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Estimating the effect of brand beliefs on brand evaluations when beliefs are measured with error

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ABSTRACT

We consider the internal validity of estimates of the effects of brand beliefs on brand evaluations when beliefs are measured with error. Consumer research suggests numerous errors that may impact belief measures. However, the literature has not determined precisely why and how myriad types of error matter for the evaluation-belief relationship. Furthermore, the literature has not explicitly considered what is necessary and sufficient to control for different types of belief error when using the latent general factor approach. We show that the important distinction for empirical research is not the origin of the error per se but its relationship to affective evaluation. Error related to brand evaluation has an inflationary effect on estimates of the evaluation-belief relationship while error unrelated to brand evaluation has an attenuating effect. We use a bifactor structural equations model to decompose belief measures into general and specific dimensions. The model uses bias free variation in specific beliefs to identify effects on brand evaluation while controlling for a general belief dimension correlated with evaluation. Compared to models that do not adjust for the bias, estimates of the bias corrected marginal effects are smaller but positive and significant.

1. Introduction

Marketing managers routinely take measures of mindset metrics to monitor brand image associations, as well as overall affect for the brand. Kotler (2003) argues that mindset metrics are important lead indicators of market performance. Academic research has shown that attention to metrics capturing attitude, such as brand liking, is indeed warranted due to the impact on sales and choice behavior (Bruce, Peters, and Naik 2012; Srinivasan, Vanhuele, and Pauwels 2010; Horsky, Misra, and Nelson 2006). While the link from attitude to purchase behavior is undoubtedly important, managers also seek to understand how putatively antecedent mindset metrics, such as brand beliefs, relate to overall evaluative mindset metrics, such as brand affect or brand attitude. For example, understanding the relationship between specific brand image beliefs (e.g., functional versus hedonic imagery) and brand attitude (e.g., liking, consideration or opinion) is a central component of most customer-based brand equity models and brand management strategies (Sharma and Kumar 2006; Keller 2001). The classic multi-attribute model formalizes the notion that consumers hold specific beliefs about a target object which determine overall evaluative judgements (Nakanishi and Bettman 1974). While conceptually straightforward, the empirical

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analysis of the multi-attribute model has proven to be fraught with problems (Beckwith and Lehmann 1975; Podsakoff et al. 2003).

A structural equation model (SEM) provides a natural way to specify an empirical multi-attribute model of overall evaluation and brand beliefs (Iacobucci 2009; Bagozzi 2010; Bentler 2010).[1] If, however, the belief dimensions are highly correlated the statistical identification of the model coefficients is complicated. As an illustrative example, consider our data on consumer beliefs about three brands in the luxury automobile category. For each brand, eight items measure beliefs about functional and hedonic brand image associations with four items corresponding to each belief dimension.[2] Fig. 1 depicts the factor scores estimated from a confirmatory factor model applied to the data. The scores on functional and hedonic brand imagery are very highly correlated; posterior mean estimates of the correlation coefficients exceed 0.80. This presents a challenge to the manager seeking to quantify the effect of functional and hedonic brand imagery on brand evaluation. Multivariate regression, in theory, estimates the impact of an independent variable on a dependent variable, holding all other independent variables constant. With a correlation coefficient in excess of 0.8, there is little independent variation in the data with which to estimate such an effect.[3] Grewal et al. (2004) find that such high multi-collinearity poses a material threat to theory testing in SEM.

High multi-collinearity in factor scores may be symptomatic of a general factor permeating the specific brand belief factor scores (Ansari and Jedidi, 2000). One potential source of a general factor is that measures of brand beliefs may merely reflect overall affect towards the brand. This type of error is typically termed "halo" error in the literature (Beckwith and Lehmann 1975). Another potential source is systematic cognitive error, potentially stemming from heuristics or implicit theories not necessarily affective in nature (Podsakoff et al. 2003). A long stream of papers in marketing and consumer behavior have attempted to address the problem of biased beliefs (Beckwith and Lehmann 1975; Reibstein et al. 1980; Holbrook 1983; Dillon et al. 1984; Huber and Holbrook 1979; Huber and James 1978). Much of this literature concerns itself with correcting belief measures putatively biased by affective halo error. Podsakoff et al. (2003) catalogue a host of potential sources of error, including affective and non-affective errors that may bias belief measures. The wide variety of affective and non-affective errors speaks to the challenges faced by researchers in estimating the relationship between specific dimensions of brand beliefs and brand evaluation. In cases where the source of the bias cannot be clearly identified a priori Podsakoff et al. (2003) advocate for using a general factor, or bi-factor, model. Using constrained components analysis Dillon et al. (2001) estimate a model similar in spirit to a bi-factor model to decompose brand ratings into brand specific associations and a general brand impression.

Two important research gaps exist in terms of modeling the effect of brand beliefs on evaluations when the beliefs measures are prone to error. First, the general factor in a bi-factor model captures the amalgamation of myriad potential biases present in the data that stem from common methods, fundamental features of consumer psychology and other possible sources. However, the literature has not precisely clarified the impact of disparate sources of common variance captured in a general factor on estimates of the evaluation-belief relationship. This is important as the nature of the error has implications for how to control potential bias in model estimates, which gives rise to the second gap. The literature is incomplete on how best to specify a general belief dimension to control for bias in a bi-factor model of the evaluation-belief relationship. Extant research has addressed this question with a post-hoc two-step model of affective evaluation as a function of both general and specific belief dimensions (Dillon et al. 2001). The two-step approach introduces additional measurement error in that it treats both the latent general and specific dimensions as data. Furthermore, modeling overall affective response as a function of the general factor suggests that the general factor gives rise to affective response. In contrast, consumer behavior theories, such as affective attribute substitution, (Kahneman and Frederick 2002) suggest that an overall affective response spills over into measures of specific belief dimensions giving rise to a common source of variance across distinct belief dimensions.

In this paper, we investigate the impact of biased belief measures on affective brand evaluations using a secondary data set on brand image provided by a cooperating firm. Our study makes two substantive contributions to the marketing literature on brand beliefs and brand evaluations. First, we show that from the vantage of empirical estimation of the evaluation-belief relationship the important distinction is not necessarily the origin of the belief error. A simulation study in conjunction with our empirical application shows that it is the correlation (or lack thereof) between any belief error and overall evaluation that ultimately matters most. Belief error correlated with overall evaluation has an inflationary effect while belief error unrelated to consumers' overall evaluation has an attenuating effect. Second, we show the importance of simultaneous estimation of the affect-belief relationship. We model beliefs arising from general and specific dimensions and allow the general dimension to correlate with affective evaluation in a simultaneous bi-factor structural equation model. We compare our estimates to those arising from a two-step approach to estimation, which we show is prone to bias. From a methods perspective our Bayesian approach to inference also enables estimation of the bias adjusted correlation in the specific dimensions of

[1] While the general consensus seems to be that multiple item measurement is preferred to single item measurement it is not a unanimously held view (Iacobucci et al. 2007).

[2] More details on the data are available in the Data section of the manuscript. The items and the average scores on each item for each brand are reported in Table 5.

[3] In the extreme, if the latent variables are perfectly collinear the structural regression could only identify the sum of the marginal effects of each individual variable.

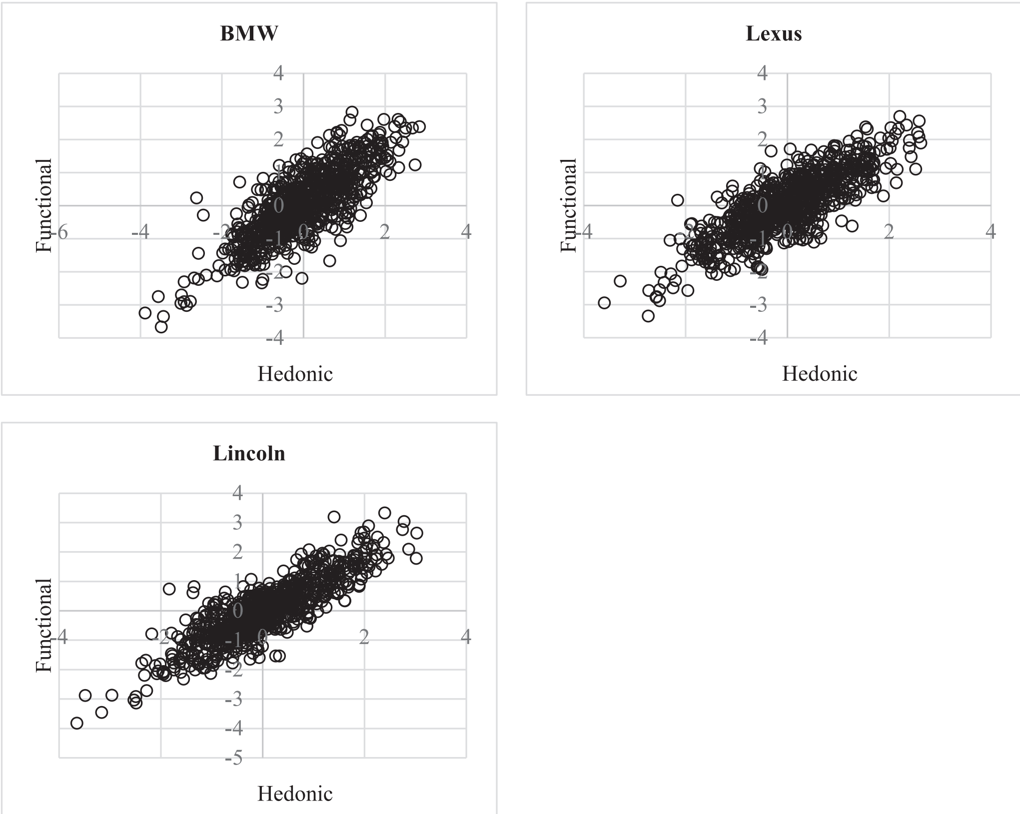

Fig. 1. Confirmatory Factor Model Scores—Brand Image Data.

brand beliefs in the context of categorical and ordinal data. In a frequentist approach to bi-factor analysis of categorical data, orthogonality of the specific dimensions reduces the likelihood evaluation to a two-dimensional integration facilitating estimation. Data augmentation allows us to estimate the correlation in the specific factors in a computationally tractable manner when dealing with categorical data, as is often the case with attitude and belief metrics.

The remaining sections of the paper proceed as follows. First, we present the conceptual foundation for our approach to analyzing the multivariate structure of the brand belief and brand evaluation data. We then present our generalized linear bi-factor structural equations modeling approach, including a discussion of specification error and bias and the development of our hypotheses. In the penultimate section we present the empirical application of our model, including a detailed description of the data and the estimation results. We conclude with a brief summary, a highlight of our intended contributions, and a note of some of the limitations of our study that may be addressed by future work.

2. Conceptual foundation

2.1. Consumer psychology of biased beliefs

Consumer belief data are often treated as direct reports of consumers' mental contents. However, these data frequently include measurements of beliefs not retrieved from long-term memory but instead constructed in response to researchers' inquiries (Feldman and Lynch 1988). The voluminous research on heuristics and biases suggests that people intuitively engage in *attribute substitution* when constructing and reporting specific beliefs; they assess target attributes by mapping the values of more easily accessible heuristic attributes onto the target scale (Kahneman and Frederick 2002). This process occurs when beliefs about a target attribute are inaccessible, an associatively related heuristic attribute is accessible, and the substitution of the heuristic attribute for the target attribute is not rejected by subsequent cognitive reflection. Since the target and heuristic attributes are different, attribute substitution imparts error in the measurement of the target belief.

At its core, attribute substitution relies on the accessibility of an associatively related heuristic attribute. Importantly, the most accessible heuristic attribute may be either cognitive or affective in nature (e.g., Kahneman and Frederick 2002; Slovic et al. 2007). Consumers asked to report their beliefs about a vehicle's reliability and performance may rely on the vehicle's exterior styling as a heuristic attribute, thereby conflating different cognitive judgments. They may also use affective reaction to the vehicle as a heuristic attribute thereby deriving beliefs from their evaluation of the vehicle.[4] These conceptually distinct varieties of attribute substitution carry different implications for estimating the relationships between specific beliefs and overall evaluations. Attribute substitution utilizing affective heuristic attributes is, almost by definition, likely to result in correlation between the overall evaluation and the error in the target belief. Attribute substitution utilizing cognitive heuristic attributes also introduces error into belief measures but the error may be less likely to be related to overall evaluation.[5]

Regardless of the specific origin of any heuristic, attribute substitution results in error seeping into the measures of the specific belief dimensions. Additionally, other biases such as common method biases may impact belief measures. In the context of a structural equation that models overall brand evaluation as a function of beliefs, the key question is whether any error in the beliefs relates to overall evaluation, either directly or indirectly, regardless of its origins. Error that is orthogonal to the overall evaluation contaminates the belief measures with noise. This is akin to the classic case of measurement error in a dependent variable, inducing regressor-error dependence that attenuates estimates of the relationship between beliefs and evaluation (Greene 2003). The magnitude of the attenuation is a direct function of the variance of the error. In contrast, error correlated with overall evaluation creates a simultaneity bias between evaluation and beliefs also resulting in regressor-error dependence. If the correlation between the error and overall evaluation is positive, this results in an upwards bias in estimates of the relationship between the specific belief dimensions and evaluation. It is possible that both processes operate simultaneously implying that some portion of any putative error may relate to overall evaluation while another portion may not. This underscores the need for careful empirical analysis. We now turn our attention to the different ways the literature has considered de-biasing measures of specific belief dimensions.

2.2. Empirical models of biased brand beliefs

Beckwith and Lehmann (1975) are among the first to note that evaluation-belief models are prone to exhibit reverse causality in the sense that direct measures of beliefs are biased by brand affect. In their model, average beliefs are assumed to be the true beliefs unbiased by the simultaneity of individual affect and beliefs. Johansson et al. (1976) argue that the Beckwith and Lehmann (1975) model is mis-specified and that average beliefs cannot be interpreted as true or unbiased beliefs. Building on the idea that beliefs and evaluations may be simultaneous, Reibstein et al. (1980) specify a structural equations model of stated behaviors, attitude, and beliefs about public transportation. They find evidence of simultaneity between stated usage behaviors and beliefs. In a similar vein Holbrook (1983) specifies a structural equations model of halo that includes simultaneity between beliefs and attitude. In contrast to much of the previous literature Holbrook (1983) finds relatively weaker effects of simultaneity between attitude and beliefs and posits that this may be due to the stimuli being manipulated via a factorial experimental design (resulting in larger objective differences across profiles to be rated). While these approaches address the possibility that belief error arises from affect, they do not account for the possibility that some of the belief error may be due to non-affective sources. Huber and James (1978) suggest a measure of bias as the residual term from a regression of the perceived level of the attribute on an objective, physical level. This limits beliefs under study to those that have a truly objective, physical state that can be properly measured (i.e., weather).

A related stream of literature addresses the problem of how to remove biases from belief ratings without requiring objective, physical attributes. Holbrook and Huber (1979) partial out attitudinal overtones from beliefs and uses the residual scores in a principal components analysis to investigate the structure of the attribute space. In contrast to much of the extant literature that finds bias corrected beliefs do not predict evaluations, Holbrook and Huber (1979) find that the bias free joint space of the attributes resulting from their partialing out strategy ably predicts evaluation. As with the previous set of approaches, a critique of this approach assumes that the cause of the bias is solely affective (or can otherwise be identified a priori). An alternative is to conduct a first stage principal components analysis (PCA) of the belief ratings and simply discard the first component in second stage analysis (Dillon 1984; Huber and Holbrook 1979). Closely related to PCA, Dillon et al. (2001) propose a constrained component analysis (CCA) measurement model to evaluate brand ratings by treating the observed ratings as arising from global and specific components. However, these approaches all treat the estimated components as error free data in second stage analyses. As such, none of these analyses consider a simultaneous measurement model of evaluation and beliefs that addresses how and why biased beliefs impact the evaluation-belief relationship. These analyses also typically treat ordinal belief ratings as continuous which is prone to biased parameter estimates and incorrect standard errors (Rhemtulla et al. 2012).

In summary, the literature to date has addressed the problem of biased beliefs either with a structural equations model that attempts to accommodate the potential simultaneity in evaluation and beliefs or with models that attempt to isolate an affective halo or general belief dimension then chaining the "de-biased" data to a second stage analysis. Our approach unifies these two strands by specifying a joint model of evaluations and beliefs that addresses the problem of brand beliefs biased by information that may or may not be related to brand evaluations. To accomplish this we specify a Bayesian generalized linear bi-factor structural equations model that treats multi-item measures of beliefs as arising from a common general belief

[4] The process of affective attribute substitution gives a more precise behavioral accounting of the mechanism behind what some researchers refer to as halo effects. Kahneman and Frederick (2002) argue that the affect heuristic belongs alongside the representativeness and availability heuristics in terms of impact on consumer decision making.

[5] We note that a cognitive heuristic attribute may indirectly induce a correlation between an overall evaluation and error if the cognitive heuristic attribute also drives an affective response.

dimension and specific belief dimensions.[6] There is a long history of bi-factor models in social science research; these models have experienced a recent renaissance among psychometric researchers (Reise 2012). The bi-factor model allows us to parse and control for information in the general belief dimension correlated with brand evaluations versus information orthogonal to brand evaluations without requiring an a priori stance on the origin of the bias. Gibbons and Hedeker (1992) and Gibbons et al. (2007) extend the bi-factor model to binomial and ordinal data. By imposing orthogonality among the specific factors, the evaluation of the likelihood reduces to a two-dimensional integration facilitating estimation in the context of categorical data. Our Bayesian approach to estimation contributes to the statistic and psychometric literatures on bi-factor analysis by showing how to estimate bias corrected estimates of the correlation in the specific belief dimensions. Table 1 summarizes the extant literature and the position of our work.

3. A model of brand evaluation and brand beliefs

We begin by describing our model of the multivariate structure of brand evaluation and brand belief data collected from a sample of consumers.[7] Consumers rate an object (i.e., on a 5 or 7 point Likert-type rating) on a set of items designed to measure an overall brand evaluation and a set of items designed to measure multiple specific dimensions of brand beliefs. We model the measurement items via generalized linear factor models with ordered probit link functions.[8]

Assume a set of categorical ratings obtained from I individual consumers on a J-dimensional vector,y_i, designed to measure a uni-dimensional affective brand evaluation. For the same set of consumers, we also observe a K-dimensional vector,x_i, designed to measure consumer beliefs. The j^{th} value of y_i resides in the set $\{1, 2, \ldots, C_j\}$ while the k^{th} value of x_i resides in the set$\{1, 2, \ldots, D_k\}$. The observed values of y_{ij} and x_{ik} are assumed to be determined by a set of latent variables $\left\{y^*_{ij}\ x^*_{ik}\right\}$ and a set of cutpoints $\{\mu^y\ \mu^x\}$ as follows

$$\begin{aligned} y_{ij} &= c \text{ if } \mu^y_{j(c-1)} < y^*_{ij} < \mu^y_{jc} \\ x_{ik} &= d \text{ if } \mu^x_{k(d-1)} < x^*_{ik} < \mu^x_{kd} \end{aligned} \tag{1}$$

For identification, we set$\mu^y_{j0} = \mu^x_{k0} = -\infty$,$\mu^y_{j1} = \mu^x_{k1} = 0$, and$\mu^y_{jC_j} = \mu^x_{kD_k} = \infty$.

In order to capture a general factor in the structure of the multi-dimensional belief data, the measurement models are written as

$$\begin{aligned} y^*_i &= \alpha^y + \Lambda\eta_i + \varepsilon^y_i \\ x^*_i &= \alpha^x + \Psi\theta_i + \Gamma\xi_i + \varepsilon^x_i \end{aligned} \tag{2}$$

The vectors α^y and α^x contain the means of the latent variables. The $J \times 1$ factor loading matrix Λ maps the scalar evaluative factor score, η_i ontoy^*_i. The measurement model for the beliefs splits the consumer beliefs into a single general dimension and multiple specific dimensions. The factor loading matrix Ψ is a $K \times 1$ matrix of general factor loadings, so called because each element of the latent x^*_i loads onto this factor. The scalar θ_i is the general belief factor score. The factor loading matrix Γ is a $K \times L$ matrix of specific belief factor loadings, where L is the dimension of the specific belief factor space and$L < K$. Each element of the latent x^*_i loads onto one of the L specific factors. The factor scores for the specific beliefs are denoted byξ_i. The vectors ε^y_i and ε^x_i are normally distributed errors each with mean zero and diagonal covariance matrices Σ^y andΣ^x, respectively.

The behavioral aspects of our model are captured by the structural equations as follows

$$\begin{aligned} \eta_i &= \beta'\xi_i + v_i \\ \xi_i &\sim MVN(0, \Upsilon) \\ \begin{bmatrix} v_i \\ \theta \end{bmatrix} &\sim MVN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_v & \sigma_{v\theta} \\ \sigma_{\theta v} & \sigma^2_\theta \end{pmatrix}\right] \end{aligned} \tag{3}$$

The effect of the bias-corrected specific belief dimensions on brand evaluation is captured by the parameter vectorβ. The matrix Υ captures the covariance in the bias-corrected specific belief dimensions. The structural equations allow for dependence between the general belief factor and the evaluation factor. The residual term from the evaluation equation and the general belief factor score are specified according to a multivariate normal distribution with covariance$\sigma_{v\theta} = \sigma_{\theta v}$. This allows for the possibility that some of the information in the general belief score is correlated with brand evaluation while the remainder is attributable to non-evaluative information.

[6] Steenkamp and Maydeu-Olivares (2021) propose a random intercept factor model that can be rewritten as model with general factor and m specific factors. Restrictions on the general factor loadings in their model results in models nested by the more general bi-factor model.

[7] The interested reader may find graphs of our proposed model and alternative models in Figs. 2-5 presented later in this section.

[8] Note that one may ignore the ordinal nature of the observed data and simply model the ratings data as a continuous variable. However, this approach is prone to biased parameter estimates and incorrect standard errors. This is especially the case if the number of categories is small (Rhemtulla et al. 2012).

Table 1
Summary of the Literature and Manuscript Position.

Method Approach	***Authors (year)***	***De-Biasing Strategy***	***Model of Beliefs and Attitude/Usage/Choice***	**Notes**
Simultaneous Equation Approaches	Beckwith and Lehmann (1975)	None	Simultaneous model of attitudes and beliefs	Models belief data as simultaneously determined with affect. Does not explicitly de-bias belief data. Does not account for non-affective errors in belief measures.
	Reibstein et al (1980)	None	Simultaneous model of beliefs, attitude, and stated usage	
	Holbrook (1983)	None	Simultaneous model of attitude and beliefs	
Partial Out Approaches	Huber and James (1978)	Measures residual between objective and perceived attribute levels.	Correlates residuals with affect in two step approach.	Require objective measures of perceptions, which is not feasible for brand beliefs such as luxury. Also uses two step approach to study belief affect relationship.
	Holbrook and Huber (1979)	Forms ad-hoc affective index and partials index out of remaining belief dimensions.	Uses partialed out belief data to predict overall evaluation in two step approach.	Uses affect measures to partial out affective overtones that seep into belief dimensions. This assumes that belief errors are only affective in nature. May over-correct data by removing all affect related information from belief measures. Also uses two step approach to study belief affect relationship.
	Huber and Holbrook (1979)	Forms ad-hoc affective index and partials index out of remaining belief dimensions.	Compares analysis of raw belief data versus belief data that has been partialed out.	
Factor Model Approaches	Dillon et al. (2001)	Constrained components model to decompose beliefs into general and specific dimensions.	Uses general and specific factor scores in two step regression of brand liking.	Specifies a general belief factor that arises from affective and/or cognitive errors. Uses two step approach to measure effect of belief dimensions on brand liking. Also specifies general factor as determining affect. This may be incorrect if affect itself causes the general factor as is posited by affective attribute substitution. Minor concerns include imposition of equality constraints in general factor loadings and treatment of categorical data as continuous.
	Sonnier and Ainslie (2011)	Generalized linear higher order factor model that treats correlated specific dimensions as arising from a single general dimension and orthogonal specific residuals.	Simultaneous estimation of brand choice and higher order factor model.	The higher order factor model is a special case of the more general bi-factor model in present study. Higher order model imposes orthogonality in specific factor score residuals and proportionality constraints in factor loadings. Specifies choice as a function of general and residual specific factor scores. This may be incorrect if affect itself causes both choice and the general factor.
	Present Study	Generalized linear bi-factor model that treats belief data as arising from a general factor and bias adjusted correlated specific factors.	Full SEM bi-factor model of attitude and beliefs.	The bifactor model captures a general source of bias, which we partition into pieces correlated with and orthogonal to affect. Using simulated data show that belief error correlated with affect results in inflationary bias while error orthogonal to affect results in attenuation bias. Shows that two-step estimation used by previous belief bias adjustment procedures are prone to error. Demonstrates estimation of bias adjusted correlated specific factors

3.1. Model identification

The model in equations (2) and (3) requires some further restrictions for identification typical to generalized linear factor models. The scale of the specific belief factors must be fixed which is accomplished by specifying Υ to be a correlation matrix. The residual variance of the overall evaluative attitude factor score, σ_v^2 is fixed to one. The variance of the general belief factor score,σ_θ^2, is also fixed to 1. The covariance of v_i and θ_i is $\sigma_{v\theta}$ and is restricted to lie on the interval $(-1, 1)$(i.e., we model the correlation between v_i and θ_i). The covariance matrix of the latent y_i^* is$\Lambda\Lambda' + \Sigma^y$. For identification, the scale of the latent y_i^* must be fixed. We accomplish this by setting the diagonal elements of the covariance matrix to 1 implying the constraint$\Sigma^y = I_J - \text{diag}[\Lambda\Lambda']$. Thus, the elements of the j^{th} row of the loading matrix Λ must lie on the interval$(-1, 1)$. The covariance matrix of the latent x_i^* is$\Psi\Psi' + \Gamma\Upsilon\Gamma' + \Sigma^x$. The scale of the latent x_i^* must also be fixed for identification. We set the diagonal elements of the covariance matrix to 1, implying the constraint$\Sigma^x = I_K - \text{diag}[\Psi\Psi' + \Gamma\Upsilon\Gamma']$. Thus, the elements of the k^{th} row of the loading matrix $[\Psi\ \Gamma]$ lie on the unit circle. Finally, the model likelihoods are invariant to sign switches across the factor loadings and scores. To remove this indeterminacy we restrict the elements of Λ to lie on the interval $(0, 1)$ and the elements of $[\Psi\ \Gamma]$ to lie on the positive quadrant of the unit circle. Following Edwards and Allenby (2003) we take draws of the conjugate but unidentified variances and co-variances and then margin down to the space of identified parameters.

In addition to the identification constraints typical to generalized linear factor models, the bi-factor model requires some additional constraints to ensure parameter identification. For the generalized linear bi-factor model, the data must contain at least two specific dimensions each containing at least three items with non-zero general factor loadings. Furthermore, if there are only two specific dimensions, the loading matrix of one of the dimensions must be able to be partitioned into two disjoint subsets of items with linearly independent general and specific factor loadings. For the generalized linear bi-factor model that allows correlation in the specific factors, we also require at least two specific dimensions but more importantly, the general and specific factor loadings must be linearly independent.(Fang, Xu, Guo, Ying, & Zhang, 2020) In Web Appendix B, we show that with the above conditions met we are able to recover all the data generating parameters, including most importantly the structural parameters, the specific factor correlations, and the correlation between the general factor and the affective factor. We show the model recovers for positive, negative, and zero correlation in the specific factor scores. We also show that when the specific and general factors are linearly related, model identification breaks down. More details are available in Web Appendix B.

3.2. The general belief dimension and brand evaluation

To assess the impact of biased beliefs it is useful to consider alternative SEM models to compare to our proposed model. Perhaps the simplest alternative model to consider is a model that contains only a single specific belief dimension and no general belief dimension. Given a high level of collinearity in factor scores, such as that shown in Fig. 1, it seems prudent to consider whether this simplest model of the belief data that uses only a single factor is sufficient to capture the structure of the data. The diagram for this model, denoted as M0, is shown in Fig. 2. Model M0 can be expanded by allowing for multiple specific dimensions of beliefs but without introducing a general belief dimension. Such a model is consistent with a standard confirmatory factor structure for the belief data. The diagram for this model, denoted as M1, is shown in Fig. 3. Model M1 can be expanded by adding the general dimension to the belief model. Model M2 adds the general dimension but sets the covariance between the general dimension and evaluation to zero. The diagram for this model is shown in Fig. 4. Our proposed model, model M3, adds a general dimension to the belief model and allows for non-zero covariance between the general dimension and attitude. The diagram for M3 is shown in Fig. 5. In each model diagram, the solid arrows denote directional relationships between either the factors and manifest items (i.e., the factor loadings in the measurement models) or the brand evaluation and specific belief factors (i.e, the structural path coefficients). The dashed arrows denote the covariance (or correlation) between factors.

Since affective evaluation is a natural assessment evoked by nearly any stimulus and possibly occurring even outside of awareness (Zajonc 1980) we might expect the existence of affective response as a matter of course. While affective error is of concern, it is also possible that some or all of any putative error in the specific belief dimensions is unrelated to affective brand evaluation. Furthermore, it is also entirely possible that cognitive errors may impact both measures of affective evaluation and specific dimensions of brand belief in a similar way (i.e., yea saying across all measures, tendencies to use the same parts of the scale across all measures) resulting in correlation between the affective evaluation and belief measures. Podsakoff et al. (2003) catalogue nearly twenty potential sources of method biases in behavioral research including errors due to scale type, response format, and general context. Bagozzi and Yi (1991) argue that common method biases may be viewed more broadly through the lens of response biases, such as halo effects, social desirability, acquiescence, leniency effects, or yea (or nay) saying. We reiterate that it is not our goal to parse each of these myriad sources of biases. As pointed out in the literature it is difficult, if not impossible, to do so (Podsakoff et al 2003). We argue that it is also unnecessary in light of assessing the evaluation-belief relationship. The more important objective is to understand the impact of errors in beliefs that may or may not be correlated with affective evaluation.

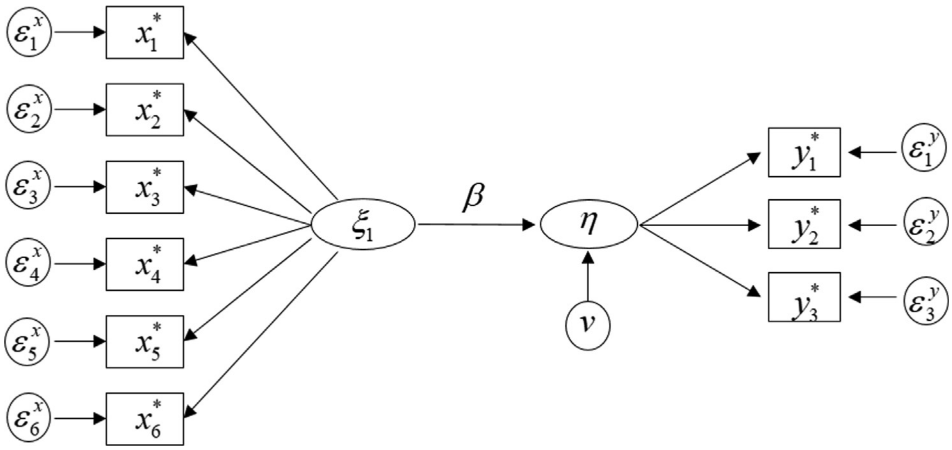

Fig. 2. Graph of Model M0: Single Belief Dimension. Model is denoted as M0 in the text. Solid lines represent direct relationships (i.e., factor loading, path coefficients or error terms) between variables. Dashed lines represent correlations between variables.

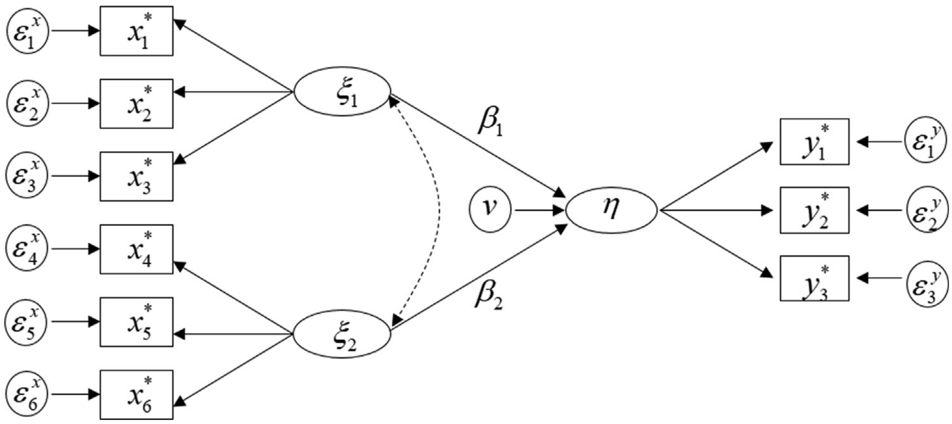

Fig. 3. Graph of Model M1: Multiple Specific Belief Factors; No General Belief Factor. Model is denoted as M1 in the text. Solid lines represent direct relationships (i.e., factor loading, path coefficients or error terms) between variables. Dashed lines represent correlations between variables.

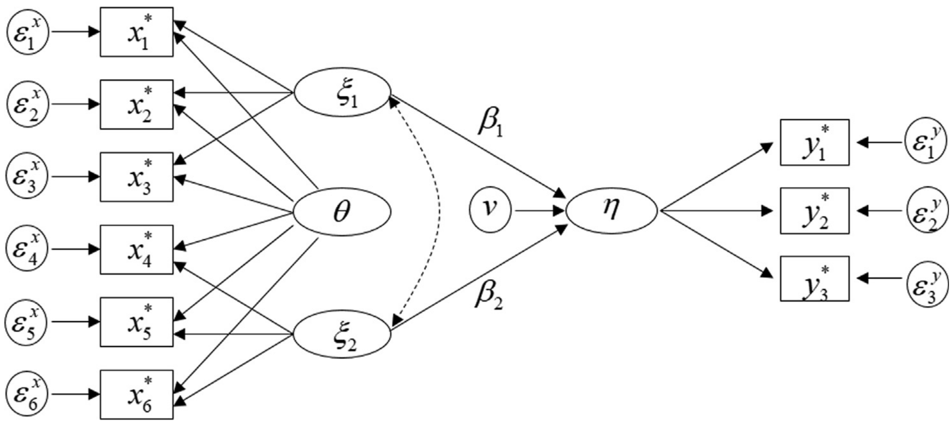

Fig. 4. Graph of Model M2: Multiple Specific Belief Factors; General Belief Factor Orthogonal to Brand Evaluation Factor. Model is denoted as M2 in the text. Solid lines represent direct relationships (i.e., factor loading, path coefficients or error terms) between variables. Dashed lines represent correlations between variables.

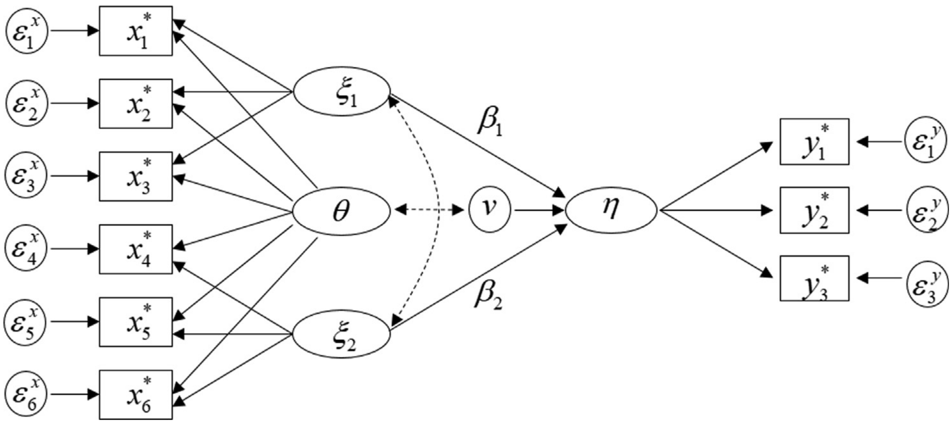

Fig. 5. Graph of Model M3: Multiple Specific Belief Factors; General Belief Factor Correlated with Brand Evaluation Factor. Model is denoted as M3 in the text. Solid lines represent direct relationships (i.e., factor loading, path coefficients or error terms) between variables. Dashed lines represent correlations between variables.

Consider the effect of incorrectly omitting the general factor from the measurement model for the beliefs as in M1. The mis-specified information from the general factor will contaminate estimates of the specific beliefs with error. If the error is independent of brand evaluation the problem is analogous to the case of measurement error in the linear regression model. Estimates ofβ, the vector of path coefficients linking specific beliefs to brand evaluation, will be attenuated and the magnitude of the attenuation is an increasing function of the variance of the error (Greene 2003). If, however, the error relates in part or whole to brand evaluation this would also induce regressor-error dependencies in terms of estimating the evaluation-belief relationship. We would expect this simultaneity bias to inflate the magnitude of the effect of brand beliefs on brand evaluation. It is, of course, possible that some of the information in the general factor is related to evaluations while some of the information is unrelated to evaluations.

Now consider specifying a general factor in the measurement model for beliefs where the correlation between the evaluation factor and general factor is set to zero, as in Model M2. It is tempting to think the mere inclusion of the general factor is sufficient to control for any error in beliefs. However, it is not sufficient to control for any error related to evaluation. The covariance parameter $\sigma_{v\theta}$ governs the strength of the relationship between the endogenous general belief and evaluation. While assuming this covariance is zero certainly simplifies model estimation, this is not a harmless assumption. Under the structural model in equation (3) the posterior distribution of the specific belief factors, ξ_i, depends on the likelihood for the evaluation factor η_i.[9] If the covariance between the general belief factor,θ_i, and the evaluation factor is not zero, then the distribution of the evaluation factor conditional on the general belief factor is $\eta_i|\theta_i\tilde{\text{N}}\left(\mu^{\eta|\theta}, \Sigma^{\eta|\theta}\right)$ where $\mu^{\eta|\theta} = \beta'\xi_i + \frac{\sigma_{v\theta}}{\sigma_\theta^2}\theta_i$ and$\Sigma^{\eta|\theta} = \sigma_v^2 - \frac{\sigma_{v\theta}^2}{\sigma_\theta^2}$. If any non-zero covariance is not properly accounted for, information from the general belief factor will be contained in the posterior distribution of the specific belief factors,ξ_i. When $\sigma_{v\theta}$ is non-zero information correlated with affective evaluation is contained in the posterior distribution of the specific belief dimensions. Thus, some or all of the effect of specific beliefs on evalution would be due to this mis-specified information. We predict the estimates of the evaluation-belief relationship from M2 compared to M1 will be larger in magnitude.

Our proposed model, M3, which contains a general dimension and allows for non-zero covariance between the general belief dimension and brand evaluation will control for both the attenuation and the simultaneity bias due to error orthogonal to and correlated with evaluation, respectively. Comparing the estimates of the evaluation-belief relationship from M3 to those from M1 is a comparison of a model that controls for both types of error (i.e., M3) versus a model that leaves both types unchecked (i.e., M1). Thus, the results will depend on how much information in the general belief score is related to evaluation. If the majority of the information in the general score is due to evaluative spillover we would expect β from M3 compared to M1 to be smaller in magnitude. If most of the information in the general score is unrelated to brand evaluation we would expect estimates of the evaluation-belief relationship from M3 compared to M1 to be larger in magnitude. Thus, it is of significant interest to consider decomposing the general factor score into the component due to evaluative and non-evaluative sources. To accomplish this, the general factor score can be written as conditional on the residual evaluation score as follows

$$\theta_i = \frac{\sigma_{v\theta}}{\sigma_v^2} v_i + \tilde{\theta}_i \text{ where } \tilde{\theta}_i\tilde{\text{N}}\left(0, \sigma_\theta^2 - \frac{\sigma_{v\theta}^2}{\sigma_v^2}\right) \tag{5}$$

[9] The Technical Appendix contains the complete expressions for the full conditional posterior distribution of these model parameters.

From equation (4) we can decompose the total variance in the general factor score into the component due to affective evaluative spillover, $\frac{\sigma^2_{v\theta}}{\sigma^2_v}$, and the component unrelated to affective evaluation, $\sigma^2_\theta - \frac{\sigma^2_{v\theta}}{\sigma^2_v}$.

It is natural to consider writing the structural equation for brand evaluation as directly dependent on the general belief dimension, $\eta_i = \beta_\theta\theta_i + \beta'_\xi\xi_i + \omega_i$. This is essentially the functional form employed by Dillon et al. (2001) in their post hoc regression analysis of the relationship between brand liking, general brand impression and brand specific associations. Dillon et al. (2001) do not use a measurement model for brand liking. Model M4, shown in Fig. 6, depicts a bi-factor SEM version of the constrained components analysis (CCA) proposed in Dillon et al. (2001). Similar to model M4, Allan et al. (2015) specify a bi-factor SEM of negative affect (NA) as a function of general and specific dimensions of anxiety sensitivity (AS).[10] In their model, NA is a linear function of the general AS factor and orthogonal specific dimensions of AS. This specification may be entirely sensible in the domain of anxiety sensitivity and negative affect. In our context the specification in M4 takes the stance that the general dimension gives rise to affective evaluation. However, the notion of "halo" effects (Beckwith and Lehmann 1975), formalized by the theory of affective attribute substitution (Kahneman and Frederick 2002), suggests overall affective evaluation substitutes into the measures of target attributes (i.e., specific beliefs). This suggests affective response gives rise to the general factor, which casts doubt on the internal validity of β_θ in Model M4. The inclusion of θ in the equation for η, though, may be sufficient to control for the correlation between affect and the general factor enabling estimation of β_ξ, the effect of the specific belief dimensions on overall affective evaluation. Even if this is so, two-stage estimation that treats the factor scores as error free data is likely biased.

In Web Appendix C, we evaluate the properties of models M1-M4. We generate synthetic data sets with a bi-factor SEM structure that captures an overall evaluative factor and set of beliefs that arise from a general belief factor and two correlated specific belief factors. For each data set, we specify the overall evaluative factor as a linear additive function of the specific belief factors. The difference in the five data sets is the degree to which the overall evaluative factor correlates with the general belief factor. We simulate data where the general belief factor is exogenously orthogonal to the overall evaluative factor or endogenously correlated with the overall evaluative factor where the correlation ranges from 0.25 to 0.9.

We find that when the data are generated with zero correlation between the general and evaluative factors, Model M1, which omits the general factor altogether, results in severe attenuation of the structural coefficients and inflated estimates of the correlation between the specific factors. As the correlation between the general and evaluative factors increases in the data generating process, we see a mix of attenuation and inflationary bias, with the inflationary bias increasing in the specific factor correlation in the data generating process. Model M2 specifies a general factor in the model orthogonal to the overall evaluation factor. With zero correlation between the general and evaluative factors, M2 recovers the structural parameters as well as the correlation in the specific factors. However, inclusion of the general factor alone is not sufficient to ameliorate bias when there is correlation between the general and evaluative factors. Model M3 specifies the general factor as correlated with the overall evaluative factor. Model M3 ably recovers the structural parameters, the correlation in the specific factors, and the correlation in the overall and evaluative factors for all data generating scenarios. Two-step estimation of Model M4 results in biased estimates of the structural coefficients and cannot be a recommended estimation strategy. Simultaneous estimation of Model 4 is able to recover the data generating parameters but requires a rescaling of the estimated parameters to do so. More details on the simulation are available in Web Appendix C.

3.3. Empirical application

Our empirical application utilizes a secondary data set provided to us by a cooperating market research firm that conducts brand tracking studies for the luxury automobile category. The data represent a slice of a larger tracking study where different cross sections of customers are surveyed over time. As the sample of consumers differ from wave to wave it is not possible to examine the data as an individual-level panel data set. This is commonly the case for tracking studies.

3.4. Automotive brand image data

According to Kantar Media the automotive category is among the leading categories in terms of the dollar volume of ad spending in the U.S. Automotive firms also routinely track the beliefs consumers hold about the imagery associated with their brands (e.g., quality, dependability, luxury, prestige, etc.) via brand tracking surveys. Such surveys typically include measures of overall evaluative attitude towards brands (e.g., opinion, consideration, pride of ownership, etc.). A key question of interest for many firms that spend heavily on advertising, especially television advertising, is the extent to which unique and favorable brand image associations drive brand evaluations (Keller 1993). Such information helps firms guide the creative content of advertising copy. However, extant research in marketing has shown the clear potential for consumer associations between automotive brands and specific dimensions of imagery to be impacted by a general dimension (Dillon et al. 2001).

The data provided to us are from a 2013 brand study of the luxury automotive category. The data capture brand image beliefs and brand evaluations amongst consumers intending to purchase a luxury sedan. In total, there are eight brand image attributes designed by the research firm to measure "hedonic" and "functional" brand imagery. In addition, the firm collects

[10] We thank an anonymous reviewer for bringing this paper to our attention.

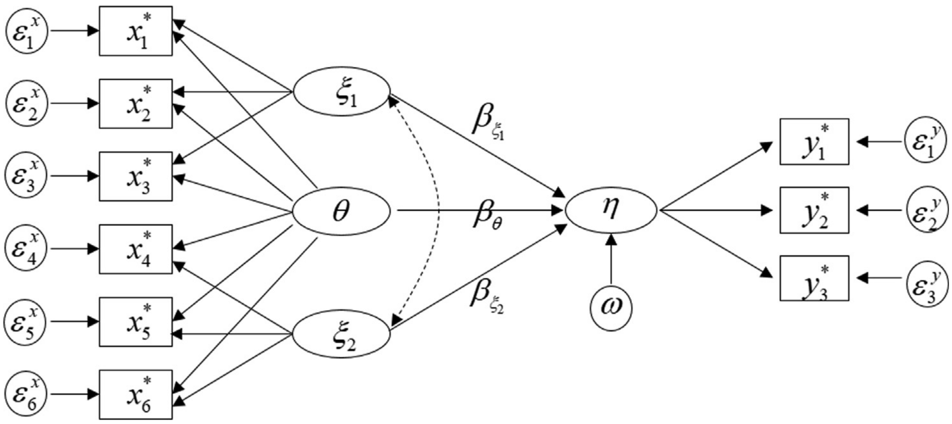

Fig. 6. Graph of Model M4: Multiple Specific Belief Factors; Brand Evaluation Factor a Direct Function of General Belief Factor. Model is denoted as M4 in the text. Solid lines represent direct relationships (i.e., factor loading, path coefficients or error terms) between variables. Dashed lines represent correlations between variables.

Table 2
Brand Image Data.

Factor	Item[a]	*BMW (n = 814)*	*Lexus (n = 830)*	*Lincoln (n = 781)*
Evaluation Factor	*Overall Opinion*	4.17	4.30	3.63
	Purchase Interest	2.70	2.90	2.19
	Proud to Own	4.29	4.25	3.61
Hedonic Factor	*Prestigious*	4.44	4.38	3.82
	Luxurious	4.22	4.34	4.03
	Distinctive	4.10	3.79	3.54
	Attractive	4.15	4.03	3.55
Functional Factor	*Quality*	4.18	4.26	3.53
	Durable	4.10	4.17	3.54
	Dependable	4.01	4.24	3.63
	Trustworthy	4.18	4.28	3.76

[a] Purchase Interest is measured on a 4 point scale. All other items measured on a 5 point scale.

measures of brand evaluation including overall opinion, purchase interest, and pride of ownership. We observe these data for three brands, BMW (a German brand), Lexus (a Japanese brand) and Lincoln (an American brand). Table 2 presents the average rating for each item for each of the three brands. Tables 3a-3c presents the polychoric correlations.

4. Results

We estimate different specifications of our structural equations model using the data set on brand imagery. We adopt a Bayesian approach to model inference which provides a number of advantages. First, in the context of SEM's Bayesian methods readily allows for sampling of the joint distribution of the latent factor scores and parameters that define the structural equations (Palomo et al. 2007). Second, Bayesian methods easily accommodate categorical data by using data augmentation to sample from the conditional posterior distribution of the continuous latent items and cutpoints that determine the observed ordinal data. Third, data augmentation also allows us to identify the residual correlation in the specific dimensions after controlling for the general dimension. Details on the prior and posterior distributions and the sampler for the model, are available in the Web Appendix.

4.1. Model fit

We estimate six different specifications of the structural equations models previously discussed. The results are presented in Table 4. Model M0 treats the belief data as arising from a single factors structure. Model M1 allows for multiple specific belief dimensions while setting the general factor to zero in estimation (i.e., the belief data are modeled as arising from a standard confirmatory factor structure with multiple correlated dimensions). Models M2 and M3 are consistent with the measurement model and structural equations described in equations (2) and (3). Both of these models specify the structure of the belief data to contain a single general dimension and multiple specific dimensions. We estimate two versions each for

Table 3a
Polychoric Correlations for Brand Image Data: BMW.

	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11
v1	1.00	0.44	0.50	0.52	0.50	0.47	0.49	0.65	0.52	0.57	0.57
v2		1.00	0.40	0.32	0.27	0.37	0.40	0.39	0.34	0.36	0.34
v3			1.00	0.69	0.68	0.68	0.75	0.66	0.61	0.66	0.74
v4				1.00	0.74	0.68	0.65	0.66	0.59	0.51	0.61
v5					1.00	0.71	0.66	0.65	0.62	0.58	0.64
v6						1.00	0.75	0.59	0.55	0.56	0.61
v7							1.00	0.61	0.55	0.61	0.68
v8								1.00	0.73	0.72	0.72
v9									1.00	0.73	0.72
v10										1.00	0.88
v11											1.00

v1. Overall Opinion, v2. Purchase Interest, v3. Proud to Own, v4. Prestigious, v5. Luxurious, v6. Distinctive, v7. Attractive, v8. Quality, v9. Durable, v10. Dependable, v11. Trustworthy.

Table 3b
Polychoric Correlations for Brand Image Data: Lexus.

	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11
v1	1.00	0.51	0.55	0.58	0.53	0.37	0.45	0.64	0.59	0.54	0.58
v2		1.00	0.50	0.40	0.34	0.32	0.38	0.43	0.43	0.42	0.47
v3			1.00	0.72	0.71	0.63	0.74	0.68	0.67	0.71	0.79
v4				1.00	0.76	0.54	0.60	0.68	0.64	0.59	0.68
v5					1.00	0.63	0.66	0.71	0.66	0.67	0.69
v6						1.00	0.76	0.52	0.51	0.47	0.54
v7							1.00	0.56	0.56	0.60	0.67
v8								1.00	0.77	0.76	0.78
v9									1.00	0.79	0.79
v10										1.00	0.90
v11											1.00

v1. Overall Opinion, v2. Purchase Interest, v3. Proud to Own, v4. Prestigious, v5. Luxurious, v6. Distinctive, v7. Attractive, v8. Quality, v9. Durable, v10. Dependable, v11. Trustworthy.

Table 3c
Polychoric Correlations for Brand Image Data: Lincoln.

	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11
v1	1.00	0.50	0.60	0.59	0.54	0.51	0.58	0.62	0.53	0.59	0.59
v2		1.00	0.54	0.40	0.40	0.48	0.58	0.47	0.41	0.49	0.50
v3			1.00	0.72	0.70	0.71	0.77	0.70	0.68	0.71	0.77
v4				1.00	0.76	0.67	0.72	0.65	0.61	0.61	0.65
v5					1.00	0.70	0.70	0.62	0.59	0.63	0.66
v6						1.00	0.76	0.63	0.60	0.65	0.63
v7							1.00	0.69	0.66	0.73	0.73
v8								1.00	0.73	0.78	0.76
v9									1.00	0.75	0.73
v10										1.00	0.88
v11											1.00

v1. Overall Opinion, v2. Purchase Interest, v3. Proud to Own, v4. Prestigious, v5. Luxurious, v6. Distinctive, v7. Attractive, v8. Quality, v9. Durable, v10. Dependable, v11. Trustworthy.

Model M2 and M3. Both versions of M2 specify the general belief dimension and brand evaluation as orthogonal. The two versions of M2 each specify the specific belief dimensions as either correlated or orthogonal, respectively. Both versions of M3 specify the general belief dimension and brand evaluation as correlated. The two versions of M3 each specify the specific belief dimensions as either correlated or orthogonal, respectively. We compute two Bayesian measures of penalized model fit, the Deviance Information Criteria (DIC) and the Log Marginal Density (LMD). For all three data sets, M3 provides the best fit to the data. For the BMW and Lexus data, allowing correlation in the specific belief dimensions provides the best fit. For the Lincoln data, restricting the specific belief dimensions to be orthogonal provides the best fit.[11]

[11] We also estimate a higher order factor SEM model, which is a restricted version of our more general bi-factor SEM model (Yung et al. 1999). Both the DIC and LMD statistics favor the bi-factor SEM.

Table 4
Penalized Model Fit Statistics: DIC[a] and LMD[b].

Model	General Factor	General Factor Correlation with Evaluation	Correlation in Specific Factors	BMW DIC	BMW LMD	Lexus DIC	Lexus LMD	Lincoln DIC	Lincoln LMD
M0[c]	–	–	–	−7,468	15,582	−7,257	15,240	−7,979	16,603
M1[d]	–	–	Yes	−6,977	14,880	−6,748	14,571	−7,441	15,932
M2[e]	Yes	No	Yes	−6,752	14,648	−6,515	14,291	−6,949	14,906
M2[f]	Yes	No	No	−6,789	14,980	−6,512	14,468	−7,181	15,695
M3[g]	Yes	Yes	Yes	**−6,581**	**14,249**	**−6,169**	**13,502**	−7,126	15,227
M3[h]	Yes	Yes	No	−6,725	14,333	−6,338	13,565	**−6,921**	**14,855**

[a] Deviance Information Criteria (Spiegelhalter et al. 2002).
[b] LMD is Log Marginal Density (Newton and Raftery 1994).
[c] Model M0: Belief Measurement Model: Confirmatory Factor Model w/ One Factor.
[d] Model M1: Belief Measurement Model: Confirmatory Factor Model, Correlated Specific Factors.
[e] Model M2: Belief Measurement Model: Hierarchical Factor Model, Exogenous General Factor and Correlated Specific Factors.
[f] Model M2: Belief Measurement Model: Hierarchical Factor Model, Exogenous General Factor and Orthogonal Specific Factors.
[g] Model M3: Belief Measurement Model: Hierarchical Factor Model, Endogenous General Factor and Correlated Specific Factors.
[h] Model M3: Belief Measurement Model: Hierarchical Factor Model, Endogenous General Factor and Orthogonal Specific Factors.

4.2. The multivariate structure of consumer beliefs: General and specific dimensions

Table 5 presents the general and specific factor loadings for the best fitting model, M3, for the brand image data. We find that much of the variation in the manifest items is due to the general factor. Across the data sets, the general factor loadings are generally larger in magnitude compared with the specific factor loadings. In the case of Lincoln, the 95 % coverage intervals on the loadings for the items "distinctive" and "attractive" contain zero suggesting that for Lincoln the hedonic dimension is less well defined after accounting for the general belief dimension. The square of the factor loadings indicate how much of the variance in each item is explained by the general and specific factors. For the brand image data the general factor accounts for between 43 % and 67 % of the variance in the manifest items (across the brands) while the specific factors account for, on average, between 14 % and 36 % of the variance in the manifest items (across the brands).

4.3. The structural model: The general belief dimension and brand evaluation

Estimates of the correlation between the general belief and brand evaluation factors are reported in Table 6. In all cases, the evaluation and general belief factors exhibit strikingly strong positive correlation. The posterior mean estimates of the correlation coefficients range from 0.93 to 0.98. The 95 % coverage intervals on the estimates of the covariance do not span zero in any case. Using the decomposition based on (4) we compute the percentage of the variation in the general belief score attributable to evaluative spillover. For the brand image data the posterior mean estimates are 96 %, 87 % and 94 %, respec-

Table 5
Factor Loadings for Model M3, Brand Belief Data[a].

		BMW (n = 814) M3[a],[b]		**Lexus (n = 830) M3[b]**		**Lincoln (n = 781) M3[c]**	
Factor	**Item**	**General**	**Specific**	**General**	**Specific**	**General**	**Specific**
Hedonic	*Prestigious*	0.84*	0.27*	0.84*	0.25*	0.79*	0.60*
		(0.04)	(0.08)	(0.03)	(0.05)	(0.02)	(0.03)
	Luxurious	0.79*	0.34*	0.81*	0.35*	0.82*	0.24*
		(0.03)	(0.07)	(0.03)	(0.05)	(0.02)	(0.03)
	Distinctive	0.67*	0.53*	0.49*	0.63*	0.85*	0.04
		(0.04)	(0.06)	(0.04)	(0.04)	(0.02)	(0.03)
	Attractive	0.52*	0.80*	0.45*	0.87*	0.92*	0.02
		(0.06)	(0.05)	(0.04)	(0.02)	(0.01)	(0.02)
Functional	*Quality*	0.70*	0.50*	0.77*	0.42*	0.78*	0.34*
		(0.04)	(0.05)	(0.03)	(0.04)	(0.02)	(0.04)
	Durable	0.62*	0.54*	0.70*	0.52*	0.75*	0.35*
		(0.04)	(0.05)	(0.03)	(0.04)	(0.02)	(0.03)
	Dependable	0.47*	0.82*	0.61*	0.72*	0.80*	0.55*
		(0.05)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)
	Trustworthy	0.56*	0.73*	0.67*	0.71*	0.82*	0.44*
		(0.06)	(0.05)	(0.04)	(0.03)	(0.02)	(0.03)

[*] Coefficients with 95% coverage intervals that do not contain zero are denoted by*.
[a] Table entries are posterior means and posterior standard deviations (in parentheses).
[b] Specific factors specified as correlated.
[c] Specific factors specified as orthogonal.

Table 6
Correlation Between the General Belief and Brand Evaluation Factors[a]

	Brand Image BMW	Lexus	Lincoln
$\sigma_{v\theta}$	0.98* (0.01)	0.93* (0.01)	0.97* (0.01)

[a] Table entries are posterior means and posterior standard deviations (in parentheses).
* Coefficients with 95% coverage intervals that do not span zeros are denoted by *.

tively, for BMW, Lexus, and Lincoln. Taken together with the information reported in Table 6 we conclude that across all the data sets the general factor accounts for a significant portion of the explained variance in the manifest items and that an overwhelming proportion of the variance in the general factor is due to information correlated with brand evaluation.

4.4. The structural model: Brand beliefs and brand evaluation

Table 7 presents the estimates of the structural parameters that link the specific belief dimensions and brand evaluation for our data sets. Parameter estimates from M1, which does not include a general belief dimension, suggest both hedonic and functional imagery drive consumers' evaluations of the brands. Under M1, hedonic imagery has a larger impact on evaluations than functional imagery. The factor score correlation coefficient estimates suggest the hedonic and functional scores are highly correlated. Model M2 adds the general factor but treats it as orthogonal (i.e., unrelated) to brand evaluation. Similar to M1, the estimates from M2 suggest hedonic imagery has a larger impact on evaluation. Estimates of the factor correlations are nearly identical. Under M3 we find smaller estimates of β compared to both M2 and M1. Under M3 both hedonic and functional beliefs have an effect on evaluations for all three brands. For BMW, hedonic and functional beliefs have equal impact on evaluation. For Lexus and Lincoln functional beliefs have a larger impact. However, for Lincoln the overall magnitude of the effects are much smaller. This is consistent with the general factor accounting for more of the variation in the belief items for the Lincoln data. Compared with M1 and M2 we find a smaller correlation between hedonic and functional beliefs for both BMW and Lexus.

4.5. Marginal effects of improvements in brand beliefs

We examine the marginal effects of a standard deviation improvement in the belief scores on the probabilities of the evaluative manifest items. In the case of the brand data, recall that the items are *overall opinion, purchase interest, and pride of ownership.* Ostensibly, managers are interested in strengthening favorable beliefs in an effort to improve customers' evaluations of their brands. Thus, it is useful to examine how improvements in the consumers' belief scores impact the evaluative items. After substituting equation (3) into equation (2) we have

$$y_i^* = \alpha^y + \Lambda[\beta'\xi_i + v_i] + \varepsilon_i^y \tag{7}$$

In any discrete choice model, including the ordered probit model used here, the marginal effects on the probabilities are not equal to the model coefficients. From equation (7) we see that the marginal effect of an improvement in a dimension of ξ_i will be determined by Λ and β. This raises an interesting methodological issue to consider in light of our different SEM's. Different combinations of Λ and β can lead to similar probabilities and marginal effects for changes in the dimensions of ξ_i. This suggests the differences in β, the estimate of the evaluation-belief relationship, across the different models reported in Table 7 may be illusory. For the sake of completeness, estimates of Λ are reported in Table 8.

Using our estimates of the model parameters, we simulate the change in response probabilities of the evaluative items given a one standard deviation improvement in each specific dimension of the belief scores. We find that the differences in β across models do not appear to be illusory; relative to the best fitting model the mis-specified models greatly overstate the effect of improving beliefs on the evaluative items. As an example, consider a 5-point response scale for an evaluative item. For any item measured with such a scale there are five response probabilities, one for each scale point. The probabilities must, by definition, sum to one which implies the marginal effects due to improvements in a dimension of ξ_i on the probabilities must sum to zero. Given an improvement in a dimension of ξ_i the change in the probability that $y_{ij} = 1$ will have the opposite sign of the corresponding β (i.e., if $\beta > 0$ the probability that $y_{ij} = 1$ decreases). The change in probability that $y_{ij} = 5$ will have the same sign of the corresponding β (i.e., if $\beta > 0$ the probability that $y_{ij} = 5$ increases). Only the signs of the changes in the probabilities $y_{ij} = 5$ and $y_{ij} = 5$ are unambiguous. What happens to the remaining interior probabilities is ambiguous; they may increase or decrease (Greene 2003). Given this feature of the marginal probabilities in the ordered probit model we report the change in the probability of a top box response for each attitudinal item given a change in each specific belief dimension.

Table 7
Structural Equations: The Effect of the Specific Dimensions of Brand Image on Brand Evaluation[a,b.]

Model	General Factor Specification	Specific Factors	BMW	Lexus	Lincoln
M1	*None*	*Hedonic*	5.94*	1.91*	2.89*
			(2.52)	(0.38)	(0.70)
		Functional	4.95*	1.68*	1.27*
			(2.41)	(0.35)	(0.35)
		Factor Correlation	0.80*	0.82*	0.86*
			(0.02)	(0.02)	(0.01)
	General Factor Specification	**Specific Factors**	**BMW**	**Lexus**	**Lincoln**
M2	*Orthogonal to Evaluation*	*Hedonic*	4.58*	3.27*	3.28*
			(1.52)	(1.05)	(0.71)
		Functional	3.32*	2.22*	1.11*
			(1.03)	(0.60)	(0.34)
		Factor Correlation	0.79*	0.82*	0.86*
			(0.02)	(0.02)	(0.01)
	General Factor Specification	**Specific Factors**	**BMW**	**Lexus**	**Lincoln**
M3	*Correlated with Evaluation*	*Hedonic*	0.47*	0.38*	0.09*
			(0.09)	(0.06)	(0.03)
		Functional	0.47*	0.42*	0.13*
			(0.09)	(0.06)	(0.03)
		Factor Correlation	0.58*	0.55*	n/a
			(0.06)	(0.05)	—

[*] deviations (in parentheses). Coefficients with 95% coverage intervals that do not span zeros denoted by *.
[a] The dependent variable in the analysis is the brand evaluation factor score. Table entries are posterior means and posterior standard For BMW and Lexus, the specific factors are specified as correlated. For Lincoln, the specific factors are specified as orthogonal.

Table 8
Brand Evaluation Factor Loadings—Brand Image Data.

Model	Item[a]	*BMW (n = 814)*	*Lexus (n = 830)*	*Lincoln (n = 781)*
M1	*Overall Opinion*	0.09*	0.25*	0.24*
		(0.04)	(0.04)	(0.04)
	Purchase Interest	0.06*	0.17*	0.18*
		(0.03)	(0.03)	(0.03)
	Proud to Own	0.20*	0.53*	0.46*
		(0.09)	(0.08)	(0.08)
M2	*Overall Opinion*	0.12*	0.18*	0.23*
		(0.04)	(0.04)	(0.04)
	Purchase Interest	0.07*	0.12*	0.18*
		(0.02)	(0.03)	(0.03)
	Proud to Own	0.26*	0.36*	0.44*
		(0.08)	(0.08)	(0.08)
M3	*Overall Opinion*	0.55*	0.58*	0.70*
		(0.04)	(0.03)	(0.02)
	Purchase Interest	0.36*	0.44*	0.58*
		(0.04)	(0.03)	(0.03)
	Proud to Own	0.80*	0.87*	0.90*
		(0.03)	(0.02)	(0.02)

[a] Table entries are posterior means and posterior standard deviations (in parentheses). Coefficients with 95% coverage intervals that do not span zeros are denoted by *.

The results of the marginal effects analyses appear in Table 9. Both the model that omits the general factor (M1) and the model that specifies the general factor as unrelated to evaluation (M2) greatly overstate the impact of improving brand beliefs on the evaluative items. Our proposed model (M3), which has the best fit to the data, suggests the probability of top box response improves by a factor of 2 % to 11 % depending on the brand and item. Estimates of the marginal effects from M1 and M2 are 3 To 5 times as high.

Table 9
Marginal Effect of a Standard Deviation Improvement in Brand Beliefs on Top Box Evaluation Ratings[a]

			BMW			Lexus			Lincoln		
Model	**General Factor Specification**	**Specific Factors**	*Opinion*	*Purchase Interest*	*Proud to Own*	*Opinion*	*Purchase Interest*	*Proud to Own*	*Opinion*	*Purchase Interest*	*Proud to Own*
M1	*None*	*Hedonic*	0.15* (0.02)	0.09* (0.01)	0.20* (0.02)	0.14* (0.02)	0.11* (0.01)	0.19* (0.02)	0.16* (0.02)	0.12* (0.01)	0.24* (0.02)
		Functional	0.12* (0.02)	0.07* (0.01)	0.16* (0.02)	0.12* (0.02)	0.09* (0.01)	0.17* (0.02)	0.06* (0.01)	0.04* (0.01)	0.09* (0.02)
	General Factor Specification	**Specific Factors**	*Opinion*	*Purchase Interest*	*Proud to Own*	*Opinion*	*Purchase Interest*	*Proud to Own*	*Opinion*	*Purchase Interest*	*Proud to Own*
M2	*Orthogonal to Evaluation*	*Hedonic*	0.15* (0.02)	0.09* (0.01)	0.21* (0.02)	0.15* (0.02)	0.12* (0.01)	0.21* (0.02)	0.17* (0.02)	0.13* (0.02)	0.25* (0.03)
		Functional	0.11* (0.02)	0.07* (0.01)	0.16* (0.02)	0.11* (0.02)	0.08* (0.01)	0.15* (0.02)	0.05* (0.01)	0.04* (0.01)	0.08* (0.02)
	General Factor Specification	**Specific Factors**	*Opinion*	*Purchase Interest*	*Proud to Own*	*Opinion*	*Purchase Interest*	*Proud to Own*	*Opinion*	*Purchase Interest*	*Proud to Own*
M3	*Correlated with Evaluation*	*Hedonic*	0.09* (0.01)	0.06* (0.01)	0.13* (0.02)	0.08* (0.01)	0.06* (0.01)	0.11* (0.02)	0.02* (0.01)	0.01 (0.01)	0.03* (0.01)
		Functional	0.09* (0.01)	0.06* (0.01)	0.12* (0.02)	0.09* (0.01)	0.07* (0.01)	0.12* (0.01)	0.02* (0.01)	0.02* (0.01)	0.04* (0.01)

[a] Table entries are posterior means and posterior standard deviations (in parentheses). Coefficients with 95% coverage intervals that do not span zeros are denoted by *.

5. General discussion

While the marketing literature has established the link between brand evaluations and sales, much of this literature relies on temporal variation in aggregate mindset metric data for model estimation. While aggregate time series data offers some advantages in terms of assessing causality, an issue with aggregate data is that it may overestimate the relationship between variables at the individual customer level ((Larivière Bart et al., 2016)). In order to understand the relationship between consumer beliefs and overall evaluation, we require an individual level analysis. Normative prescriptions suggest how researchers might collect data differently to ameliorate bias but such prescriptions are not always feasible or practical for managers, especially if using syndicated versus custom brand image studies. In this paper we take a more positive approach to deal with the question of how bias affects the belief-evaluation relationship in individual-level cross sectional data on brand image routinely available to managers and what managers can do to remedy the problems via statistical methods. We propose a Bayesian structural equations model to disentangle the impact of errors in measures of specific brand belief dimensions in the context of estimating the brand evaluation-brand belief relationship at the individual level. Our approach allows for specification and estimation of different structural relationships between the latent evaluation and belief factor scores as a natural part of the model hierarchy. Based on our model we derive predictions regarding the effect of errors in the belief dimensions that may be orthogonal to or correlated with consumers' overall brand evaluations. We test our model using simulated data with known properties. We then apply our proposed model to a secondary data set in the domain of brand image that contains items that measure overall brand evaluations (e.g., liking, interest, willingness to recommend, favorability, etc.) as well as multiple specific dimensions of beliefs about brand image associations for three automotive brands.

5.1. Contributions to the literature

We show that from an empirical estimation viewpoint the important distinction is not the origin of the belief error per se (i.e., affective or cognitive) but the *correlation or lack thereof* between any belief error and overall evaluation. Error in beliefs correlated with overall evaluation has an inflationary effect while belief error unrelated to consumers' overall evaluation has an attenuating effect. Using simulated data we show that error orthogonal to overall evaluation attenuates estimates of the effect of brand beliefs on overall evaluation and that specifying a general factor in the belief model is sufficient to control for bias. However, if the error is correlated with overall evaluation, treating the general factor as orthogonal to overall evaluation is not sufficient to control for bias. Our approach corrects for bias and does not require the analyst to identify the origin or type of error. Simulation results show that our model recovers the data generating parameters when the error is orthogonal to overall evaluation and when it is correlated with overall evaluation. We show that extant approaches that specify overall evaluation as a linear function of general and specific factors in a two-step approach cannot control for bias. However, this appears primarily due to the additional bias imparted by two step estimation. We show that simultaneous estimation of a model with overall evaluation as a linear function of general and specific factors can recover the unbiased correlation and structural parameters after applying a rescaling procedure. From a methods perspective, we show how the Bayesian approach to inference allows for the identification of the correlation in the specific factors in a bi-factor model. Frequentist approaches rely on the orthogonality of the specific dimensions to facilitate evaluation of the likelihood required for estimation.

The managerial implications of our contributions to the literature are highlighted by our empirical application. Penalized model fit statistics favor models that specify a general belief dimension correlated with brand evaluation in the structure of all three data sets. Information in the general belief dimension that is correlated with affective evaluation and left unaccounted results in biased estimates of the effect of beliefs on evaluation. In particular, we show that models that fail to account for the evaluative spillover into beliefs dramatically overstate the impact of improvements in the associations with different dimensions of brand beliefs. Our results suggest that while skepticism of simple analyses of the evaluation-belief relationship is warranted, it does not mean managers are wasting resources with efforts to improve brand beliefs (Sharp 2018). Our bias corrected model shows that strengthening the association with unique and favorable brand beliefs has a positive impact on overall brand evaluation, which research shows drives firm performance. Our bias-correction approach should be practically useful to managers attempting to measure the effect of interventions designed to improve brand evaluations.[12] As efforts to improve beliefs are costly, it is valuable to understand precisely how these beliefs may or may not affect overall brand evaluations.

5.2. Limitations and opportunities for future research

In terms of limitations of our findings, a limitation of our secondary data is that we do not observe individual level panel data on beliefs and evaluation over time. Such data are difficult to obtain but would potentially allow researchers to make stronger causal claims compared to individual cross sectional data. A second limitation of our secondary data is that the respondents, by design, all possessed a fair amount of familiarity with the objects rated. It would be interesting to estimate our model on data from respondents with lower levels of familiarity. As affective response may occur outside of cognition it is unclear what the ultimate findings would entail for such consumers. In terms for future research, as tools that allow researchers to infer attitude and beliefs from social media become more prevalent researchers should pay careful attention to the multivariate structure of these data. Our model should also prove useful to researchers investigating the effect of mindset metrics on brand sales or brand choice, a topic which is receiving much contemporary interest from marketing academics. Future research in this domain should carefully consider the role a general brand effect might play in mindset metric data as our results clearly demonstrate the potential for a general factor to bias estimates of the effect of mindset metrics on marketing outcomes such as brand sales or brand choice.

Data availability

The data that has been used is confidential.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.ijresmar.2023.02.002.

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[12] We are not suggesting that managers themselves might implement our model, although technically inclined managers might do so. Rather, we are suggesting that statistical consultants that help managers to implement marketing mix models or choice-based conjoint models, for example, could make use of our approach to help guide managerial decisions.

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Further reading

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