

The Evolution of Internal Market Structure

Oliver J. Rutz

Yale School of Management, Yale University, New Haven, Connecticut 06520, oliver.rutz@yale.edu

Garrett P. Sonnier

Red McCombs School of Business, University of Texas at Austin, Austin, Texas 78712,
garrett.sonnier@mcombs.utexas.edu

We present a dynamic factor-analytic choice model to capture evolution of brand positions in latent attribute space. Our dynamic model allows researchers to investigate brand positioning in new categories or mature categories affected by structural change such as entry. We argue that even for mature categories not affected by structural change, the assumption of stable attributes may be untenable. We allow for evolution in attributes by modeling individual-level time-specific attributes as arising from dynamic means. The dynamic attribute means are modeled as a Bayesian dynamic linear model (DLM). The DLM is nested within a factor-analytic choice model. Our approach makes efficient use of the data by leveraging estimates from previous and future periods to estimate current period attributes. We demonstrate the robustness of our model with data that simulate a variety of dynamic scenarios, including stationary behavior. We show that misspecified attribute dynamics induce temporal heteroskedasticity and correlation between the preference weights and the error term. Applying the model to a panel data set on household purchases in the malt beverage category, we find considerable evidence for dynamics in the latent brand attributes. From a managerial perspective, we find advertising expenditures help explain variation in the dynamic attribute means.

Key words: choice modeling; Bayesian estimation; dynamic models; factor-analytic models

History: Received: August 4, 2009; accepted: October 20, 2010; Eric Bradlow served as the editor-in-chief and Greg Allenby served as associate editor for this article. Published online in *Articles in Advance* February 9, 2011.

1. Introduction

Factor-analytic choice models are often used to represent the internal structure of a market. Using data on consumer choice among competing brands, these models infer the positions of brands in a latent attribute space and consumer preferences for those attributes.¹ In most applications, internal market structure is investigated by taking a short-run (e.g., six months in Elrod and Keane 1995) snapshot of a mature category, assuming that brand positions are constant over time (Elrod 1988, Elrod and Keane 1995). Erdem and Winer (1999) extend the internal market structure approach to model the correlation in latent attributes across different product categories. Erdem (1996) accounts for consumer choice dynamics by modeling consumers as potentially state-dependent in their choices. Consumers may exhibit either habit persistence or variety-seeking behavior. More specifically, habit persistence or variety seeking is captured as a function of the difference between

the latent brand positions of the current and previous choice. In a recent paper, Inman et al. (2008) combine Erdem's (1996) model with Fader and Hardie's (1996) stock-keeping unit (SKU) attribute approach to estimate the latent position of SKU attributes along with attribute-level choice dynamics.

A critical assumption of extant models of internal market structure is that the brand positions in the latent attribute space do not evolve over time. In this paper, we argue that latent brand attributes, and hence the competitive positions defined by these attributes, may evolve over time.² Although this seems likely for a new category, it may also hold for a mature and stable category. If so, static models of internal market structure may yield misleading insights. A potential source of dynamics, for example, is advertising. Consider the two roles of advertising espoused in the economics and marketing literatures, information and persuasion (Ackerberg 2001, Narayanan and Manchanda 2009). These are inherently dynamic constructs and hold the potential to change consumer perceptions about a brand over time. For example, marketing communications on the health benefits of a particular brand of orange juice

¹ The term *market structure* has been used to describe the representation of brand positions in an attribute space (e.g., Elrod 1988, Elrod and Keane 1995, Erdem 1996). We build on this stream of research and have thus adopted this terminology. Market structure has also been used to describe substitution patterns in a market as evidenced by cross-price elasticities (Cooper 1988, Kamakura and Russell 1989).

² For ease of exposition, we will use the term "brand attributes" to refer to the latent brand attributes.

may influence a consumer's utility by informing her about health properties of the brand (Stigler and Becker 1977). Even once educated, the consumer may still be open to persuasion. Indeed, models of decaying stocks of "goodwill" capture the notion that perceptions about a brand are inherently dynamic (for example, see Nerlove and Arrow 1962).

The competitive positioning of brands in an attribute space is one of the key strategic decisions faced by marketing managers. The goal of positioning is to utilize the marketing mix to locate the brand in a distinctive place in the mind of the consumer (Kotler and Keller 2006). A brand's competitive position is largely influenced by its points of differentiation, which can be established on more *tangible* physical features or more *intangible* attributes. For example, Coca-Cola and Pepsi are differentiated on the physical dimension of sweetness. In blind taste tests, consumers express a preference for Pepsi. However, preference for Coca-Cola is enhanced by the Coke brand, due in part to branding efforts (Ariely 2008). These branding efforts often focus on intangible attributes. For example, Coca-Cola's current marketing communications use the slogan "Open happiness." It is unclear which tangible attribute or attributes provide "happiness."

Differentiation by tangible attributes is difficult to achieve for many consumer packaged goods. Frequently, consumers cannot differentiate between products in blind tastings (Keller 2003). From the consumers' perspective, these products are often differentiated primarily by intangible assets such as brand identity (Bronnenberg et al. 2009). Thus, firms continually invest in market communications to strengthen current positions or move to more favorable ones. Existing models of internal market structure are not well suited to capture evolution in product positioning. Although evolution in positioning can involve changes to the physical attributes of a product (e.g., successive generations of a durable good), changes in marketing communications alone may also trigger a change in a brand's position. Consider, for example, Anheuser-Busch's \$50 million-dollar "drinkability" campaign for Bud Light (Beirne 2008). The light beer category is well established in the United States and is dominated by three players with relative stable market shares: Anheuser-Busch, Coors, and Miller. Thus, it seems reasonable to posit that the light beer category is a good example of a stable and mature category. Bud Light's focus on the subjective attribute drinkability is not the result of a reformulation of the physical product. It appears that the firm is attempting to affect consumer perceptions about the product.

Although product position may evolve in mature categories, it is more likely that evolution occurs in

new product categories or categories impacted by structural shocks. Van Heerde et al. (2004) show that entry shifts existing brands' price and advertising elasticities (with respect to sales) immediately after introduction of a new brand, and over time, the elasticities stabilize. Building on their findings, we posit that in new categories or categories affected by structural change, latent brand positions are potentially evolving. If so, existing models of internal market structure are limited in their ability to study new and evolving categories because of the assumption of stable latent brand attributes over the observation window. Especially in these categories, the need for managers to understand positioning relative to competitors is important for growth and development of a strong brand. A dynamic factor-analytic choice model, as proposed in this paper, can help us to understand the positioning dynamics in a new category.

To summarize, in both mature and new categories the potential for evolving brand attributes exists. The main contribution of this paper is to develop a dynamic model of internal market structure that allows for evolution of latent brand attributes.³ Although applicable to new product categories, the method also enables researchers to revisit brand position dynamics in mature categories (perhaps because of firm repositioning efforts). The model can also accommodate the introduction of new brands over the analysis horizon.⁴ The model is specified as a factor-analytic multinomial probit and is estimated via Markov chain Monte Carlo (MCMC) methods. We use data augmentation to draw a set of latent utilities consistent with the observed choice data. Conditional on the draws of the latent utilities, we estimate individual-level dynamic brand attributes, individual-level preference weights for these attributes, and individual-level parameters for both marketing mix variables and consumer state dependence. We estimate the dynamic attributes via a forward-filtering, backward-smoothing algorithm. By capturing dynamics in brand attributes and the competitive positions defined by these attributes, our model allows us to understand the evolution of market structure over time. Our model also allows us to investigate the immediate and long-run effects of advertising spending on the brand attributes.

To demonstrate the efficacy and robustness of the modeling approach, we apply the model to two simulation scenarios. In the first scenario, the brand attributes are relatively static over time. This is

³ For the remainder of this paper, we will use the phrase "dynamic parameters" to describe time-varying parameters.

⁴ Although the model is able to address entry, we do not consider it in this paper.

the classic case of a stable and mature market in which implementation of a static model would be warranted. The second scenario contains a mix of dynamic conditions, with stationary brands, evolving brands, and converging brands. We show that our proposed model does not impute dynamics where none exists and ably recovers different dynamic parameter paths, as well as the weights and marketing mix coefficients. The second simulation scenario allows us to investigate the effect of misspecified dynamic attributes. We show analytically as well as empirically that estimating a static model on dynamic data induces both temporal heteroskedasticity in the error term as well as correlation between the error term and the preference weights. As a result, estimates of the attributes and marketing mix coefficients suffer from bias and attenuation.

In our empirical application, we estimate the proposed model with four years of panel purchase history data on consumer purchases in a new adult beverage category. The category is characterized by changing market shares of the major competitive brands over time; thus it is well suited to test the implications of our modeling approach. We capture the initial positioning of the brands in the latent attribute space and how the brand attributes are evolving as the category matures. Model estimates show a mix of dynamic behavior, with most of the brands evolving over the four years while one is relatively static. We find that advertising spending explains some of the evolution of the latent brand attributes. Comparing our proposed model to a set of competitive models, both static and dynamic, we find that our proposed model that captures the evolution of internal market structure best fits the data both in-sample and out-of-sample.

The remainder of this paper is structured as follows. We briefly review static models of internal market structure. Next, we introduce our dynamic model to study the evolution of internal market structure. Using a simulation study, we demonstrate the model's ability to handle a variety of market dynamics as well as investigate the effect of misspecified dynamics. An empirical application of the model to a new product category follows. We demonstrate that our proposed model yields superior in-sample and out-of-sample fit compared with alternative static and dynamic models. We also consider the implied patterns of price responsiveness across the models as well as the effect of changing advertising spending. Finally, we summarize and conclude.

2. Review of the Static Factor-Analytic Choice Model

Assume we observe $i = 1, \dots, I$ individuals choosing among $j = 0, \dots, J$ alternatives on each of $t = 1, \dots, T$

weeks. Let the utility of alternative j to individual i on occasion t be given by $U_{ijt} = \gamma_{ij} + x'_{ijt}\beta_i + \varepsilon_{ijt}$, where $\varepsilon_{ijt} \sim N(0, 1)$. The J vector y_{it} indexes the choice that corresponds to the alternative with maximum utility for individual i at week t . The parameter γ_{ij} is an individual-specific brand intercept. The vector x_{ijt} is a vector of independent variables, such as price, display, and feature, and can also include measures of state dependence. Pioneered by Elrod (1988), factor-analytic choice models decompose the intercept γ_{ij} into latent attributes and preference weights for the attributes. Thus, each product in the choice set has a position defined by the latent attributes. Elrod (1988) decomposes γ_{ij} as follows:

$$\gamma_{ij} = a_j^1 w_i^1 + a_j^2 w_i^2, \quad (1)$$

where a_j^k is the level of the k th attribute for brand j and w_i^k is individual i 's preference weight for the k th attribute.

Given that the brand attributes as well as weights are latent, identification restrictions are necessary.⁵ First, adding a constant to the k th attribute of all the brands does not affect the probability of brand choice for any individual. To remove this indeterminacy, we fix the location of one of the brands by setting the a s to zero. Second, the attribute preference weights can be scaled by a constant without affecting the brand choice probabilities. Thus, the w s are restricted to have a unit variance. Finally, the attribute and preference vectors can be simultaneously rotated without affecting the choice probabilities. A number of options are available to remove this indeterminacy (Elrod 1988, Elrod and Keane 1995, Erdem 1996). Following Erdem (1996), we set $\mu_w^k = \mu_w \geq 0 \forall k$. It follows that $w_i^k \sim N(\mu_w, 1) \forall k$.

Thus far, the model specifies that individuals are homogenous with respect to brand attributes. Chintagunta (1994) introduces a latent-class heterogeneity structure on the attributes. Erdem (1996) allows for individual-level heterogeneity in the attributes, which can be understood as allowing for differences in individual perceptions with respect to the brand attributes. Erdem (1996) also introduces individual choice dynamics by allowing utility to depend on the difference between attributes of the current brand choice and the attributes of the brand purchased on the last choice occasion. Although this addresses state dependence from the individual's perspective, it does not account for dynamic brand attributes. We now turn our attention to modeling heterogeneous dynamic brand attributes.

⁵ These restrictions are standard procedure in the factor-analytic choice model literature, and we discuss them without proof. For a comprehensive discussion, see Erdem (1996).

3. A Dynamic Factor-Analytic Choice Model

3.1. The Nature of Latent Brand Attributes

Extant factor-analytic choice models assume that a brand's attributes and hence its position in attribute space are constant over time. This limits the application of the model to categories where the researcher is confident that the brand attributes are indeed static and poses a problem with using the model to study new categories or categories affected by structural shocks. Additionally, as discussed in §1, the assumption of static brand attributes may be untenable for even mature categories.

A static factor-analytic choice model may be viewed through the lens of individual learning models. In learning models, products are treated as having a true level of an imperfectly observed attribute that is stable over time, essentially, the γ_{ij} in Equation (1).⁶ Individuals have expectations about γ_{ij} and learn about the true level over time through, for example, advertising signals centered on the true level. Temporal variation in the expected value of the attribute is due to individual uncertainty, which diminishes as the consumer learns. Once individuals have extinguished uncertainty about the true attribute levels, the expectation converges to the true static level (Narayanan and Manchanda 2009). At this point, the brand choice probabilities are driven by the learned attribute levels, which create differentiation. Assuming that the category is stable and mature, static factor-analytic choice models ostensibly estimate the true static attribute levels and implicitly assume learning has occurred and individuals are no longer uncertain about the attribute levels.

It is useful to more closely consider the nature of these estimated attributes. If the attributes are "hard" and objective (e.g., detergent cleansing power), only a change in physical attributes should change a brand's position in attribute space (given that learning has occurred). If, however, the estimated attributes capture "soft" perceptions that inherently change over time, then the notion of learning the true level of an objective attribute is no longer meaningful. Many studies point toward "soft" perceptions in the domain of consumer packaged goods (CPG). For many CPG products, individuals cannot differentiate between products in blind tastings (e.g., Keller 2003). In other words, the products are not differentiated on any objective attribute at all. Other consumer products such as vodka or bottled water are inherently tasteless and odorless. Hence, if the objective attributes of a set of products are indeed indistinguishable

and these attributes are what is measured by models of internal market structure, then the analysis should show little to no differentiation. However, published applications of internal market structure models consistently demonstrate differentiation on the latent attributes. Furthermore, competing firms in CPG industries often command different prices and achieve different market shares despite any discernable quality differences (Bronnenberg et al. 2009). This suggests that soft individual perceptions are playing a major role in defining the differentiated positions in the attribute space measured by internal market structure analysis.

Many CPG products are indeed differentiated not by objective quality but primarily by their brands (Bronnenberg et al. 2009). Brand image associations, consumer perceptions about a brand, and other brand-related constructs are likely to vary over time. Firms invest in their brands to establish and maintain unique and favorable associations, influence consumer perceptions, and ultimately position and reposition their brands in the competitive landscape. This is inherently a *dynamic* process. A recent example is Cheerios, which two years ago attempted to strengthen their health positioning by claiming that by eating Cheerios "you can lower your cholesterol 4% in 6 weeks."⁷ One may be able to measure the objective amount of whole grains in a serving of Cheerios, but the perceived amount of "health" in a serving of Cheerios is likely more subjective and may wax and wane.

In summary, we believe that the latent brand attributes estimated by factor-analytic choice models are most likely soft attributes. As such, internal models of market structure may benefit from relaxing the assumption of static brand attributes. As discussed, there are many reasons to suggest that brand attributes, especially soft attributes, are likely to be dynamic. Ultimately, we view this as an empirical question whose answer may very well differ from category to category, product to product, and also likely depends on the observation window. We now turn our attention to developing a model to allow researchers to investigate market structure dynamics.

3.2. Model Specification

We introduce an extension to the static factor-analytic choice model that accommodates a variety of dynamic behavior in the brand attributes, including a lack of dynamics. In terms of specifying dynamic parameters in a weekly brand choice model, the ideal case

⁶ Erdem (1996) cites the richness of peanut butter or the cleansing power of detergents as examples of brand attributes.

⁷ The U.S. Food and Drug Administration (FDA) recently warned General Mills, maker of Cheerios, that Cheerios was being sold as an unapproved new drug. The FDA warned General Mills that it must either change the way it markets Cheerios or apply for federal approval to sell Cheerios as a drug (FDA 2009).

for the researcher would be to observe a sufficient number of observations per week to enable estimation of any dynamic parameters at the weekly level. Unfortunately, this will rarely, if ever, be the case. Thus, the researcher must choose the period of calendar time (e.g., monthly, quarterly, or annually) on which the dynamic parameters will be specified. This creates two different time scales, one for the purchase occasions and one for the system dynamics (Lachaab et al. 2006). We allow for the attributes to evolve across periods we label q . The specification of q (e.g., months, quarters, or years) will depend on the empirical application. We discuss this in more detail in our application. Generally, each week, t , can be uniquely assigned to a period in which it occurs, q , where $q = 1, \dots, Q$. As before, we begin with a latent utility equation, $U_{ijt} = \gamma_{ijt} + x_{ijt}^k \beta_i + \varepsilon_{ijt}$. We model γ_{ijt} as

$$\gamma_{ijt} = a_{ijq}^1 w_i^1 + a_{ijq}^2 w_i^2. \tag{2}$$

The same restrictions on the w s as in the static factor-analytic choice model apply here as well and as in the static model, $w_i^k \sim N(\mu_w, 1) \forall k$. We assume that individuals are heterogeneous in their perceptions of the dynamic brand attributes and model the individual-level brand attributes a_{ijq}^k as

$$a_{ijq}^k = a_{jq}^k + c_{ij}^k, \tag{3}$$

where $c_{ij}^k \sim N(0, \sigma_k^2)$ and $k = 1, 2$. Equation (3) models the heterogeneous dynamic attribute as arising from a time-varying mean and a time-invariant individual shock (Liechty et al. 2005). Note that although restricting the individual shock to be time invariant imposes a stronger shrinkage structure, this allows us to investigate finer intervals of q than otherwise possible.⁸

We specify the evolution of the mean brand attribute level as a naturally state-dependent process

$$a_{jq}^k = \delta_j^k a_{jq-1}^k + \omega_{jq}^k, \tag{4}$$

where δ is a parameter to be estimated and ω is an error term (to be defined below). Equation (4) allows us to empirically assess the stickiness of a brand's mean attribute a_{jq}^k . If current mean attributes are not

influenced by previous period mean attributes, then δ_j will be close to zero. However, if changes in a brand's attribute do not occur instantaneously but rather evolve slowly over time, then δ_j will be closer to one (Kort et al. 2006, Bass et al. 2007). To estimate the model, we stack the attribute means over brands and latent attributes into a $(J - 1)K \times 1$ vector, a_q .⁹ The evolution of a_q is modeled as a first-order autoregressive process as follows

$$a_q = \Delta a_{q-1} + \omega_q, \tag{5}$$

with $\omega_q \sim \text{MVN}(0, \Omega)$. The $(J - 1)K \times (J - 1)K$ parameter matrix Δ (to be estimated) contains the state dependence parameters. The covariance matrix Ω allows for possible correlation in the dynamic parameter paths.

To summarize, the model hierarchy is as follows:

$$\begin{aligned} U_{ijt} &| w_i^k, a_{jq}^k, c_{ij}^k, \beta_i, x_{ijt}, y_{ijt}, \\ w_i^k &| \mu_w, \\ a_q &| \Delta, a_{q-1}, \Omega, \\ c_{ij} &| \sigma^2, \\ \beta_i &| \bar{\beta}, \Sigma_\beta. \end{aligned} \tag{6}$$

The model is estimated with MCMC methods, which sample from the full conditional distributions for model parameters. We use a data augmentation step to draw the latent utilities U_{ijt} . Next, Gibbs and Metropolis-Hastings steps are used to draw the remaining model parameters, with the exception of the time-varying attribute means a_{jq}^k . A complication arises since for $0 < q < Q$ the conditional distribution of a_q depends on a_{q-1} as well as on a_{q+1} . This creates a circular reference problem that precludes us from specifying and sampling from the conditional distribution of a_q in the usual manner. One way to address this problem is to conceptualize the model as a Bayesian dynamic linear model (DLM) that can be estimated using forward-filtering, backward-sampling algorithm to account for the above-mentioned problem. The general specification of a DLM is

$$\begin{aligned} y_t &= F_t^T \theta_t + \kappa_t, \\ \theta_t &= G_t \theta_{t-1} + \eta_t, \end{aligned} \tag{7}$$

where y_t is the observed data, θ_t are the unobserved dynamic states of interest, and $\kappa_t \sim N(0, \Sigma_\kappa)$ and $\eta_t \sim N(0, \Sigma_\eta)$ (West and Harrison 1997). G_t describes the evolution of the unobserved states θ ,¹⁰ and F_t matches

⁸ An alternative specification for Equation (3) is $a_{ijq}^k = a_{jq}^k + v_{ijq}^k$. Allowing the individual shocks to be time varying implies that the researcher can only use observations for the individual that occur within a given q . Clearly, there is a trade-off between the amount of information and the level of q ; the finer the latter, the less of the former. Fixing the individual shocks to be time invariant allows us to consider finer levels of temporal aggregation within our dynamic model. We also model the transitions across time as a smooth process. Alternatively, we could also allow for discrete movements that could be triggered by an advertising shock or competitive entry. This could be implemented by adding covariates, e.g., indicator variables, to Equation (7).

⁹ As discussed, the attributes of one brand are set to zero for identification.

¹⁰ Note that in our application, $G_t = G \forall t$ as follows from Equation (7).

Table 1 Simulation Study, Selected Parameters

			95% Coverage interval			
			Data-generating value ^a	Mean	Lower bound	Upper bound
(a) Simulation-stationary attributes, dynamic model						
Price coefficient	Mean	−1.00	−0.95	−1.04	−0.86	
	Variance	0.25	0.30	0.24	0.37	
Importance weight	Mean	1.00	0.97	0.89	1.05	
(b) Simulation-mixed attributes, dynamic model						
Price coefficient	Mean	−1.00	−0.97	−1.05	−0.89	
	Variance	0.25	0.25	0.19	0.32	
Importance weight	Mean	1.00	0.97	0.89	1.05	
(c) Simulation-mixed attributes, static model						
Price coefficient	Mean	−1.00	−0.70	−0.77	−0.64	
	Variance	0.25	0.15	0.12	0.19	
Importance weight	Mean	1.00	0.97	0.86	1.08	

^aCell entries are posterior mean estimates.

the unobserved states with the observed data. In our case, we model the dynamic latent attribute means a_q as the unobserved states of interest and use the current draw of the latent utilities U_{ijt} as data. In other words, at step s in the sampler we use the forward-filtering, backward-sampling algorithm to draw a new set of mean brand attributes $\{a_q\}_s$ and construct a new $\{a_{ijq}^k\}_s$ based on Equation (3).

To implement the sampler, we require a proper but diffuse prior distribution for the initial period, $a_0 \sim N(m_0, C_0)$, as well as priors for other model hyperparameters. Details are included in the appendix. Before presenting our empirical application, we turn our attention to the performance of the dynamic factor-analytic choice model in a variety of evolutionary settings in a simulation study.

4. Simulation

Our simulation investigates parameter recovery in a variety of dynamic settings, including a lack of dynamics. This simulation was guided by three questions. First, can the proposed model ably recover dynamic parameters? Second, does the proposed model correctly recover static data and not impose dynamics where none exist? Finally, can the proposed model recover different types of dynamics? We also consider the effects of erroneously applying a static factor-analytic choice model to dynamic data. We generate data according to two scenarios. Both scenarios are composed of four brands, each of which is described by a two-dimensional brand attribute vector and a price. The two scenarios are differentiated by the evolutionary path of the latent brand attributes. Note that for identification, we set the brand attributes for brand 1 to zero. To focus more precisely on the dynamic aspects of the model, our

simulated data assume homogeneous brand image attributes. We simulate data with weekly observations over four years. For the simulation, we estimate the dynamic attributes at the quarterly level.

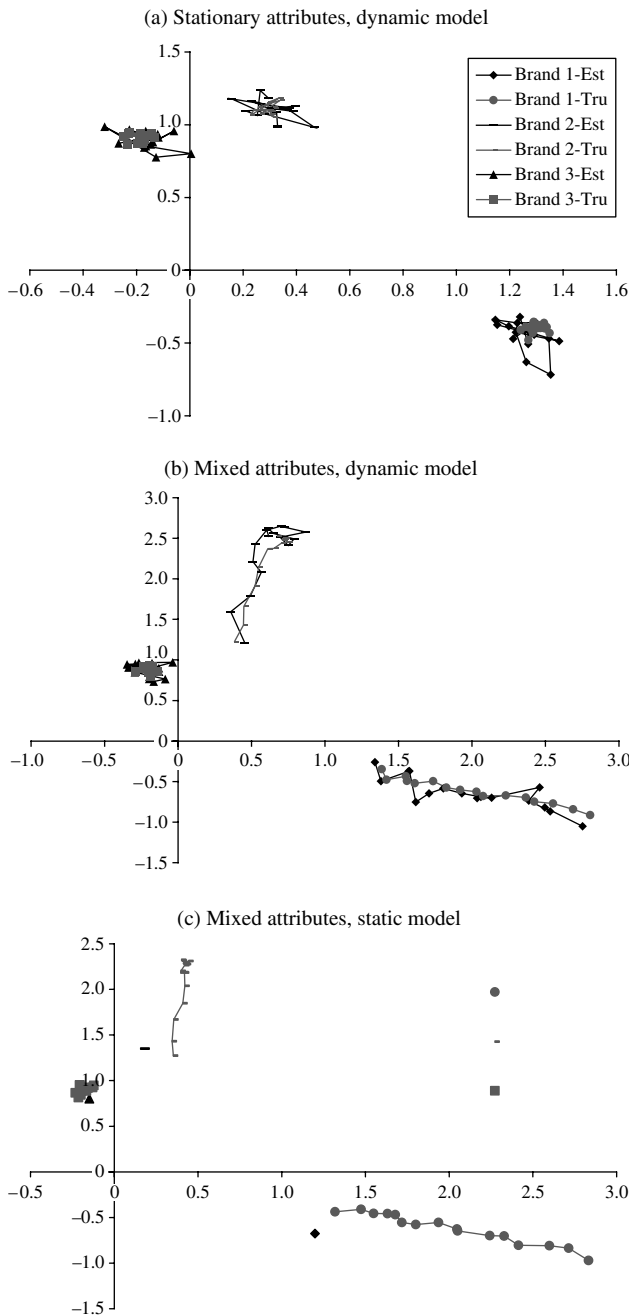
In the first scenario, the brand attributes are approximately stationary over time. This is intended to describe the case where the category is stable, with little to no shifts in brand attributes. In this setting, the static factor-analytic choice model is appropriate. An important consideration is how the proposed model handles a situation where the brand attributes are stationary or approximately stationary. The main concern is whether the dynamic model imputes dynamic behavior where none exists. In the second scenario, we mix different dynamic behaviors. We simulate one static brand and two dynamic brands along with the base brand. For one dynamic brand, the attributes are evolving over the entire observation period. This is consistent with a new product category where the “dust settling” has yet to occur. For the second dynamic brand, the attributes are evolving in initial time periods, but settling into a stationary position in later periods.¹¹

4.1. Results

To assess the ability of the proposed model to recover the brand attributes, we apply the model to two synthetic data sets consistent with each of the aforementioned scenarios. Table 1, panels a and b, present the data-generating values and the 95% coverage intervals of the estimates for the mean and variance of the

¹¹ We also conducted simulations in which all three brands exhibit the same dynamic behavior as well as a scenario in which a structural shock occurs, introducing dynamics in a formerly static category. In all our simulation, the proposed model ably recovers the parameters. The simulations are available from the authors upon request.

Figure 1 Simulation Study: Brand Attributes



price coefficient and the mean importance weight. We see that for both scenarios, the 95% coverage intervals for each parameter contain the data-generating values. Figures 1(a) and 1(b) present plots of the data-generating values and estimates of the brand attributes for each of the five scenarios. In all cases, the model estimates are reasonably close to the data-generating values. In Figure 1(a), we see that the model does not appear to impose dynamic behavior where none exists. The estimates cluster around the approximately stationary brand attributes. In Figure 1(b), we see that the model ably handles the

different types of dynamics. Overall, the proposed model appears to be quite robust to a variety of evolutionary scenarios.

4.2. Investigating Misspecified Dynamics

It is of interest to investigate the behavior of the static factor-analytic model under the second scenario where the true brand attributes are dynamic. Consider a simplified true model with a single dynamic homogeneous brand attribute and evolution described by a random walk. Let the true latent utility model be

$$U_{ijt} = a_{jq}w_i + x'_{ijt}\beta_i + \varepsilon_{ijt},$$

$$a_{jq} = a_{jq-1} + v_{jq},$$
(8)

with error terms distributed $\varepsilon_{ijt} \sim N(0, 1)$ and $v_{jq} \sim N(0, \sigma_v^2)$, and initial condition a_{j0} . At any time t , we can express the utility as $U_{ijt} = (a_{jq-1} + v_{jq})w_i + x'_{ijt}\beta_i + \varepsilon_{ijt}$. Through repeated substitution, we can express the utility as $U_{ijt} = (a_{j0} + \sum_{q=1}^{q_t} v_{jq})w_i + x'_{ijt}\beta_i + \varepsilon_{ijt}$, where q_t is the quarter corresponding to week t . Rearranging terms yields

$$U_{ijt} = a_{j0}w_i + x'_{ijt}\beta_i + \xi_{ijt},$$

$$\xi_{ijt} = \left(\sum_{q=1}^{q_t} v_{jq} \right) w_i + \varepsilon_{ijt}.$$
(9)

From an econometric perspective, there are three things to note about ξ_{ijt} . First, the variance of ξ_{ijt} is greater than the variance of ε_{ijt} . Because the utility function is scaled by the error variance, this will deflate the choice model parameter estimates (Griliches and Yatchew 1985, Swait and Louviere 1993). Second, the presence of the summation in the equation for ξ_{ijt} implies that the variance of ξ_{ijt} is not constant over time but increasing with each time period. Finally, note that ξ_{ijt} is correlated with w_i ; thus the standard assumption that $E(\xi_{ijt} | w_i) = 0$ is violated, which will bias the estimate of a_{j0} . Estimating the model given in (12) by a static factor-analytic choice model of the form $U_{ijt} = a_j w_i + x'_{ijt}\beta_i + \varepsilon'_{ijt}$ with $\varepsilon'_{ijt} \sim N(0, 1)$ will, at best, approximately recover the initial state of the brand attributes. At worst, attenuation, inefficiency, and estimation bias in the latent attributes will result from the misspecification of the true dynamic model as static. Note that this is similar to the effects of misspecified individual-level preference heterogeneity (Chintagunta et al. 1991).

To assess the bias resulting from estimating a static factor-analytic choice model on dynamic data, we estimate a static model on our second synthetic data sets described above. Table 1, panel c, presents the data-generating values and the 95% coverage intervals of the estimates for the mean importance weight and the price coefficient. Estimates of the mean and the

variance of the price coefficient are attenuated. Figure 1(c) presents plots of the data-generating values and estimates of the latent brand attributes for each of the two scenarios. In Figure 1(c), consistent with our intuition, we see that the misspecified static model approximates a point in the neighborhood of the early periods.

5. Empirical Application

To illustrate the dynamic factor-analytic choice model, we apply it to a panel data set of household purchases in the flavored malt beverage category over the period 2002–2005. The data are from the IRI academic data set (Bronnenberg et al. 2008). We believe that this category is well suited to investigating dynamics in latent brand attributes. First, the category is relatively new, emerging in the U.S. market in the late 1980s as an alternative to wine coolers. The category grew significantly in the mid- to late 1990s, with the introduction and success of brands such as Zima and Smirnoff. A third major brand, Bacardi, entered the market the year before our observation period. Second, over the observation period, we see large shifts in market share. Table 2 presents the four brands included in our analysis and their market shares within the panel of $N = 250$ households. We model the most popular package size in the category, a six-pack of 12-ounce bottles. To be included in the sample, households must have at least one purchase in each of the four years of the observation period. Table 3 presents the average market prices and percentage changes in price over the observation period for the four brands. The relatively small price changes over the observation period lead us to conclude that price competition is, most likely, not a key driver of the significant market share changes.

As discussed in §3, to estimate the model described in Equation (6), the researcher must specify the time periods over which the mean brand positions are evolving. Whereas the researcher should desire the finest level of temporal aggregation possible (weekly, in our case), attention must be paid to the trade-off between the level of temporal aggregation and information. For our empirical application, brand choice is modeled on the weekly level, whereas the brand attributes are allowed to vary monthly. We note that with enough data per time period, we could simply estimate an individual static model for each time

Table 2 Brands and Market Shares

	2002 (%)	2003 (%)	2004 (%)	2005 (%)
Smirnoff	45	65	58	62
Bacardi	21	15	28	12
Zima	25	9	4	16
Twisted Tea	9	12	11	11

Table 3 Market Prices

	2002		2003		2004		2005	
	Price ^a (\$)	% Δ	Price ^a (\$)	% Δ	Price ^a (\$)	% Δ	Price ^a (\$)	% Δ
Smirnoff	7.11	N/A	7.19	1	7.23	1	7.15	-1
Bacardi	7.33	N/A	7.33	0	7.36	0	6.96	-5
Zima	5.88	N/A	6.01	2	5.96	-1	6.08	2
Twisted Tea	6.55	N/A	6.63	1	6.53	-2	6.87	5

^aAverage price paid including promotional discounts.

period.¹² In our case, we could attempt to estimate a static model for each month. However, this would leave us with a rather limited amount of data. Our dynamic model is efficient in the sense that it uses data from previous periods as well as from future periods to estimate current period brand attributes. Static snapshots are not able to use data in this way and present a less efficient and stable means to estimate the brand attributes.

We include in the vector of explanatory variables x_{ijt} price (prices paid including promotions) and an indicator variable for last brand purchased. Including the last brand purchased helps control for state dependence and follows the standard approach of most of the marketing literature. Finally, it is well known that correlation between price and error terms may lead to endogeneity bias in the price coefficient. Using data on barley prices, we implement an instrumental variable procedure (e.g., Kuksov and Villas-Boas 2008) to alleviate any concerns about price endogeneity.¹³

5.1. Model Comparison

To estimate the model described in Equation (6), we run the sampler for a burn-in period of 15,000 iterations. To conduct posterior inference, we run the sampler for an additional 75,000 iterations and thin the chain by a factor of 5. Convergence diagnostics (e.g., Raftery and Lewis 1995) indicate that the chain has converged and that autocorrelation is not a significant problem. For comparison, we estimate a set of alternative models. The total set of estimated models can be grouped into standard probit choice models with no factor structure and factor-analytic probit choice models. Under each type of model (the standard and factor analytic), we estimate static models, models where the intercepts or brand attributes vary as a polynomial function of time and models with a first-order state-dependent process on

¹² This is also true for choice models with heterogeneous household response coefficients. With enough observations per household, one can simply estimate models household by household. Of course, this condition is rarely, if ever, met in practice.

¹³ Barley prices are obtained from <http://www.econstats.com>.

Table 4 Model Fit Statistics

		LMD ^a	Hit rate ^b
Standard probit	Dynamics on intercepts		
	Static	-1,168	0.63
	Polynomial	-1,118	0.60
	Dynamic linear	-1,109	0.77
Factor-analytic probit	Dynamics on attributes		
	Static	-1,145	0.69
	Polynomial	-989	0.76
	Dynamic linear	-956	0.81

^aEstimated via harmonic mean of model likelihoods computed with the GHK simulator.

^bEstimated with last choice for each household which is held out of estimation.

the intercepts or attributes (i.e., the DLM approach).¹⁴ This results in a total of six models for comparison, three standard models and three factor-analytic models. Table 4 reports the harmonic mean estimator of the log-marginal density (LMD).¹⁵ The likelihoods are computed via the Geweke, Hajivassiliou, and Keane (GHK) algorithm. We also hold out the last choice of each household and use these holdout choices to compute the out-of-sample hit rate. Estimates of the LMD and out-of-sample hit rate strongly favor our proposed model over the alternative models. Two additional points emerge from the model comparison. First, comparing each factor-analytic probit to its standard probit counterpart (based on dynamic specification of the intercepts or attributes), we see that the factor-analytic probits outperform the standard probits. This underscores the value of modeling internal market structure. Second, within each model type (standard probit versus factor-analytic probit), model comparison results demonstrate the value of modeling dynamics.

Table 5 reports the posterior mean estimates of the mean and variance of the price and last brand purchased coefficients, as well as the posterior mean estimate of the mean importance weight. The coefficients are all of the expected sign and estimated with reasonable precision. The estimates of the dynamic attribute means are plotted in Figure 2.¹⁶ Table 6

¹⁴ We illustrate the polynomial function of time approach for the standard dynamic probit. In this case, the intercepts are modeled $\alpha_{ij} = \alpha_{ij} + c_{ij}$, where $c_{ij} \sim N(0, \sigma^2)$. The mean of the brand intercept is modeled as $\alpha_{ij} = \alpha_{0j} + \alpha_{1j}q + \alpha_{2j}q^2$. The polynomial function of the time approach is implemented in the same manner for the factor-analytic probit.

¹⁵ We find that a model with a full covariance structure on the mean brand attributes Ω does not outperform a model in which Ω is restricted to be a diagonal matrix. Consequently, we present the results from the latter model.

¹⁶ Because of the large number of parameters (48 parameters for each of the three brands and each of the two attributes), we do not report these values in tabular form. The results are available in tabular form upon request.

Table 5 Selected Model Parameters

	Mean ^a	Variance
Price	-0.50* (0.12)	0.80 (0.29)
Last brand purchased	0.22* (0.08)	0.41 (0.15)
Importance weight	1.25* (0.21)	1.00 —

^aCell entries are posterior mean and posterior standard error (in parentheses).

*The 95% coverage interval does not span zero.

Table 6 Latent Brand Attributes: Variance of Individual-Level Shocks

σ_1^2	σ_2^2
0.32 (0.17)	0.28 (0.15)

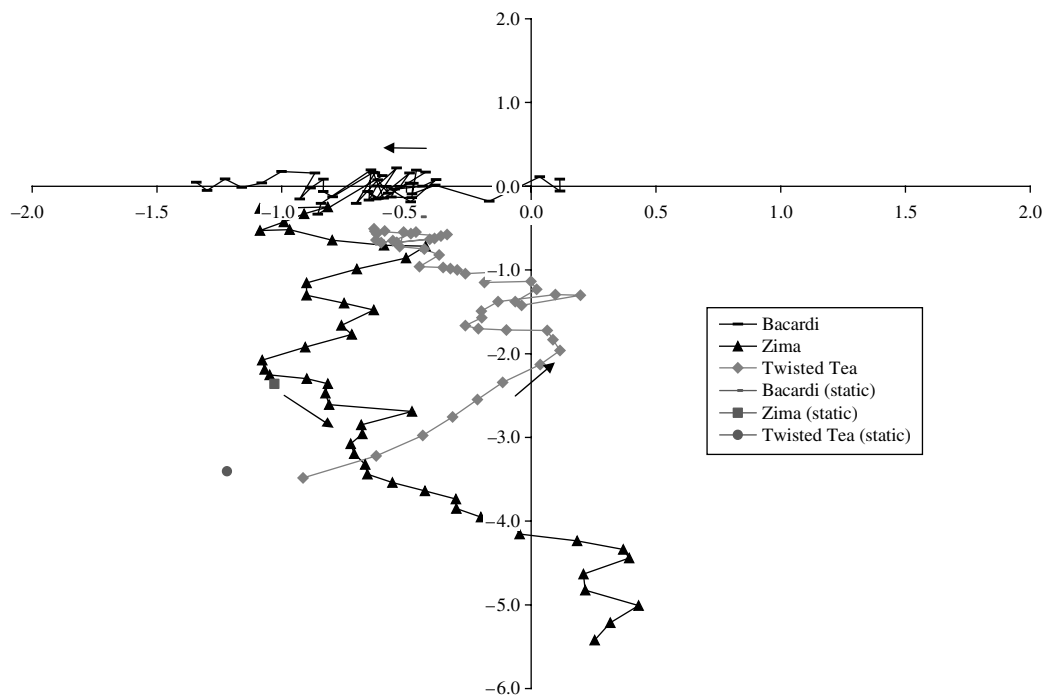
Note. Cell entries are posterior mean and posterior standard error (in parentheses).

reports the variances of the individual-level shocks to the dynamic attributes. Recall that the identifying brand is the market share leader, Smirnoff.

From Figure 2, we see that Bacardi is moving in a westerly direction on the horizontal dimension (attribute 1) and is essentially unchanged on the vertical dimension (attribute 2). Zima is moving in a southeasterly direction. From its initial condition in the southwest quadrant of the map, Zima moves past the origin on the horizontal dimension (attribute 1) and away from the origin on the vertical dimension (attribute 2). Twisted Tea exhibits significant movement over the observation window. From its initial position in the southwest quadrant, Twisted Tea first moves northeast toward the origin on both dimensions, then turns northwest, away from the origin on the horizontal dimension (attribute 1). Figure 2 also presents the attribute means implied by the static model; the dynamic model outperforms the static model in terms of penalized model fit. More importantly, the static factor-analytic choice model cannot reflect the evolution of the brand attributes over time. As in our simulation, the static model implies attribute means that are in the neighborhood of the initial periods of the dynamic model. Furthermore, the mix and state dependence coefficients are almost certain to be adversely affected by bias and attenuation.¹⁷

¹⁷ For comparison purposes, the mean price and state dependence coefficients for the static model are -0.39 and 0.34, respectively. Both parameters are significant; i.e., the 95% coverage intervals do not contain zero. Recall from Table 5 these values are -0.50 and 0.22 for the dynamic model.

Figure 2 Plot of Dynamic Attribute Means



Note. The origin of the map is the market share leader, Smirnoff.

Table 7 presents the carryover parameters for the dynamic means (δ) as well as the error variances (Ω). The 95% coverage intervals for δ coefficients for all brands on the first attribute do not span zero. For the second attribute, the 95% coverage intervals for the δ coefficient for Twisted Tea does not span zero, whereas the coverage intervals for Bacardi and Zima do span zero. Overall, we find significant evidence for dynamic behavior in the latent brand attributes.

Table 7 State Equation Parameters

	Δ	Ω
Attribute 1		
Bacardi	0.85* (0.31)	0.032 (0.016)
Zima	0.97* (0.04)	0.032 (0.016)
Twisted Tea	0.94* (0.10)	0.032 (0.016)
Attribute 2		
Bacardi	0.04 (0.21)	0.034 (0.017)
Zima	0.03 (0.20)	0.031 (0.015)
Twisted Tea	0.45* (0.21)	0.031 (0.015)

Note. Cell entries are posterior mean and posterior standard error (in parentheses).

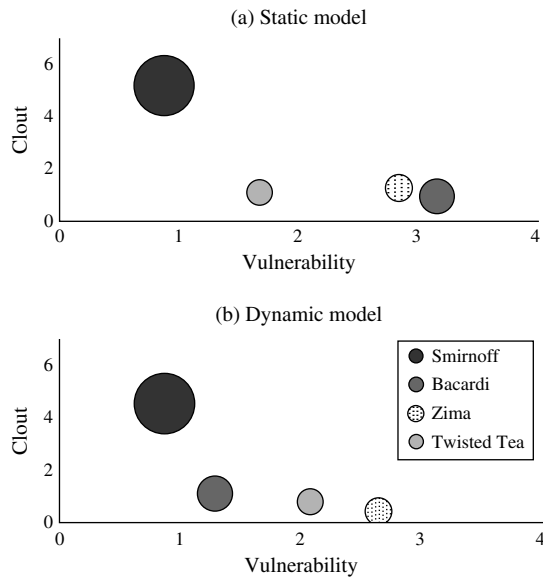
*Indicates the 95% coverage interval does not span zero.

We compare the aggregate own- and cross-price elasticities implied by our proposed model with those implied by the static factor-analytic probit. For both models, we estimate the elasticities by computing the overall market shares at the observed prices and the then recomputing the shares after increasing price by an increment of 10% (sequentially for each brand holding all else constant). Table 8 reports the elasticities. First, as can be seen from Table 8, compared with our proposed model, the elasticities implied by the static factor-analytic probit understate the own price elasticity for the market share leader, Smirnoff, and overstate the own-price elasticities for the three smaller market share brands. Second, to compare the cross-price elasticities, we compute the aggregate clout and vulnerability implied by the static

Table 8 Aggregate Own- and Cross-Price Elasticities

	Smirnoff	Bacardi	Zima	Twisted Tea
(a) Static factor-analytic probit				
Smirnoff	-0.30	0.41	0.22	0.24
Bacardi	2.16	-3.12	0.50	0.50
Zima	2.27	0.21	-2.54	0.36
Twisted Tea	0.79	0.33	0.56	-3.94
(b) Dynamic factor-analytic probit				
Smirnoff	-0.52	0.33	0.23	0.30
Bacardi	0.87	-1.93	0.19	0.23
Zima	1.78	0.60	-1.72	0.27
Twisted Tea	1.89	0.18	0.00	-3.71

Figure 3 Clout and Vulnerability



and dynamic factor-analytic models (Kamakura and Russell 1989). These maps represent the structure of price competition implied by each model. From Figure 3, we see that the order of the smaller market share brands is quite different across the two models. In particular, note that the static model implies Bacardi is the most vulnerable brand, whereas the dynamic model implies Bacardi is much less vulnerable. Also, note that Zima’s clout is overstated in the static model. In the dynamic model, Zima is the weakest brand, with the smallest clout and largest vulnerability. Given that our proposed model has the best fit, these results imply that the static factor-analytic probit yields misleading results with respect to both own- and cross-price response. Especially for the smaller market share brands, the differences across the static and dynamic models are quite large.

5.2. Extensions

We consider two extensions to our dynamic factor-analytic probit model. First, we consider dynamic importance weights. Second, we investigate whether or not advertising spending explains the evolution of the dynamic attribute means.

5.2.1. Extension 1. Economic theory, for the most part, assumes stable consumer preferences (e.g., Stigler and Becker 1977). Recent research in behavioral economics, however, challenges this notion (e.g., Ariely et al. 2006). We extend our proposed model to allow for dynamic evolution of the importance weights. A complication arises because of the identification restriction on the mean of the importance weights μ_w , which is constrained to be strictly non-negative. This restriction prohibits application of the

forward-filtering, backward-sampling algorithms utilized for the evolution of the attribute values as a result of the mixture of nonconjugate distributions (i.e., we use a log-normal for μ_w). To navigate around this problem, we propose a Metropolis step to draw the dynamic weights that allows for dynamics in w . To extend the model to dynamic weights, we express γ_{ijt} as

$$\gamma_{ijt} = a_{ijq}^1 w_{iq}^1 + a_{ijq}^2 w_{iq}^2. \tag{10}$$

We model the a_{ijq}^k as before. Similarly, we model the dynamic weights as

$$w_{iq}^k = w_q^k + v_i^k, \tag{11}$$

where $v_i^k \sim N(0, 1)$ for $k = 1, 2$. As before, we assume that $w_q^k = w_q \forall k$. We specify the prior as $w_q^* \sim N(\lambda w_{q-1}^*, \sigma_w^2)$ where $\ln(w_q) = w_q^*$. Note that the posterior distribution will include information from the likelihood of the $q + 1$ period. For more details, please see the appendix. We estimated our proposed model allowing for dynamic weights. The estimate of the LMD for the model allowing dynamic weights is -969 , worse than the estimated LMD for the model that assumes static importance weights, -956 . Thus, we conclude that the assumption of static importance weights is appropriate for our data.

5.2.2. Extension 2. Our empirical results indicate that the latent brand attributes are evolving over time. We have demonstrated that the dynamic factor-analytic probit best fits the data and that failure to model dynamics may yield misleading inferences with respect to price competition. However, as currently specified, the model does not indicate how managers might influence the evolution of the latent attributes. Assuming that the dynamic attributes are indeed capturing soft attributes, advertising spending seems to have the potential to explain the evolution of the attributes. Our proposed model allows us to investigate this issue in a natural fashion. We extend our proposed model to allow advertising spending to influence the evolution of the dynamic attribute means,

$$a_{jq}^k = \delta_j^k a_{jq-1}^k + z_{jq} \gamma_j^k + \omega_{jq}^k, \tag{12}$$

where z_{jq} is advertising spending. We obtain monthly advertising expenditures for our brands from Taylor Nelson Sofres’ Competitive Media Report.

Previous research in marketing has demonstrated the need to account for dynamic effects of advertising. Such effects can be modeled with advertising lags. In Equation (12), past advertising spending is parsimoniously captured in the lagged value of the attributes, alleviating the need to include lags in the specification. We estimate the proposed model with advertising and find that advertising indeed affects the evolution of the attributes. The estimated LMD also

Table 9 Advertising Parameters

	γ
Attribute 1	
Bacardi	-0.020 (0.061)
Zima	0.035* (0.010)
Twisted Tea	0.015 (0.038)
Attribute 2	
Bacardi	0.142* (0.057)
Zima	0.0001 (0.002)
Twisted Tea	0.195* (0.051)

Note. Cell entries are posterior mean.

*Indicates the 95% coverage interval does not span zero.

improves to -950 . The estimates of γ are reported in Table 9. We find that advertising affects the first dimension for Zima while affecting the second dimension for Bacardi and Twisted Tea. We see this as further evidence that our model captures soft attributes as variation in advertising explains some of the variation in the attributes. Although our data do not capture advertising themes, this is an interesting topic for future research. Different types of advertising themes, for example, “hip” or “sophisticated,” can easily be incorporated into our model by using indicator variables in combination with gross rating points.

Although understanding the effect of advertising on the attributes is interesting, managers are also interested in the ultimate effect on shares. To investigate this issue, we shock advertising for one brand and trace the changes in market share over time. To implement the shock, we average each brand’s advertising and then, in turn, shock each brand by adding the average of its advertising to the current advertising in a particular period.¹⁸ We find that the effect of the advertising shock has mostly dissipated after about two periods. Interestingly, we find that the shares for the Smirnoff and Bacardi are more responsive than the shares for Zima and Twisted Tea. The implied advertising elasticity for Smirnoff and Bacardi is 0.19 and 0.18, respectively. For Zima and Twisted Tea, we find that the implied advertising elasticity is much smaller, 0.06 for both. Note that Smirnoff and Bacardi command higher prices and that Smirnoff is the clear market share leader. Using

¹⁸ We also used the maximum of advertising and find similar results. We implemented the shock in the 20th month of the observation window. Similar results were obtained with different months as the shock period.

the Ailawadi et al. (2003) concept of price premiums as a measure of brand equity, our results suggest the higher-equity brands reap greater benefit from marketing efforts. These results are consistent with the Slotegraaf and Pauwels (2008) finding that marketing response is stronger for high-equity brands.

6. Summary and Conclusions

We present a dynamic factor-analytic choice model to capture evolution of brand positions in latent attribute space. This represents an important extension to static factor-analytic choice models, which require the assumption that the brand attributes are stable over time. This assumption has limited empirical applications of the static factor-analytic choice model to mature product categories for which the researcher can be reasonably certain that the brand attributes are static over the observation period. Our dynamic model allows researchers to investigate brand positioning in new product categories or mature categories affected by structural change. We argue that even for stable and mature product categories, the assumption of stable brand attributes may be a strong assumption. The marketing and economics literatures have shown that even once learning has occurred in new product markets, advertising and other marketing efforts continue to play a dynamic persuasive role in individual choice. Furthermore, many CPG products are differentiated not by objective quality, but primarily by their brands (Bronnenberg et al. 2009). Thus, differentiation is defined primarily on softer (intangible) attributes rather than physical (tangible) characteristics. Given this, it seems reasonable to argue that factor-analytic choice models estimated on CPG data that find differentiation on brand attributes are likely measuring soft attributes. If factor-analytic choice models measure hard, objective product attributes that do not vary over time, the static assumption is perhaps more defensible. However, if these models are capturing differentiation based on softer attributes, the static assumption seems untenable as firms continually invest in advertising and marketing communications in an attempt to position and reposition their brands in the competitive landscape.

With enough data per unit of time, the researcher may estimate a sequence of static snapshots to capture dynamic in brand attributes over time. However, sparse data are more likely to be the norm. In this case, an efficient modeling solution is required. Our proposed model uses a forward-filtering, backward-sampling algorithm to model latent brand attributes as an evolving state-dependent process. More specifically, we model individual- and time-specific brand

attributes as arising from dynamic mean brand attributes. These means are specified as first-order autoregressive processes, capturing the idea that changes in a brand's attribute do not occur instantaneously, but rather evolve over time. The dynamic brand attributes and means are estimated in a Bayesian DLM framework nested within a factor-analytic choice model.

We investigate the properties of our dynamic factor-analytic choice model with a simulation study. We simulate data according to a variety of evolutionary scenarios, including a lack of evolution (the stationary scenario). It is of interest to ensure that the dynamic model does not impute dynamic behavior where none exists. We also investigate a variety of dynamic scenarios. In all cases, the dynamic factor-analytic choice model ably captures the behavior of the latent brand attributes. We also investigate the effects of misspecified dynamics in the brand attributes. Analytically, we show that misspecified brand attribute dynamics induce temporal heteroskedasticity and correlation between the preference weights and the error term. A nonconstant error variance affects parameter interpretation as probit model parameters are identified relative to the error variance.¹⁹ Furthermore, the correlation between the weights and the error term may bias estimates of the brand attributes. In a second simulation study, we estimate static factor-analytic choice models with dynamic data. Our results demonstrate that estimates of the brand attributes and the marketing mix coefficients are adversely impacted by misspecified dynamics.

We apply the dynamic factor-analytic choice model to a panel data set on household purchases in the malt beverage category. The category is well suited to exploring dynamics as it is a relatively new and turbulent category. We compare our proposed model with competing standard and factor analytic probit choice models. Within each type (e.g., standard and factor analytic), we estimate static and dynamic models. We find that our proposed model, which captures the evolutionary behavior of the latent brand attributes, performs best in terms of in-sample and out-of-sample fit. Although factor-analytic choice models are often used to describe market structure, market structure has also been captured by patterns of cross-price response. We compare our proposed model with a static factor-analytic choice model in terms of both own- and cross-price response. The elasticities implied by the static factor-analytic model understate the own-price elasticity for the market share leader and overstate the own-price elasticities for the smaller market share brands. In terms of clout and vulnerability, we also find pronounced differences across the static and dynamic factor-analytic

models for the smaller market share brands. Given that our proposed model has the best fit, these results imply that the static factor-analytic model yields misleading results with respect to both own- and cross-price response.

We also consider two extensions to our proposed model, allowing for dynamic importance weights and investigating the role of advertising. Although economic theory, for the most part, assumes stable consumer preferences, the consumer behavior literature has questioned this assumption. We extend our proposed model to account for dynamics in the importance weights as well as the attributes. We find that the model with static importance weights better fits the data. In terms of advertising, we extend our proposed model to allow advertising to affect the dynamic attributes. We find that advertising spending explains the evolution of brand attributes for some of our brands across both dimensions. We also estimate the ultimate impact of changes in advertising spending on market shares under our proposed model and find that increases in advertising spending benefit higher-equity brands disproportionately.

In closing, we discuss some limitations of our model and some possibilities in terms of future research. First, our empirical application considers a relatively new category. It would be straightforward and of interest to apply the model to a number of stable and mature categories to assess the robustness of the assumption of stable attributes. Second, our model treats the attribute space as fixed over time. Entry or exit may cause the attribute space to expand or shrink. Also, repositioning may dynamically affect the dimension of the attribute space. To accommodate changing dimensionality of the attribute space, the number of dimensions could be included as a dynamic parameter in the model. However, this would significantly complicate the application of the DLM to the problem, as the dimension of the latent attribute space could shrink or expand from period to period with relatively little information per period to inform the estimate. Third, we do not model the firm production process with respect to the latent attributes. Arguably, the firm combines product characteristics and marketing efforts to produce profit-maximizing attributes (Hauser and Simmie 1981). Ultimately though, consumers observe firm choices with respect to product characteristics and marketing effort and engage in their own attribute production process (Stigler and Becker 1977). Thus, the econometrician would need to specify firm and consumer roles in the joint production of the brand attributes. Notably, this may require jointly modeling of interdependent supply-side decisions and demand-side responses dynamically in a nonstationary environment. As noted in Van Heerde et al. (2004), this would be a considerable modeling challenge.

¹⁹ This is, of course, also true for a logit choice model.

Appendix

We detail the sampler used to estimate the dynamic factor-analytic choice model described in Equation (9).

$$\begin{aligned} U_{ijt} &| w_i, a_{jq}, c_{ij}, \beta_i, x_{ijt}, y_{ijt}, \\ w_i &| \mu_w, \\ a_q &| \Delta, a_{q-1}, \Omega, \\ c_{ij} &| \sigma^2, \\ \beta_i &| \bar{\beta}, \Sigma_\beta. \end{aligned}$$

Recall that we observe $i = 1, \dots, I$ individuals choosing from among $j = 0, \dots, J$ alternatives on each of $t = 1, \dots, T$ choice occasions. Each choice occasion, t , can be uniquely assigned to a month, q , where $q = 1, \dots, Q$ (see §5 for details). The dimension of the vector x_{ijt} is given by L , while the dimension of the latent brand attribute space is given by K . Note that $a_{ijq}^k = a_{jq}^k + c_{ij}^k$.

(1) Generate $U_{ijt} | w_i, a_{ijq}, \beta_i, x_{ijt}, y_{ijt}$ and $\sigma_\varepsilon^2 = 1$ for $i = 1, \dots, I$ and $t = 1, \dots, T$.

Start with $j = 1$:

if $y_{ijt} = 1$, then

$$U_{ijt} \sim \text{TN}(a_{ijq}^1 w_i^1 + a_{ijq}^2 w_i^2 + x'_{ijt} \beta_i, 1, U_{ijt} > U_{imt} \forall m \neq j);$$

if $y_{ijt} = 0$, then

$$U_{ijt} \sim \text{TN}(a_{ijq}^1 w_i^1 + a_{ijq}^2 w_i^2 + x'_{ijt} \beta_i, 1, U_{ijt} < \max(U_{imt}) \forall m \neq j)$$

increment j and return to the top.

(2) Generate $\beta_i | X_i, U_i$.

$$\beta_i | X_i, U_i \sim \text{MVN}(b, [X_i' X_i + \Sigma_\beta^{-1}]^{-1}),$$

$$b = [X_i' X_i + \Sigma_\beta^{-1}]^{-1} [X_i' \tilde{U}_i + \Sigma_\beta^{-1} \bar{\beta}],$$

$$\tilde{U}_{ijt} = U_{ijt} - [a_{ijq}^1 w_i^1 + a_{ijq}^2 w_i^2],$$

where \tilde{U}_i is a $JT \times 1$ vector consisting of the elements of \tilde{U}_{ijt} .

(3) Generate $\bar{\beta} | \beta_i, \Sigma_\beta$.

$$\bar{\beta} | \beta_i, \Sigma_\beta \sim \text{MVN}(\bar{b}, [(\Sigma_\beta^0)^{-1} + N(\Sigma_\beta)^{-1}]^{-1}),$$

$$\bar{b} = [(\Sigma_\beta^0)^{-1} + N(\Sigma_\beta)^{-1}]^{-1} [(\Sigma_\beta^0)^{-1} \bar{\beta}^0 + N(\Sigma_\beta)^{-1} \left(\frac{1}{N} \sum_{i=1}^N \theta_i \right)],$$

$$\Sigma_\beta^0 = 10^6 \times I_L,$$

$$\bar{\beta}^0 = 0_L.$$

(4) Generate $\Sigma_\beta | \beta_i, \bar{\beta}$.

$$\Sigma_\beta | \beta_i, \bar{\beta} \sim \text{IW} \left(v + N, \left(S + \sum_{i=1}^N (\beta_i - \bar{\beta})(\beta_i - \bar{\beta})' \right) \right),$$

$$v = 2 + L,$$

$$S = v I_L.$$

(5) Generate $w_i^k | a_{ijq}^k, U_{ijt}$, for $k = 1, 2$.

$$w_i^k | \mu_w \sim N \left[\left(\sum_t \sum_j (a_{ijq}^k)^2 + 1 \right)^{-1} \left(\sum_t \sum_j \tilde{U}_{ijt}^k a_{ijq}^k + \mu_w \right), \right.$$

$$\left. \left(\sum_t \sum_j (a_{ijq}^k)^2 + 1 \right)^{-1} \right],$$

$$\tilde{U}_{ijt}^k = U_{ijt} - [a_{ijq}^k w_i^k - x'_{ijt} \beta_i].$$

(6) Generate $\mu_w | w_i^1, w_i^2$.

Given the constraint $\mu_w^k = \mu_w \geq 0 \forall k$, we use a truncated normal prior for μ_w . The full conditional distribution is expressed as

$$\begin{aligned} \mu_w | w_i^1, w_i^2 &\propto \exp \left[-\frac{1}{2} \sum_{z=1}^{KN} (\tilde{w}_z - \mu_w)^2 \right] \times \frac{(1/\sigma^0) \phi((\mu_w - \mu_w^0)/\sigma^0)}{1 - \Phi(\alpha)}, \\ \mu_w^0 &= 0, \\ \sigma^0 &= \sqrt{10^6}, \end{aligned}$$

with $\alpha = ((0 - \mu_w^0)/\sigma^0)$, and \tilde{w}_z is the z th element of $\text{vec}(w)$.

We use a random-walk Metropolis-Hastings algorithm to generate draws from this nonstandard distribution.

(7) Generate $c_{ij}^k | \sigma_k^2$.

$$c_{ij}^k | \sigma_k^2 \sim N(c_j, [W_i^k W_i^k + \sigma_k^2]^{-1}),$$

$$c_j = [W_i^k W_i^k + \sigma_k^2]^{-1} [W_i^k \tilde{U}_{ij}^k],$$

$$\tilde{U}_{ij}^k = U_{ijt} - [a_{jq}^1 w_i^1 - a_{jq}^2 w_i^2 - c_{ij}^k w_i^k - x'_{ijt} \beta_i],$$

where \tilde{U}_{ij}^k is a $T \times 1$ vector consisting of the elements of \tilde{U}_{ijt}^k , and W_i^k is a $T \times 1$ vector with w_i^k in each row.

(8) Generate $\sigma_k^2 | c_{ij}^k$ for $k = 1, 2$.

$$\sigma_k^2 | c_{ij}^k \sim \text{IG} \left(c_\sigma + \frac{1}{2} \sum_i (c_i^k)' (c_i^k), d_\sigma + \frac{NJ}{2} \right),$$

where $c_\sigma = 2 + LJ$ and $d_\sigma = 1$ and c_i^k consists of the elements of c_{ij}^k stacked into a $J \times 1$ vector.

(9) Generate $a_{jq} | U_{ijt}, a_{jq-1}$.

We estimate the latent dynamic mean brand positions using a DLM (for further details on the Bayesian DLM, please see West and Harrison 1997). The observation equation of our DLM matches the observed outcomes—in our case, the heterogeneous brand attributes—with the latent dynamic variables of interest—in our case, the latent dynamic mean brand positions. The observation equation is given by

$$\tilde{U}_{ijt}^k = a_{jq}^k w_i^k + \varepsilon_{ijt}, \quad (13)$$

where $\tilde{U}_{ijt}^k = U_{ijt} - a_{jq}^k w_i^k - c_{ij}^k w_i^k - x'_{ijt} \beta_i$ and $k = 1, 2$.

The transformation equation described the evolution of the mean brand attribute level as a naturally state-dependent process and is given by

$$a_{jq}^k = \delta_j^k a_{jq-1}^k + \omega_{jq}^k. \quad (14)$$

To facilitate estimation, we stack the latent dynamic means into a vector, \bar{A}_q , and collect each δ_j^k into the diagonal matrix Δ , such that

$$\bar{A}_q = \begin{pmatrix} a_{1q}^1 \\ \vdots \\ a_{1q}^1 \\ \vdots \\ a_{1q}^K \\ \vdots \\ a_{Jq}^K \end{pmatrix} \quad \text{and}$$

$$\Delta = \text{diag}[\delta_1^1 \quad \dots \quad \delta_J^1 \quad \dots \quad \delta_1^K \quad \dots \quad \delta_J^K].$$

Similarly, we collect the elements of \tilde{U}_{ijt}^k for the purchase occasions that occur in month q into the vector \tilde{U}_{iq} :

$$\tilde{U}_{iq} = \begin{pmatrix} \tilde{U}_{11t \in q}^1 \\ \vdots \\ \tilde{U}_{1j1t \in q}^1 \\ \vdots \\ \tilde{U}_{11T \in q}^K \\ \vdots \\ \tilde{U}_{1jT \in q}^K \end{pmatrix} \quad \text{and} \quad \tilde{U}_q = \begin{pmatrix} \tilde{U}_{1q} \\ \vdots \\ \tilde{U}_{Nq} \end{pmatrix}.$$

To match the \bar{A}_q onto \tilde{U}_q , we require a mapping matrix. A complication arises because of the collection of weekly purchase occasions into the appropriate quarter; \tilde{U}_q will be of differing lengths for different quarters depending on the number of weekly purchase occasions occurring within the quarter. Thus, the mapping matrix F_q will also be of differing size for each quarter. The matrix F_q contains the appropriate w_i^k according to Equation (13). The covariance matrix Σ_ε is given by $I_{S_q} \otimes [v \otimes I_K]$, where $v = I_j$ and S_q is number of weekly purchase occasions occurring within the quarter. We now rewrite the observation and state equations for our DLM in matrix form:

$$\begin{aligned} \tilde{U}_q &= F_q \bar{A}_q + \varepsilon_q, \\ \bar{A}_q &= \Delta \bar{A}_{q-1} + \omega_q, \end{aligned} \tag{15}$$

where $\varepsilon_q \sim \text{MVN}(0, \Sigma_\varepsilon)$ and $\omega_q \sim \text{MVN}(0, \Omega)$. We use a forward-filtering, backward-smoothing algorithm (West and Harrison 1997) to sample the dynamic latent means, \bar{A}_q . Conditional on $A_q, F, \Sigma_v, \Delta, \Omega$, the distribution of $\bar{A} = \{\bar{A}_1, \dots, \bar{A}_Q\}$, $p(\bar{A} | A, F, \Sigma_v, \Delta, \Omega)$ follows the standard normal DLM with a known covariance matrix, where $A = \{A_1, \dots, A_Q\}$. The sampler has the following two steps.

Step 1. We sample \bar{A}_q from the posterior distribution $p(\bar{A}_q | D_q)$ based on the forward-filtering algorithm (see West and Harrison 1997) for all $q = 1, \dots, Q$, where D_q is the set of all information available at quarter q .

Step 2. We sample \bar{A}_{q-1} from the posterior distribution $p(\bar{A}_{q-1} | D_q)$ based on the backward-smoothing algorithm (see West and Harrison 1997) for all $q = 1, \dots, Q$. The resulting draws from Steps 1 and 2, $\bar{A} = \{\bar{A}_1, \dots, \bar{A}_Q\}$, are draws from the full posterior distribution of the latent dynamic means.

(10) Generate $\Delta | \bar{A}, \bar{A}_0, \Omega$.

We draw each diagonal element of Δ one by one.

$$\begin{aligned} \delta_j^k | \bar{A}^k, \bar{A}_0^k, \omega_j^k &\sim \text{N} \left[\bar{\delta}_j^k, \left(\frac{1}{\sigma_\delta^0} + \frac{1}{\sigma_{\omega_{kj}}^2} \tilde{A}_{j0}^{k'} \tilde{A}_{j0}^k \right)^{-1} \right], \\ \bar{\delta}_j^k &= \left(\frac{1}{\sigma_\delta^0} + \frac{1}{\sigma_{\omega_{kj}}^2} \tilde{A}_{j0}^{k'} \tilde{A}_{j0}^k \right)^{-1} \left(\frac{\bar{\delta}_j^0}{\sigma_\delta^0} + \frac{1}{\sigma_{\omega_{kj}}^2} \tilde{A}_{j0}^{k'} \tilde{A}_j^k \right), \\ \bar{\delta}_j^0 &= 0, \\ \sigma_\delta^0 &= 10^6. \end{aligned}$$

The vector \tilde{A}_j^k is the stacked vector of the elements of \tilde{A}_{jq}^k from $q = 1, \dots, Q$, and \tilde{A}_{j0}^k is the vector with \tilde{A}_{j0}^k in the first position and the stacked elements of \tilde{A}_{jq}^k from

$q = 1, \dots, Q - 1$ in the remaining positions. The scalar $\sigma_{\omega_{kj}}^2$ is the appropriate diagonal element of Ω .

(11) Generate $\Omega | a_q, a_{q-1}, \Delta$.

We draw each diagonal element of Ω one by one

$$\sigma_{\omega_{kj}}^2 | a_{jq}^k, a_{ijq}^k \sim \text{IG} \left(c_\sigma + \frac{1}{2} (\tilde{A}_j^k - \delta_j^k \tilde{A}_{j0}^k)' (\tilde{A}_j^k - \delta_j^k \tilde{A}_{j0}^k), d_\sigma + \frac{Q}{2} \right),$$

where $c_\sigma = 3$ and $d_\sigma = 1$. The vectors \tilde{A}_j^k and \tilde{A}_{j0}^k are defined as in Step 10.

We now consider the dynamic importance weights described in Equations (10) and (11). We detail the Metropolis step used to estimate the dynamic mean of the importance weights. Recall that $w_{iq}^k = w_q^k + v_i^k$, where $v_i^k \sim \text{N}(0, 1)$ for $k = 1, 2$, $w_q^k = w_q \forall k$, and $w_q^* \sim \text{N}(\lambda w_{q-1}^*, \sigma_w^2)$, where $\ln(w_q) = w_q^*$. We require the posterior distribution for $q = 1, 1 < q < Q$, and $q = Q$. The dynamic mean at $q = 1$ is given the prior distribution $w_1^* \sim \text{N}(\bar{w}_1, \sigma_w^2)$. Denote the probability that $y_{ijt} = 1$ as P_{ijt} and the choice probabilities belonging to month q as $P_{ijt \in q}$.

(12) Generate $w_1 | a_{ij1}, w_2, v_i, \lambda, \sigma_w^2, \beta_i$.

The choice likelihood for $q = 1$ is $\prod_i \prod_j \prod_{t \in 1} P_{ijt \in 1}$. For our probit model, this likelihood can be computed via the GHK algorithm. The likelihood for w_2^* is $w_2^* \sim \text{N}(\lambda w_1^*, \sigma_w^2)$. Finally, the prior for w_1^* is $w_1^* \sim \text{N}(\bar{w}_1, \sigma_w^2)$. The mixture of probit and log-normal data densities and the log-normal prior density results in a nonconjugate posterior for w_1 . We use a random-walk Metropolis algorithm to sample from the posterior.

(13) Generate $w_q | a_{ijq}, w_{q-1}, w_{q+1}, v_i, \lambda, \sigma_w^2, \beta_i$ for $1 < q < Q$.

The choice likelihood for q is $\prod_i \prod_j \prod_{t \in q} P_{ijt \in q}$. For our probit model, this likelihood can be computed via the GHK algorithm. The likelihood for w_{q+1}^* is $w_{q+1}^* \sim \text{N}(\lambda w_q^*, \sigma_w^2)$. Finally, the prior for w_q^* is $w_q^* \sim \text{N}(\lambda w_{q-1}^*, \sigma_w^2)$. The mixture of probit and log-normal data densities and the log-normal prior density results in a nonconjugate posterior for w_q . We use a random-walk Metropolis algorithm to sample from the posterior.

(14) Generate $w_Q | a_{ijQ}, w_{Q-1}, v_i, \lambda, \sigma_w^2, \beta_i$.

The choice likelihood for Q is $\prod_i \prod_j \prod_{t \in Q} P_{ijt \in Q}$. For our probit model, this likelihood can be computed via the GHK algorithm. The prior for w_Q^* is $w_Q^* \sim \text{N}(\lambda w_{Q-1}^*, \sigma_w^2)$. The mixture of the probit data density and the log-normal prior density results in a nonconjugate posterior for w_Q . We use a random-walk Metropolis algorithm to sample from the posterior.

References

Ackerberg, D. A. 2001. Empirically distinguishing informative and prestige effects of advertising. *RAND J. Econom.* 32(2) 100–118.

Ailawadi, K. L., D. R. Lehmann, S. A. Neslin. 2003. Revenue premium as an outcome measure of brand equity. *J. Marketing* 67(4) 1–17.

Ariely, D. 2008. *Predictably Irrational: The Hidden Forces That Shape Our Decisions*. Harper Collins, New York.

Ariely, D., G. Lowenstein, D. Prelec. 2006. Tom Sawyer and the construction of value. *J. Econom. Behav. Organ.* 60(1) 1–10.

- Bass, F., N. Bruce, S. Majundar, B. P. S. Murthi. 2007. Wearout effects of different advertising themes: A dynamic Bayesian model of the advertising-sales relationship. *Marketing Sci.* **26**(2) 179–195.
- Beirne, M. 2008. Bud Light touts “drinkability.” *Adweek* (October 3), http://www.adweek.com/aw/content_display/news/agency/e3i149edf3e04ae0168b00cab13b4836a92.
- Bronnenberg, B. J., M. W. Kruger, C. F. Mela. 2008. The IRI marketing data set. *Marketing Sci.* **27**(4) 745–748.
- Bronnenberg, B. J., S. K. Dhar, J.-P. Dubé. 2009. Brand history, geography, and the persistence of CPG brand shares. *J. Political Econom.* **117**(1) 87–115.
- Chintagunta, P., D. C. Jain, N. J. Vilcassim. 1991. Investigating heterogeneity in brand preferences in logit models for panel data. *J. Marketing Res.* **28**(4) 417–428.
- Chintagunta, P. K. 1994. Heterogeneous logit model implications for brand positioning. *J. Marketing Res.* **31**(2) 304–311.
- Cooper, L. G. 1988. Competitive maps: The structure underlying asymmetric cross elasticities. *Management Sci.* **34**(6) 707–723.
- Elrod, T. 1988. Choice map: Inferring a product-market map from panel data. *Marketing Sci.* **7**(1) 21–40.
- Elrod, T., M. P. Keane. 1995. A factor-analytic probit model for representing the market structure in panel data. *J. Marketing Res.* **32**(1) 1–16.
- Erdem, T. 1996. A dynamic analysis of market structure based on panel data. *Marketing Sci.* **15**(4) 359–378.
- Erdem, T., R. S. Winer. 1999. Econometric modeling of competition: A multi-category choice-based mapping approach. *J. Econometrics* **89**(1–2) 159–175.
- Fader, P. S., B. G. S. Hardie. 1996. Modeling consumer choice among SKUs. *J. Marketing Res.* **33**(4) 442–52.
- Griliches, Z., A. Yatchew. 1985. Specification error in probit models. *Rev. Econom. Statist.* **67**(1) 134–139.
- Hauser, J. R., P. Simmie. 1981. Profit maximizing perceptual positions: An integrated theory for the selection of product features and price. *Management Sci.* **27**(1) 33–56.
- Inman, J. J., J. Park, A. Sinha. 2008. A dynamic choice map approach to modeling attribute-level varied behavior among stockkeeping units. *J. Marketing Res.* **45**(1) 94–103.
- Kamakura, W. A., G. J. Russell. 1989. A probabilistic choice model for market segmentation and elasticity structure. *J. Marketing Res.* **26**(4) 379–390.
- Keller, K. L. 2003. *Strategic Brand Management: Building, Measuring, and Managing Brand Equity*. Prentice Hall, Upper Saddle River, NJ.
- Kotler, P., K. L. Keller. 2006. *Marketing Management*. Prentice Hall, Upper Saddle River, NJ.
- Kort, P. M., J. P. Caulkins, R. F. Hartl, G. Feichtinger. 2006. Brand image and brand dilution in the fashion industry. *Automatica* **42**(8) 1363–1370.
- Kuksov, D., J. M. Villas-Boas. 2008. Endogeneity and individual consumer choice. *J. Marketing Res.* **45**(6) 702–714.
- Lachaab, M., A. Ansari, K. Jedidi, A. Trabelsi. 2006. Modeling preference evolution in discrete choice models: A Bayesian state-space approach. *Quant. Marketing Econom.* **4**(1) 57–81.
- Liechty, C., D. K. H. Fong, W. S. DeSarbo. 2005. Dynamic models with individual level heterogeneity applied to evolution during conjoint studies. *Marketing Sci.* **24**(2) 285–293.
- Narayanan, S., P. Manchanda. 2009. Heterogeneous learning and the targeting of marketing communication for new products. *Marketing Sci.* **28**(3) 424–441.
- Nerlove, M., K. J. Arrow. 1962. Optimal advertising policy under dynamic conditions. *Economica* **29**(114) 129–142.
- Raftery, A. E., S. M. Lewis. 1995. Implementing MCMC. W. R. Gilks, S. Richardson, D. J. Spiegelhalter, eds. *Markov Chain Monte Carlo in Practice*. Chapman & Hall, London, 115–130.
- Slotegraaf, R. J., K. Pauwels. 2008. The impact of brand equity and innovation on the long-term effectiveness of promotions. *J. Marketing Res.* **45**(3) 293–306.
- Stigler, G. J., G. S. Becker. 1977. De gustibus non est disputandum. *Amer. Econom. Rev.* **67**(2) 76–90.
- Swait, J., J. Louviere. 1993. The role of the scale parameter in the estimation and comparison of multinomial logit models. *J. Marketing Res.* **30**(3) 305–314.
- U.S. Food and Drug Administration (FDA). 2009. Letter to General Mills, Inc. (May 5), FDA, Washington, DC. <http://www.fda.gov/ICECI/EnforcementActions/WarningLetters/ucm162943.htm>.
- Van Heerde, H. J., C. F. Mela, P. Manchanda. 2004. The dynamic effect of innovation on market structure. *J. Marketing Res.* **41**(2) 166–183.
- West, M., J. Harrison. 1997. *Bayesian Forecasting and Dynamic Models*. Springer, New York.