MINIMUM HOP MULTICASTING IN BROADCAST WIRELESS NETWORKS WITH OMNI-DIRECTIONAL ANTENNAS

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ABSTRACT

In this paper, we consider the problem of minimum-hop multicasting in wireless networks. We first present a Mixed Integer Linear Programming model of the problem, followed by a discussion of a (sub-optimal) sequential shortest path heuristic algorithm with "node unwrapping". This sequential algorithm is amenable to distributed implementation. The node unwrapping part of the algorithm is used to modify the weight matrix of the underlying graph after each iteration and exploits the inherently broadcast nature of wireless transmissions. Simulation results are presented which indicate that reasonably good solutions can be obtained using the proposed heuristic algorithm.

I. INTRODUCTION

We consider the problem of minimum-hop multicasting in wireless networks where individual nodes are equipped with limited capacity batteries and therefore have a restricted communication radius. Such networks are generally referred to as Multi-Hop Wireless Networks (MHWN) since establishing a broadcast/multicast tree in such networks often require co-operation of intermediate nodes which serve to relay information onwards to the intended destination node(s). In MHWNs, minimizing the number of hops in the routing tree is motivated primarily by the need to conserve bandwidth, minimize end-to-end delays, especially for delay-critical data packets, and reduce packet error probabilities. In certain military applications, employing a low-power multicast tree with minimum number of transmissions can serve to further reduce the possibility of detection/interception by enemy radar. Individual transmissions in multicast trees in MHWNs are generally low-powered, given the limitations on battery capacity. Moreover, a suitable topology control algorithm can be used to ensure a power efficient topology. For example, topologies can be constructed to minimize the maximize transmitter power needed to maintain connectivity [2] or the total transmitter power. The focus of this paper is to provide solution methodologies for minimum hop multicasting in power efficient wireless network topologies.

Previous efforts at attempting to solve the minimum-hop multicasting problem include a Hopfield neural network based approach and a couple of heuristics discussed in [1]. In this paper, we first present a Mixed Integer Linear Programming (MILP) model for optimal solution of the problem and then discuss a sub-optimal heuristic algorithm. The MILP model is based on the well-studied single-origin multiple-destination uncapacitated flow problem, tailored to reflect the inherently broadcast nature of the wireless medium, whereby a transmission from node i to node j will also be picked up by all other nodes which are closer to i than j, if line-of-sight exists and nodes are provided with omni-directional antennas. The heuristic is a sub-optimal sequential path algorithm which is amenable to a distributed implementation.

The rest of this paper is organized as follows. Section II outlines our assumptions, followed by descriptions of the MILP model and the sequential heuristic algorithm in Sections IV and Section V. Simulation results are presented in Section VI.

II. Network Model

We assume a fixed N-node wireless network with a specified source node and a broadcast/multicast application. Any node can be used as a relay node to reach other nodes in the network. All nodes are assumed to have omni-directional antennas. We also assume that all nodes are equipped with limited capacity batteries which limits the maximum transmitter power and hence the degree of connectivity (defined as the number of nodes which can be reached by a transmitting node using a direct transmission) of a node.

For any N-node network, the power matrix, $\mathbf{P}$, is an $N \times N$ symmetric matrix. The $(i,j)th$ element of the power matrix represents the power required for node i to transmit to node j and is assumed to be given by:

$$\mathbf{P}_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{\alpha/2} = d_{ij}^{\alpha} \tag{1}$$

where $\{(x_i, y_i) : 1 \le i \le N\}$ are the coordinates of the nodes in the network, α ($2 \le \alpha \le 4$) is the channel loss exponent and d_{ij} is the Euclidean distance between nodes i and j.

III. Problem Statement

Let $\mathcal{N}$ be the set of all nodes in the network, s the source node, $\mathcal{E}$ the set of all directed edges[1] and $\mathcal{D}$ the set of destination nodes, $\mathcal{D} \subseteq \{\mathcal{N} \setminus s\}$. Let the cardinality of these sets be N, E and D respectively; $i.e.$, $N = |\mathcal{N}|$, $E = |\mathcal{E}|$ and $D = |\mathcal{D}|$. Denoting the transmitter power threshold of node i by Y_i^{max}, the set of all edges, $\mathcal{E}$, is given by:

$$\mathcal{E} = \{(i \to j) \mid (i,j) \in \mathcal{N},\ i \ne j,\ \mathbf{P}_{ij} \le Y_i^{max},\ j \ne s\} \tag{2}$$

The third condition in the right hand side of (2) restricts the set of nodes reachable by a direct transmission from any transmitting node depending on its power constraint and the last condition reflects that no transmitting node needs to reach the source node.

Let $\{F_{ij} : \forall (i \to j) \in \mathcal{E}\}$ be a set of flow variables, with $\mathcal{E}$ defined as in (2) and $\{H_i : \forall i \in \mathcal{N}\}$ be a set of binary variables denoting the $hop - count$ of nodes in the network. For wired networks, the hop-count of any node i, H_i, is simply the number of links carrying positive flow out of the node. For wireless networks, however, H_i is visualized as an indicator variable which is equal to 1 if there is at least one link carrying a positive flow out of node i, and 0 otherwise. This definition of hop-count follows from the inherently broadcast nature of the wireless medium, where multiple nodes can be reached from a transmitting node using a single transmission to the farthest node. This is illustrated in Figure 1. The total hop-count, therefore, is simply the number of transmitting nodes in the multicast tree and minimizing the total hop-count is equivalent to minimizing the number of transmitting nodes in the tree.

[1] In this paper, we assume that all edges are directed. The notation $(i \to j)$ will be used to denote a directed edge from node i to j. The notation (i,j) will be used to refer to the node pair.

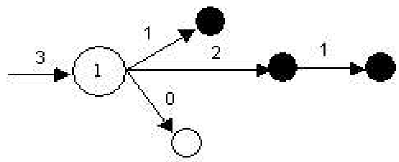

Fig. 1. Shaded circles in the above network represent the destination nodes. The numbers above the edges are the fbws. For a wired network, the hop-count of node 1 is 2, equal to the number of edges directed out of node 1 carrying a positive fbw. If the network is assumed to be wireless and if nodes are equipped with omni-directional antennas, the hop-count of node 1 is 1, since it can send a packet to the farther destination node, which will be picked up by the destination node closer to it. By this reasoning, the total hop-count in a wireless multicast tree is equal to the number of transmitting nodes in the tree.

IV. Mathematical Model

Following the above definition, the objective function of the minimum-hop multicast problem in wireless networks can now be written as:

$$minimize \sum_{i=1}^{N} H_i \tag{3}$$

The general multicast problem can be interpreted as a single-commodity, single-origin multiple-destination uncapacitated flow problem, with the source (the *supply node*) having D units of supply and the destination nodes (*demand nodes*) having one unit of demand each. For other nodes, the net in-flow must equal the net out-flow, since they serve only as relay nodes[2]. At a conceptual level, the flow model can also be viewed as a token allocation scheme where the source node generates as many tokens as there are destination nodes and distributes them along the "most efficient" (in terms of number of hops) tree such that each destination node gets to keep one token each.

The above flow problem can be solved using the usual *conservation of flow constraints* as shown below (see $e.g.$ [3]):

$$\sum_{j=1}^{N} F_{ij} = D;\ i = s,\ (i \to j) \in \mathcal{E} \tag{4}$$

[2] Note that not all of the relay nodes need to act as such.

$$\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} = 1;\ \forall i \in \mathcal{D},\ (i \to j) \in \mathcal{E} \quad (5)$$

$$\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} = 0;\ \forall i \notin \{\mathcal{D} \cup s\},\ (i \to j) \in \mathcal{E} \quad (6)$$

We now have to write down constraints linking the flow variables to the hop-count variables. A suitable equation for expressing the condition that "the hop-count of a node is equal to 1 if there is a positive flow in at least one link directed away from the node, and 0 otherwise" is given in (7):

$$D \cdot H_i - \sum_{j=1}^{N} F_{ij} \geq 0;\ \forall i \in \mathcal{N},\ (i \to j) \in \mathcal{E} \quad (7)$$

The coefficient of H_i in (7) is due to the fact that the maximum flow out of a node is equal to the number of destination nodes in the network. Equation (7) leaves open the possibility for H_i being greater than or equal to 1 if there is no flow out of node i; $i.e.$, if $\sum_{j=1}^{N} F_{ij} = 0$. However, in this case, setting $H_i \geq 1$ would unnecessarily increase the cost of the optimal solution, or, in other words, the smallest integer value of H_i which satisfies (7) when there is no flow out of node i is 0. On the other hand, if there is *any* flow out of node i, the smallest integer value of H_i which satisfies (7) is 1.[3] By the above reasoning, the set of hop-count variables, $\{H_i\}$, will always be restricted to 0-1 values in the optimal solution, and can therefore be simply declared to be non-negative integers.

The final set of constraints express the integrality of the H_i variables and non-negativity of the F_{ij} variables.

$$H_i \in \{0, 1\};\ \ \forall i \in \mathcal{N} \quad (8)$$

$$F_{ij} \geq 0;\ \ \forall (i \to j) \in \mathcal{E} \quad (9)$$

To summarize, the objective function (3) subject to constraints (4) to (9) solves the minimum-hop multicast problem in wireless networks. The number of variables is equal to $E + N$, since the number of flow variables in the formulation is equal to E while the number of hop-count variables is equal to N. Strictly speaking, however, the number of hop-count variables is equal to $N - 1$ since the multicast tree must include a transmission from the source and hence H_i must be equal to 1 for $i = source$.

A. Discussion

Note that the values of the flow variables in the optimal solution are not particularly important for determining the routing tree. What is important is the $zero - nonzero$ status of the variables. The actual routing tree can be constructed by identifying the transmitting nodes and their farthest neighbors for which there is an outward positive flow, as illustrated below.

Referring to the 10-node network in Figure 2, let node 1 be the source and the destination nodes be 2, 5, 7, 9 and 10. Assume that the degree of connectivity of each node is 3, $i.e.$, each node can communicate with only 3 of its nearest neighbors. The flow variables which appear in the optimization model (using eqn. 2) and their optimal values are shown in (10). Note that the first column in the flow matrix, **F**, is empty since node 1 is the source and reflects the condition $j \neq source$ in (2). The diagonal elements of **F** are empty because of the condition $i \neq j$ in (2). Whether flow variables corresponding to the rest of the indices exist or not is dictated by the maximum power constraint on the transmitters.

Examining the first row of the optimal flow values in (10), it can be seen that there are non-zero flows from node 1 to nodes 2, 3 and 9, of which node 3 is the farthest. This is shown as a solid line from node 1 to 3 in Figure 2. The dotted lines to nodes 2 and 9 represent that these nodes pick up the transmission by virtue of their being closer to node 1 than 3. The actual sequence of transmissions in the multicast tree is therefore: $\{1 \to 3, 3 \to 6, 6 \to 5, 7 \to 10\}$.

$$\mathbf{F} = \begin{bmatrix} - & 1 & 3 & - & - & - & - & - & 1 & - \\ - & - & 0 & - & - & - & - & - & 0 & - \\ - & 0 & - & 0 & - & 3 & - & - & - & - \\ - & - & 0 & - & 0 & 0 & - & - & - & - \\ - & - & 0 & 0 & - & 0 & - & - & - & - \\ - & - & 0 & - & 1 & - & 2 & - & - & - \\ - & - & - & - & - & 0 & - & 0 & - & 1 \\ - & - & - & - & - & - & 0 & - & 0 & 0 \\ - & 0 & - & - & - & - & - & 0 & - & - \\ - & - & - & - & - & - & 0 & 0 & 0 & - \end{bmatrix} \quad (10)$$

We conclude this section by noting that the above MILP model can also be used for obtaining *maximum power constrained* minimum hop multicast trees. In [4], a polynomial time optimal algorithm was presented for obtaining the multicast trees which maximizes minimum node lifetime, or alternately, as a special case, minimizes the maximum transmit power. Let $\hat{Y}$ be the optimal maximum transmit power obtained after solving the minimax problem. Redefining the set of valid edges as:

$$\mathcal{E} = \{(i \to j) \mid (i, j) \in \mathcal{N},\ i \neq j,\ \mathbf{P}_{ij} \leq \hat{Y},\ j \neq s\} \quad (11)$$

in place of (2) and solving the MILP model will yield a minimum hop multicast tree such that the maximum transmit power is not greater than $\hat{Y}$.

[3] This can be easily verified by substituting $\sum_{j=1}^{N} F_{ij} = 1$ and $\sum_{j=1}^{N} F_{ij} = D$ in (7).

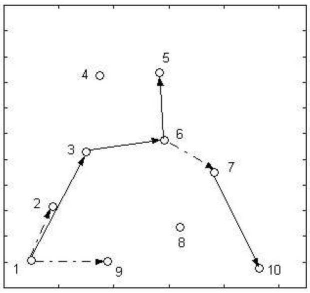

Fig. 2. Example 10-node network. Node 1 is the source and the destination nodes are 2, 5, 7, 9 and 10. The solid lines represent the actual transmissions in the multicast tree. The dotted lines represent *implicit* transmissions; $i.e.$, the associated recipient nodes pick up the transmissions by virtue of their being closer to the transmitting nodes.

B. LP-relaxation

We now briefly discuss the LP-relaxation of the above MILP model. Such relaxations usually form the basis of approximation algorithms. For an excellent discussion on approximation algorithms derived from LP-relaxations, readers are referred to [5]. Given an MILP problem **P** and an instance of the problem, I, let us denote the LP-relaxation of the instance by $LP(I)$. If the optimal solution of $LP(I)$ is integral, the problem is solved. Otherwise, the fractional optimal solution, which is a lower bound on the optimal solution of I, is usually rounded[4] (which can be deterministic or randomized) to provide a feasible integral solution. An approximation guarantee can then be obtained by comparing the costs of the fractional solution and the integral solution.

For our MILP model, the LP-relaxation is obtained by replacing constraints (8) with:

$$0 \leq H_i \leq 1; \;\; \forall i \in \mathcal{N} \tag{12}$$

From (7), it is apparent that H_i will be set equal to $\sum_j F_{ij}/D$ in the optimal solution of the relaxed model since the objective function involves minimization of the sum of all H_i's. Note that $0 \leq \sum_j F_{ij}/D \leq 1$ since the maximum flow out of any node is equal to D and the minimum is 0. Therefore, the upper bound on H_i in (12) is redundant and the variables $\{H_i\}$ can simply be declared to be non-negative, as shown in (13).

$$H_i \geq 0; \;\; \forall i \in \mathcal{N} \tag{13}$$

If there is zero flow out of node i, H_i will be equal to 0 in the optimal solution; $i.e.$, $H_i = 0$ if $\sum_j F_{ij} = 0, j \neq i$. No rounding is therefore required in this case for H_i. Also, since the net flow out of the source node is always equal to D (4), H_{source} will be equal to 1 in the optimal relaxed solution and will not require any rounding.

For any node $i \neq source$, if the total outflow is non-zero and the ratio $\sum_j F_{ij}/D$ is fractional ($i.e.$, $0 < \sum_j F_{ij} < D$), the cost associated with rounding up H_i to the nearest integral value is $1 - \sum_j F_{ij}/D$. Clearly, the round-up cost associated with the node H_i decreases as $\sum_j F_{ij} \rightarrow D$. The maximum round-up cost occurs when $\sum_j F_{ij} = 1$.

We now construct a problem instance for which the optimal relaxed solution will incur the maximum round-up cost. Following our discussion in the previous paragraph, it is clear that the round-up cost will be maximum if the following conditions are satisfied:

(a) the optimal minimum-hop multicast tree comprises of node-disjoint (except at the source node) paths to each of the destination nodes,
(b) all nodes other than source and destinations are used as relays and carry unit flow[5] each.
(c) all destination nodes are leaves in the optimal tree and are farthest from the source (in terms of number of hops). For odd N ($N \geq 5$), this condition can be met if the number of destination nodes is given by $D = (N-1)^{1/2}$.

For example, consider the 9-node, 2-destination problem instance in Figure 3. The darkly shaded nodes are the destinations and the dotted circles represent the communication range of each node. Clearly, the optimal solution for this problem instance involves 7 hops, as shown. If the LP-relaxation of this problem is solved, all directed links will be assigned unit flow as shown. Correspondingly, the optimal cost of the relaxation is equal to $1 + 6 \times (1/2) = 4$, since the hop-count of the source is equal to $\sum_j F_{ij}/D = 2/2 = 1$ and that of all relay nodes (shown lightly shaded) is equal to $\sum_j F_{ij}/D = 1/2$. In general, if the above conditions are satisfied, it can be shown that the ratio of the optimal solution to its LP-relaxation is given by $D(N-D)/(N-1) \leq D$.

V. Sequential Shortest Path heuristic

In this section, we describe a sub-optimal sequential shortest path heuristic for solving the MILP problem. Let π_D be any ordering of the destination nodes with respect to the source. For example, they can be ordered with respect to increasing or decreasing Euclidean distance[6] from the source. As the name of the heuristic suggests, the MILP problem

[4]There exists other methods for converting the fractional solution to an integral solution, $e.g.$, the primal-dual scheme [5].

[5]This must be satisfied since the paths to the destinations are node-disjoint, except at the source.

[6]Other ordering criteria are also possible. For example, the destination nodes can be ordered on the basis of a shortest path (in terms of number of hops) tree to the source.

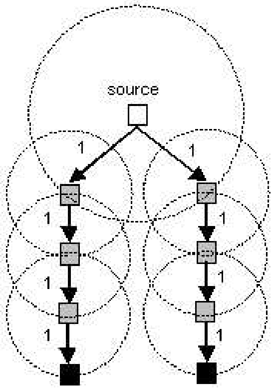

Fig. 3. A 9-node, 2-destination problem instance for which the round-up cost incurred in converting the optimal fractional solution to an integral solution is the maximum. The darkly shaded nodes are the destinations and the dotted circles represent the communication range of each node. Clearly, the optimal solution for this problem instance involves 7 hops, as shown above. If the LP-relaxation of this problem is solved, all directed links will be assigned unit flow as shown. Correspondingly, the optimal cost of the relaxation is equal to $1 + 6 \times (1/2) = 4$, since the hop-count of the source is equal to $\sum_j F_{ij}/D = 2/2 = 1$ and that of all relay nodes (shown lightly shaded) is equal to $\sum_j F_{ij}/D = 1/2$.

is solved by computing a series of shortest paths in the sequence given by π_D. Let $\mathbf{W}^{(1)}$ be the initial weight matrix used for computing the shortest path[7] between the source and $\pi_D(1)$, the first node in π_D. The $(i, j)th$ element of $\mathbf{W}^{(1)}$ is given by:

$$\mathbf{W}_{ij}^{(1)} = \begin{cases} 1, & \text{if } (i \rightarrow j) \in \mathcal{E} \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

where $\mathcal{E}$ is the initial set of edges defined in (2).

As explained in Section III, the minimum hop multicast problem in wireless networks with omni-directional antennas can be viewed as a minimization of the number of transmitting nodes. Consequently, if $\vec{tr}_1$ is the set of transmitting nodes in the shortest path obtained after the first iteration, using these nodes as relays in subsequent iterations would not incur any additional cost. In other words, the weight matrix for the second iteration can be modified as follows:

$$\mathbf{W}_{ij}^{(2)} = \begin{cases} 0, & \text{if } i \in \vec{tr}_1 \\ \mathbf{W}_{ij}^{(1)}, & \text{otherwise} \end{cases} \tag{15}$$

We refer to the weight modification procedure after each iteration as *node unwrapping*. Using $\mathbf{W}^{(2)}$, a shortest path is computed between the source and $\pi_D(2)$, the second node in π_D. This procedure is repeated till all destination nodes are reached and the final multicast tree is obtained by concatenating the shortest paths obtained at each iteration. Figure 4 provides a high level description of the sequential shortest path algorithm. We note that since distributed algorithms[8] exist for the shortest path problem, the heuristic is amenable to distributed implementation provided the multicast group members are aware of their Euclidean distance (or, any other criterion used to sort the destination nodes) from the source. The algorithm in Figure 4 can also be used for maximum power constrained minimum hop multicasting if the set of directed edges in the underlying graph is defined as in (11).

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1. Let π_D be any ordering of the destination nodes with respect to the source.
2. Let k be the iteration index.
3. Let $path(k)$ be the shortest path obtained at iteration k.
4. Set $k = 1$;
5. Compute the initial weight matrix $\mathbf{W}^{(k)}$ (see eqn. 14).
6. Find the shortest path between the $source$ and the node $\pi_D(k)$, $path(k)$.
7. **while**(not all destination nodes reached)
   - Increment $k = k + 1$;
   - Compute the new weight matrix $\mathbf{W}^{(k)}$ (see eqn. 15).
   - Find the shortest path between the $source$ and $\pi_D(k)$.

   end while
8. Concatenate the set of shortest paths $\{path(k)\}$ to obtain the multicast tree.

Fig. 4. High level description of the sequential shortest path algorithm.

Note that the above procedure would take D shortest path iterations to terminate, one iteration for every destination. However, because of node unwrapping, it may be possible to reach additional destination nodes without any additional cost, as illustrated in Figure 5. A simple modification to the algorithm in Figure 4 can be made to check whether additional destination nodes can be reached by node unwrapping. If so, those destination nodes that have not yet been reached after unwrapping can be reordered and the first node in the reordered set chosen as the destination for the next shortest path iteration.

Experimental results suggest that ordering the destination nodes with respect to decreasing Euclidean distance from the source ($i.e.$, the farthest node is the destination for the first iteration) usually results in the fewest number of iterations than if they are ordered with respect to increasing Euclidean distance, with no appreciable difference in solution quality. As implemented, if there are multiple shortest paths at any iteration with the same hop count, any one is chosen arbitrarily.

[7]See [6] or [7] for a description of shortest path algorithms.

[8]For example, distributed Bellman-Ford [7].

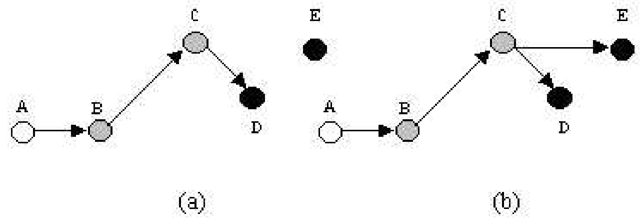

Fig. 5. (a) Shortest path at current iteration, $A \rightarrow B \rightarrow C \rightarrow D$, before node-unwrapping. (b) Node E can be reached simply by unwrapping node C. No additional iteration is required.

VI. Simulation Results

We conducted a study of the performance of the optimal and the heuristic methods for different multicast group sizes in 20, 30, 40 and 50-node networks. The networks and destination sets were chosen so that a feasible[9] solution exists. Transmitter power constraints were set so that each node was connected to its 4 nearest neighbors. The freely available linear programming software, LPSOLVE [8], which uses a branch and brand algorithm to solve MILP problems, was used to compute the optimal solutions. The sequential shortest path algorithm was implemented by ordering the destination nodes with respect to decreasing Euclidean distance from the source. The performance measures for comparing the optimal and heuristic solutions are the *mean* (PM_1), *max* (PM_2) and *standard deviation* (PM_3) of the ratio of the sequential shortest path heuristic to the optimal, over 50 randomly generated instances.

Table I provides a statistical summary of the simulation results for multicast group sizes 5, 10 and 15. As can be seen from the tables, the heuristic performs quite reasonably on average, being within 110% of the optimal in all cases. The worst performance we observed was for 20-node networks and multicast group size = 5, where the heuristic hop count is 140% of the optimal hop count.

VII. Conclusion

In this paper, we have presented a mixed integer linear programming model and a sub-optimal sequential shortest path heuristic for solving the minimum-hop multicast problem in wireless networks with omni-directional antennas. We also showed that a simple redefinition of the set of directed edges in the network graph allows for the solution of the minimum hop multicast problem subject to a maximum transmitter power constraint. The heuristic algorithm has been shown to perform reasonably well in simulations conducted on different multicast group sizes in small and medium scale networks. We are currently working on incorporating QoS (bounded delay and minimum SINR) guarantees in the MILP model.

[9]A *feasible* solution to the minimum-hop multicast problem exists if all destination nodes can be reached, given the transmitter power constraints.

TABLE I

Simulation results.

N	*Multicast Size*	PM_1	PM_2	PM_3
20	5	1.06	1.40	0.12
	10	1.05	1.25	0.08
	15	1.09	1.30	0.10
30	5	1.04	1.38	0.09
	10	1.05	1.20	0.06
	15	1.05	1.22	0.06
40	5	1.04	1.25	0.07
	10	1.04	1.20	0.06
	15	1.07	1.20	0.06
50	5	1.03	1.22	0.06
	10	1.06	1.27	0.08
	15	1.09	1.31	0.08

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