best}$;
/* *Increment t* */
• $t \leftarrow t + 1$;
end while
6. Print T^{best} and Y^{best}.

Fig. 4. High level description of the ACS algorithm.

one Type-A and one Type-B ant at node 1. Assume that the local visibilities of nodes 2, 3 and 4 from 1 are given by: $\eta_{12} = 0.5$, $\eta_{13} = 1.5$, $\eta_{14} = 2.0$. Let $\tau_{12}(t) = \tau_{13}(t) = \tau_{14}(t) = 1$. Choosing $\beta_A = 1$ and $\beta_B = 0.1$, the probabilities $\{a_{ij}\}$ (see eqn. 20) for the two types of ants are as follows:

- Type-A: $a_{12} = 0.11$, $a_{13} = 0.33$, $a_{14} = 0.56$
- Type-B: $a_{12} = 0.31$, $a_{13} = 0.34$, $a_{14} = 0.35$

Clearly, if both the ants are following their exploratory regimen (see Step 5 in Figure 3), while the Type-A ant will choose the edge $1 \to 4$ (note that node 4 is closest to node 1 since $\eta_{ij} = 1/\mathbf{P}_{ij}$) 56% of the time, the Type-B ant has almost equal chances of selecting any of the three edges. Type-B ants, therefore, can select their edges by looking $deeper$ into the network, as opposed to Type-A ants which are *mostly greedy* and tend to choose nearby nodes. Because of this reason, we will refer to Type-A ants as *narrow-vision* ants and Type-B ants as *wide-vision* ants. It may be noted that the wide-vision ants, because of their ability to make decisions by looking deeper into the network, are better suited for exploiting the broadcast nature of the wireless medium than the narrow-vision ants.

A high level description of the ACS algorithm is provided in Figure 4. For a multicast application, the tree generated using the edge-selection criterion in Figure 3 can have a lot of *redundant* edges. An edge, $(i \to j)$, is deemed to be redundant if j itself is not a destination node or none of the descendants of j is a destination node. Given a directed graph, G, the *descendants* of node i, denoted by $de(i)$, is defined as the set of nodes, $\{j\}$, such that there is a path from i to all nodes in $\{j\}$. That is,

$$de(i) \stackrel{\triangle}{=} \{j \mid i \mapsto j \text{ but not } j \mapsto i\}$$

where $(i \mapsto j)$ is a directed path from node i to node j. A pruning operation is therefore necessary to eliminate all redundant edges. Note that the pruning step can lead to a substantial reduction in the total transmission power cost of the tree, especially if $D/N << 1$.

We now discuss the edge-reinforcement mechanism in Figure 4. We have adopted a two-level pheromone update operation; a local update after computing each tree $T_k(t)$ (the update step inside the **for** loop in Figure 4) and a global update after all trees $\{T_k(t)\}$ have been computed at a given time instant t (the update step after the **for** loop). Note that the latter pheromone update is partly proportional to the quality of the best solution produced till iteration t. Better the best solution, the higher the pheromone amount that is deposited on the set of directed edges in the best multicast tree. The role of the pheromone decay coefficient, ρ, is to prevent stagnation in the search process, a situation where all or most of the ants end up choosing the same set of edges and hence generating identical trees.

TABLE I

Parameter values used in the simulations.

$Parameter$	$N = 10$	$N = 20$	$N = 30$
$Y_{i,s}^{max}$	0.050	0.030	0.015
t^{max}	50	50	50
N_A	3	3	3
N_B	2	2	2
ρ	0.2	0.2	0.2
τ^{min}	10^{-4}	10^{-4}	10^{-4}
τ^{max}	2	2	2
β_A	1	1	1
β_B	$1/\alpha^2$, if $t \leq \lfloor 0.3 * t_{MAX} \rfloor$ $1/\alpha$, if $\lfloor 0.3 * t_{MAX} \rfloor + 1 \leq t \leq \lfloor 0.6 * t_{MAX} \rfloor$ 1, if $\lfloor 0.6 * t_{MAX} \rfloor + 1 \leq t \leq t_{MAX}$		
q_0	0.3, if $t \leq \lfloor 0.3 * t_{MAX} \rfloor$ 0.6, if $\lfloor 0.3 * t_{MAX} \rfloor + 1 \leq t \leq \lfloor 0.6 * t_{MAX} \rfloor$ 0.9, if $\lfloor 0.6 * t_{MAX} \rfloor + 1 \leq t \leq t_{MAX}$		

VI. Simulation Results

We conducted a study of the performance of optimal and ACS methods in 10, 20 and 30-node networks with 3-sector antennas. Multicast group sizes were chosen to be 9 (in effect, a broadcast application) for 10-node networks and 5 for 20/30-node networks. In each case, 25 networks were randomly generated and the tree powers were averaged to obtain the mean tree power. 'α' was chosen to be equal to 2 for all cases. The open source linear programming software, LPSOLVE [8], which uses a LP-based branch and brand algorithm to solve MILP problems, was used to compute the optimal solutions. Values of the parameters used in the simulations are given in Table I.

A key point to note in Table I is the dynamic nature of the parameters q_0 and β_B with respect to t. Gradually reducing q_0 ensures that the bulk of the exploration work (Step 5 in Figure 3) is carried out during the initial stages of the algorithm, when the pheromone distribution on the edges is not too uneven and "trail conditions" are more suitable for wide-vision ants, as explained in Section V-B. Increasing β_B with respect to t has the effect of reducing the local visibility of wide-vision ants so that they start behaving more like their narrow-vision counterparts as iteration progresses. In fact, for $\lfloor 0.6 * t^{max} \rfloor + 1 \leq t \leq t^{max}$, β_B is equal to β_A, which ensures that all ants concentrate on the best routes generated and look for better solutions within local neighborhoods during the last stages of the algorithm.

Our performance measures for comparing the optimal and ACS solutions are as follows:

$$PM_1 = \text{Mean}\left[100 \times \frac{\sum_i Y_i(ACS) - \sum_i Y_i(opt)}{\sum_i Y_i(opt)}\right] \quad (24)$$

$$PM_2 = \text{Max.}\left[100 \times \frac{\sum_i Y_i(ACS) - \sum_i Y_i(opt)}{\sum_i Y_i(opt)}\right] \quad (25)$$

$$PM_3 = \text{St. Dev.}\left[100 \times \frac{\sum_i Y_i(ACS) - \sum_i Y_i(opt)}{\sum_i Y_i(opt)}\right] \quad (26)$$

Table II provides a statistical summary of the simulation results. Clearly, the solutions generated by the ACS algorithm are near-optimal, being within 4% of the mean optimal solution in all cases. The computational time required to find the solutions using the ACS algorithm was extremely small since only 250 ($t^{max} \times (N_A + N_B) = 50 \times 5$; see Table I) total solutions were generated in each case.

TABLE II

Comparison of the optimal and ACS solutions. $\alpha = 2$ for all N. See equations (24), (25) and (26) for definitions of the parameters PM_1, PM_2 and PM_3.

N	D	PM_1	PM_2	PM_3
10	9	3.68	16.78	5.23
20	5	3.75	19.71	4.42
30	5	2.25	8.31	2.49

VII. Conclusion

In this paper, we developed a mixed integer linear programming (MILP) model for solving the minimum power multicast problem with sectored antennas in energy constrained wireless networks. We also discussed an ant colony system (ACS) optimization approach for solving the MILP optimization problem. The algorithm uses a mix of narrow-vision and wide-vision ants. While a narrow-vision ant located at a particular node tends to choose a nearby unreached node to visit next, wide-vision ants are allowed to choose distant nodes to visit next. Experiments carried out on 10, 20 and 30-node networks confirm that near-optimal results can be obtained using the ACS algorithm, and in very little computation time.

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