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Production cost functions and demand uncertainty effects in price-only contracts

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The price-only contract is the simplest and most common contract between a supplier and buyer in a supply chain. In such a contract, the supplier proposes a fixed wholesale price, and the buyer chooses a corresponding order quantity. The buyer's optimal behavior is modeled using the Newsvendor model and the supplier's optimal behavior is modeled as the solution to an optimization problem. This article explores, for the first time, the impact of general production costs on the supplier's and buyer's behavior. It is revealed that increased supplier's production *efficiency*, reflected in lower marginal production costs, increases the buyer's optimal profit. Therefore, a buyer would always prefer the more efficient supplier. A higher supplier efficiency, however, may or may not increase the supplier's optimal profit, depending on the production function's fixed costs. The effect of demand uncertainty, as measured by the coefficient of variation, is shown to increase the optimal order quantity. The uncertainty effect on the firms' optimal profits is analyzed. Also, the relationship between production efficiency and the response to demand uncertainty is explored and it is shown that a higher efficiency level increases the responsiveness and volatility of the supplier's production quantities. Thus, higher-efficiency suppliers are better positioned to respond to changes in the demand uncertainty in the supply chain.

Keywords: Production, efficiency, price-only contracts, supply chain

1. Introduction

The framework of price-only contracts is one of the most common in supply chain interactions between a supplier and a buyer. In such a contract, the supplier proposes a fixed wholesale price, and the buyer chooses an order quantity, as a function of the wholesale price. The supply chain operates under uncertainty, where customer demand is modeled as a random variable with a known distribution. The Newsvendor model is a formalization of the buyer's reaction to the contract terms given by the supplier and the demand uncertainty, which maximizes the buyer's expected profit. Lariviere and Porteus (2001) were the first to study the supplier's optimal behavior under price-only contracts. They derived the supplier's optimal wholesale price that maximizes the supplier's profit, assuming an optimal behavior on the part of the buyer as modeled by the Newsvendor problem, under mild conditions.

In spite of the ubiquitous presence of price-only contacts, there has been no study of the effects of production efficiencies on the performance of the supply chain. Furthermore, the effect of demand uncertainty in price-only contracts was only studied under the assumption that the supplier has a fixed unit production cost (or a linear total production cost function), independent of the quantity produced/ordered. We investigate here, for the first time, the effects of production efficiencies and demand uncertainty in the presence of *general* total production cost functions. Although a quantity discount contract might be chosen in the context of varying marginal production costs, Altintas *et al.* (2008) showed that these contracts can amplify variability in a bullwhip-type effect, raising questions about their value. It is therefore more practical to retain price-only contracts in the face of changing production functions.

Instead of fixed unit production costs, we introduce total production cost *functions* that allow varying unit production costs. We study both convex and concave production functions. Concave production cost functions are important as they model economies of scale, where the unit cost of production decreases with the total quantity produced. Convex functions arise in situations where a manufacturing plant has a standard capacity, which can be expanded with overtime at a higher unit cost (i.e., steeper slope). As

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a concrete example, consider a plant that faces demand stochastically distributed between ℓ and u. This plant can, under their regular capacity, produce at a unit cost of p up to a quantity x_1; additional capacity, up to a quantity x_2, can be obtained by overtime shifts, whose additional labor charges result in a higher unit production cost of $p_1 > p$; finally, even more capacity (up to u) can be achieved by contracting to an outside firm, at an even higher unit production (or procurement) cost of $p_2 > p_1$. This scenario motivates our interest in convex production cost functions. Still, our results also hold for general non-decreasing production functions.

The performance of price-only contracts is studied here in the context of a simple supply chain consisting of a single product, with one supplier and one buyer. We investigate the impact of production efficiency on the optimal decentralized behavior of the supplier and buyer, the effect of demand uncertainty on the optimal order quantities and profits of the supplier and buyer, and the interaction of production efficiency with the uncertainty of the demand distribution. The behavior of these parameters can be determined if the general production cost function satisfies a condition on its production elasticity; we show that this condition is satisfied by all convex production cost functions, and many concave functions. In particular, our contributions are as follows.

1. We identify *production efficiency* as a critical characteristic of production cost functions. A more-efficient production cost function grows at a slower rate as a function of production quantity. A surprising result derived here is that increased supplier production efficiency *always* increases the buyer's optimal profit but does not necessarily increase the supplier's optimal profit. This implies that a buying manager is *always* motivated to seek out more-efficient suppliers. The impact of efficiency on the supplier is different: if the supplier can choose a more-efficient process, associated with a higher fixed cost, the supplier's optimal profit increases provided that the increase in fixed cost is bounded by a quantity we derive here. This analysis guides a supply manager to the determination of whether an investment in increased efficiency is justifiable.
2. The effect of demand uncertainty on the optimal ordering quantities in the presence of general production cost functions is analyzed. As the demand uncertainty increases, the supplier induces a higher buyer's order quantity, unless the marginal production cost decreases quickly; in particular, if marginal costs decrease more quickly than marginal revenues, the profit-maximizing optimality condition (that sets them equal) must occur at a lower order quantity. We further study the impact of demand uncertainty on the profits of the supplier and buyer.
3. We quantify the agility of the optimal supplier's wholesale price and buyer's purchase quantity to the uncertainty of demand: provided that the production cost benefits from economies of scale, the more-efficient supplier manifests higher agility in response to demand volatility. Thus, higher-efficiency suppliers are better positioned to respond to changes in the demand uncertainty in the supply chain.

Next, we provide a literature review, surveying the most relevant research.

1.1. *Literature review*

Our article studies an extension of the basic price-only contract, by looking at the effect of general production cost functions on the supplier's side. The paper most relevant to our research is Cho and Gerchak (2005), which introduced production costs for the supplier and operating costs for the buyer in a similar setting; their focus is on the buyers's operating costs, whereas our focus is on the supplier's production costs. In particular, (i) these authors assume that the production costs are concave, which we do not, and (ii) these authors do not differentiate between fixed and variable supplier production costs, which we focus on here. Gilbert and Cvsa (2003) also considered the effect of supplier production costs under a price-only contract. However, they took a different perspective on this issue, studying the tradeoff between investing to lower production costs and allowing the wholesale price to adjust to market demands. Donohue (2000) studied the effect of different production modes (an expensive fast one and a cheap slow one), in a supply chain for fashionable goods; we instead allow for a continuum of production modes. A related study of capacity, instead of production, can be found in Van Mieghem (1999), which considered subcontracting as a mechanism to increase capacity (for both the supplier and buyer). There have also been other generalizations of this basic contract, including demand updating (Cachon and Lariviere, 2005), competing manufacturers (Cachon and Kok, 2010), effort-dependent demand (Corbett and DeCroix, 2001), and multiple selling seasons (Anupindi and Bassok, 1998). For more general information and additional references, Lariviere (1999) surveyed supply chain contracts in a stochastic demand environment.

Most related to our article is Lariviere and Porteus (2001), which was the first to completely analyze the price-only contract in a supply chain consisting of a single supplier and buyer under demand uncertainty; they modeled the buyer's optimal response to the supplier's wholesale price by the Newsvendor model, which allowed them to define the supplier's profit maximization problem. Under the (mild) assumption of Increasing Generalized Failure Rate (IGFR) distributions, the supplier's optimal wholesale price is shown to be unique. However, they only considered fixed unit production costs, which imply linear total production cost functions. A portion of our contribution is that we generalize several of the results in

Lariviere and Porteus (2001) to general production cost functions.

1.2. *Overview of the price-only contract*

In a price-only contract, the supplier specifies a wholesale price w, and the buyer decides on the order quantity q. The buyer then sells the product to a final customer at a unit revenue of r. Customer demand is a random variable D with support on $[\ell, u]$ represented by a distribution F. It is assumed that F has a finite mean μ and standard deviation σ, that F^{-1} exists, and that the density f is positive on $[\ell, u]$. Demand uncertainty is measured here as the *coefficient of variation* $\rho = \sigma/\mu$.

The buyer's optimal behavior is determined via the standard Newsvendor model, in which the optimal order quantity q^* is a function of the specified wholesale price w, the unit-revenue r, and the demand distribution F. More precisely, the buyer solves $\max_q E_F[r \min\{D, q\} - wq]$, which has the solution $q^* = F^{-1}(1 - w/r)$; note that this behavior also induces the optimal price curve $w(q) = r(1 - F(q))$ (the inverse of the Newsvendor solution). The buyer guarantees the supplier payment for the quantity purchased. The supplier therefore does not face any demand uncertainty. All demand uncertainty is absorbed by the buyer, who has to make the quantity decision to balance the risk of over-ordering versus under-ordering.

The supplier, knowing that the buyer will react according to the Newsvendor model, will incorporate this knowledge into her own profit maximization problem, whose solution provides the optimal wholesale price. Note that since both the supplier and buyer are solving their respective optimization models, there is decentralized decision making in the supply chain. It is well known that this decentralization, partially due to the buyer absorbing all demand uncertainty, results in suboptimal supply chain performance; indeed, the buyer orders a quantity that is smaller than the system's optimal. It is nevertheless shown here, for general production functions, that as the risk increases, the supplier, at optimality, does absorb some of the risk indirectly by reducing the wholesale price, which, in turn, induces the buyer to increase the order quantity.

1.3. *Article outline*

In Section 2 we introduce the concept of production elasticity and characterize the supplier's optimal wholesale price under general production cost functions. We introduce in Section 3 the concept of relative efficiency of two production cost functions and demonstrate the impact of efficiency on the optimal wholesale price and order quantity. In this section, we also study the supplier's and buyer's profits, showing the surprising result that the supplier can, but not necessarily will, benefit from increased production efficiency, whereas the buyer will always benefit. In Section 4 we show how the optimal order quantity and firm profits are dependent on the uncertainty of the demand distribution, as measured by the coefficient of variation. In Section 5, it is demonstrated that the agile response to uncertainty increases with production efficiency. Concluding remarks are provided in Section 6.

2. Supplier's optimal behavior for a general production cost function

The supplier's profit maximization problem in a price-only contract was first considered by Lariviere and Porteus (2001), *for constant unit production costs and zero fixed setup costs*. They characterized the optimal wholesale price for the supplier and considered the effects on it of market size and demand uncertainty. Here we study the supplier's profit maximization decision for non-constant unit production costs and positive fixed setup costs.

The supplier produces $q > 0$ units of a single product with a *total* production cost of

$$P(q) = c + \int_0^q p(x)dx,$$

where $c \geq 0$ is the setup cost of production and $p(q) > 0$ is the continuous marginal cost production function. We assume that c is small enough to allow the supplier to extract a positive profit from the price-only contract, which is relatively innocuous as most of our results do not depend on c. Note that this total production cost function is continuous and nonlinear. We focus here on convex and concave production cost functions $P(q)$, though our results can be applied to *any* functional form. To handle piecewise linear total production cost functions, we utilize nonlinear approximations; since we only require continuity, this approximation can be arbitrarily good.

We review here in detail the buyer's optimization model, since the buyer's optimal behavior is used in defining the supplier's optimization model. Under a fixed price contract, the supplier proposes a unit wholesale price w; the buyer, facing uncertain demand $D \in [\ell, u]$, $0 \leq \ell \leq u \leq \infty$, characterized by a continuous distribution F and unit revenue r, solves

$$q(w) = \arg\max_q \; r E[\min\{q, D\}] - wq = F^{-1}(1 - w/r),$$

which gives the expected profit-maximizing order quantity for the buyer. The supplier, knowing that the buyer will behave according to this Newsvendor model, will optimize her profit by appropriately selecting a profit-maximizing wholesale price w that solves the following problem:

$$\max_w \; wq(w) - P(q(w)).$$

Using a change of variable that is the inverse of the buyer's optimal order quantity

$$w(q) = r(1 - F(q(w))),$$

we can equivalently consider the supplier's problem a profit maximization over q:

$$\max_q \; r(1 - F(q))q - P(q).$$

To solve the supplier's problem under a general production cost function, we utilize the concepts of a distribution's generalized failure rate, and an IGFR.

Definition 1. $g(q) = qf(q)/(1 - F(q))$ is the *generalized failure rate* of a distribution F.

Definition 2. F has an IGFR if $g(q)$ is increasing in q.

In what follows, we assume that the demand distribution F has an IGFR, which is a mild assumption, as many common distributions satisfy it (e.g., uniform, gamma, Pareto). This assumption is popular in the academic literature on contracting and pricing, as it usually results in an objective function being unimodal, which gives a unique optimum. Further details on IGFR distributions, including a more complete list of IGFR distributions, can be found in Banciu and Mirchandani (2013). From the practical point of view, Lariviere and Porteus (2001) showed that the IGFR $g(q)$ is the reciprocal of the price elasticity of the demand, defined as the percent decrease in the purchase quantity q per 1% increase in the price w; therefore, an IGFR is equivalent to a decreasing price elasticity.

We also introduce the new concept of production elasticity, which is similar to the price elasticity.

Definition 3. The *production elasticity* is

$$\frac{w}{p(q(w))}\frac{dp(q(w))}{dw}.$$

For simplicity, we shall also write the production elasticity without the arguments as $w/p\; dp/dw$. Note that, in our context, the buyer's purchase quantity equals the supplier's production quantity. The standard economic definition of price elasticity, discussed above, can therefore be stated as the percent decrease in the *production* quantity q for a 1% increase in the price w. The production elasticity similarly measures the impact of a price change, except with respect to marginal production costs, rather than production quantities; more precisely, the production elasticity is defined as the percentage increase in the marginal production cost $p(q)$ per 1% increase in price w. In other words, production elasticity looks at the impact on production *costs*, rather than production quantities, when wholesale prices change. A critical assumption of most of our results is that the production elasticity is less than one; this simply implies that marginal production costs grow slower than wholesale prices.

Theorem 1. *Suppose that F has an IGFR, with a finite mean and support $[\ell, u]$ and that production elasticity $w/p\; dp/dw < 1$. Then*

1. *The supplier's first-order optimality condition may be written as*

$$(1 - F(q))(1 - g(q)) = p(q)/r. \qquad (1)$$

2. *The supplier's profit function is unimodal on* $[0, \infty)$. *If a solution q^* to Equation* (1) *exists, it is unique, must lie in the interval* $[\ell, u]$, *and maximizes the supplier's profit. If no solution exists, the supplier's profit-maximizing sales quantity is ℓ.*

Proof. The supplier's profit, as a function of q, is $\Pi_s(q) = r(1 - F(q))q - P(q)$; the derivative is

$$\Pi_s'(q) = r(1 - F(q)) - rqf(q) - p(q).$$

The first-order condition can be written as

$$\frac{qf(q) + p(q)/r}{1 - F(q)} = 1. \qquad (2)$$

We let $a = qf(q)$, $b = 1 - F(q)$, and $c = p(q)/r$. We know that $a/b = g(q)$ is increasing since F has an IGFR; taking derivatives with respect to q, this is equivalent to $ba' \geq ab'$. We next show that $(a + c)/b$ is strictly increasing, which implies that if a solution to the first-order condition exists, it is unique. Taking the derivative, we see that we must only prove that $b(a' + c') > b'(a + c)$; since $ba' \geq ab'$, it is sufficient to show that $bc' > cb'$, which can be written as

$$\begin{aligned}
&(1 - F(q))p'(q) > -p(q)f(q)\\
&\quad\Leftrightarrow \frac{1}{p}\frac{dp}{dq} > -rf(q)\frac{1}{r(1 - F(q))}\\
&\quad\Leftrightarrow \frac{1}{p}\frac{dp}{dq} > \frac{dw}{dq}\frac{1}{w}\\
&\quad\Leftrightarrow \frac{w}{p}\frac{\frac{dp}{dq}}{\frac{dw}{dq}} < 1 \quad \left(\text{since } \frac{dw}{dq} < 0\right)\\
&\quad\Leftrightarrow \frac{w}{p}\frac{dp}{dw} < 1.
\end{aligned}$$

Therefore, if the first-order condition has a solution q^*, it is unique. Noting that $\Pi_s'(u) < 0$ and $\Pi_s'(q) > 0$ for $q < \ell$, we conclude that the profit function is unimodal and the solution q^* maximizes the profit.

If $(a + c)/b|_{q=\ell} = \ell f(\ell) + p(\ell)/r$ is greater than one, no solution to the first-order condition exists, and the derivative is negative everywhere on $[\ell, u]$ and positive on $[0, \ell)$. Consequently, in this case, the profit function is again unimodal and $q = \ell$ maximizes the profit. ∎

2.1. *Linear and convex production cost functions*

We next demonstrate that convex production cost functions satisfy the production elasticity condition and therefore Theorem 1 applies to any of these production functions.

Lemma 1. *All convex production cost functions have production elasticity $w/p\ dp/dw < 1$.*

Proof. Due to convexity, $p'(q) \geq 0$, the production elasticity is

$$\begin{aligned}\frac{w}{p}\frac{dp}{dw} &= \frac{w}{p}\frac{dp/dq}{dw/dq} \\ &= \frac{r(1-F(q))}{p(q)} \times \frac{p'(q)}{-rf(q)} \\ &\leq 0,\end{aligned}$$

which is less than one. ■

Corollary 1. *Suppose that F has an IGFR, with a finite mean and support $[\ell, u]$, and that the production cost function is convex. Then*

1. *The supplier's first-order condition may be written as*

$$(1 - F(q))(1 - g(q)) = p(q)/r. \tag{3}$$

2. *The supplier's profit function is unimodal on* $[0, \infty)$. *If a solution q^* to Equation* (3) *exists, it is unique, must lie in the interval* $[\ell, u]$, *and maximizes the supplier's profit. If no solution exists, the supplier's profit-maximizing sales quantity is ℓ.*

Since linear production cost functions are a special case of the general functions we consider, Corollary 1 demonstrates that the result in Lariviere and Porteus (2001), which considers only linear production cost functions, is a special case of our results.

2.2. *Concave production cost functions*

Considering concave production cost functions with $p'(q) \leq 0$, the production elasticity is

$$\begin{aligned}\frac{w}{p}\frac{dp}{dw} &= \frac{w}{p}\frac{dp/dq}{dw/dq} \\ &= \frac{(1-F(q))}{p(q)} \times \frac{|p'(q)|}{f(q)},\end{aligned}$$

which may or may not be greater than one. It is not surprising that the elasticity condition in Theorem 1 does not hold in general, as a concave production function contributes a convex expression (cost is subtracted from revenues) to a maximization problem. However, we next show that for specific concave production functions, relatively mild conditions on the demand distribution F can be derived to ensure a unique solution q^*.

Example 1. Consider a concave production cost function that is logarithmic in form, $P(q) = c + p_0 \ln(q+1)$ and $p(q) = p_0/(q+1)$, for some positive constants c and p_0. The production elasticity of $P(q)$ is

$$\frac{w}{p}\frac{dp}{dw} = \frac{w}{p}\frac{dp/dq}{dw/dq} = \frac{(1-F(q))}{f(q)(q+1)} < \frac{1}{g(q)}.$$

Therefore, $g(q) \geq 1$ is a sufficient condition to enforce $w/p\ dp/dw \leq 1$. Since F has an IGFR, $g(q)$ is increasing, and it is sufficient to have $g(\ell) = \ell f(\ell) \geq 1$. We next consider some sample demand distributions associated with the logarithmic production cost function:

1. If F is a uniform distribution, then $\ell f(\ell) \geq 1$ is equivalent to $\ell > u/2$.
2. Let F be a pareto distribution with distribution $F(q) = 1 - (q_0/q)^\alpha$ and $f(q) = \alpha q_0^\alpha / q^{\alpha+1}$, for $q \geq q_0$. Note that a finite mean only exists if $\alpha > 1$, which we assume to be true so that we can apply Theorem 1. The generalized failure rate $g(q) = \alpha > 1$. Therefore, all Pareto distributions with a finite mean satisfy the production elasticity condition for a logarithmic production cost function.

◄

Example 2. Consider a concave production cost function whose concavity has a square root form, $P(q) = c + p_0\sqrt{q}$ and $p(q) = p_0/(2\sqrt{q})$, for some positive constants c and p_0. The production elasticity of $P(q)$ is

$$\begin{aligned}\frac{w}{p}\frac{dp}{dw} &= \frac{w}{p}\frac{dp/dq}{dw/dq} \\ &= \frac{1}{2g(q)}.\end{aligned}$$

Therefore, $g(q) \geq 1/2$ is a sufficient condition to ensure that a unique profit-maximizing solution exists. Continuing the discussion of the uniform and Pareto distributions from Example 1, now associated with the square-root production cost function, a uniform distribution will allow a unique solution q^* if $\ell > u/3$ and all Pareto distributions with finite means will result in a unique solution q^*. ◄

Therefore, although our results cannot accommodate *any* concave production cost function, for specific concave production cost functions, we can derive conditions on the demand distribution F that allow a unique profit-maximizing wholesale price to exist.

3. Efficiency and its impact on supply chain behavior

For the following, we assume that the demand distribution F and the production cost function $P(q)$ fall within the confines of Theorem 1. In particular:

Assumption 1. The distribution F has a finite mean and an IGFR.

Assumption 2. The combination of the distribution F and production cost function $P(q)$ induce a production elasticity less than one.

The idea of relative efficiency of two production functions is a new, yet intuitive, concept that measures a supplier's ability to monetize production efficiency. In comparing the *marginal* production cost rates of two functions, the function with the smaller marginal cost rate is deemed more efficient. Note that the relative efficiency comparison of production cost functions does not require the functions to be either convex or concave.

Definition 4. A production function $P_1(q) = c_1 + \int_0^q p_1(x)dx$ is said to be *more efficient* than production function $P_2(q) = c_2 + \int_0^q p_2(x)dx$ if $p_1(q) < p_2(q)$ for all $q \in [\ell, u]$, regardless of the values of c_1 and c_2.

3.1. *The impact of efficiency on wholesale prices and order quantities*

This section provides an analysis of the supplier's optimal wholesale price and the buyer's optimal order quantity's dependence on the production efficiency. The next theorem demonstrates the rather intuitive result that as the production process becomes more efficient, the supplier desires a higher optimal buyer order quantity q and hence offers a lower wholesale price. Consequently, as the production process becomes more efficient, the supplier benefits as well as the buyer. The theorem holds for *any* pair of production cost functions that satisfy the production elasticity condition in Assumption 2 and a demand distribution that satisfies the IGFR of Assumption 1.

Theorem 2. *For a given distribution of demand F, and a pair of production cost functions $P_1(q)$ and $P_2(q)$, with $P_1(q)$ more efficient than $P_2(q)$:*

1. *The optimal ordering quantity for $P_1(q)$ is higher than the optimal ordering quantity for $P_2(q)$.*
2. *The optimal wholesale price for $P_1(q)$ is lower than the optimal wholesale price for $P_2(q)$.*

Proof.
Part 1: The left-hand side of the first-order condition (2) for a generic production function $P(q)$ is $(qf(q) + p(q)/r)/(1 - F(q))$. This expression is increasing faster in q for $P_2(q)$ than for $P_1(q)$, by the definition of efficiency. Therefore, the optimal solution q_1^* will be found at a larger q for $P_1(q)$ than the optimal solution q_2^* for $P_2(q)$.
Part 2: Since $w = r(1 - F(q))$, the optimal value of the wholesale price w_1^* for $P_1(q)$ is lower than w_2^*, the optimal wholesale price for $P_2(q)$. ■

Note that only marginal costs affect terms of trade between the supplier and buyer and that fixed costs do not play a role. This result is relevant in practice as it demonstrates that only production efficiency, and not fixed costs, is relevant for the supplier's pricing and the buyer's ordering decisions (although we later see that fixed costs impact the supplier's profit). The intuition is that fixed costs are sunk and do not influence any operational decisions. Note that our theorems are based on a number of assumptions: namely, (i) IGFR demand distribution; (ii) production elasticity $w/p\ dp/dw < 1$; (iii) unlimited production capacity; and (iv) the supplier is required to produce.

Note that Theorem 2 only depends on the relative efficiency of two production cost functions and not their absolute values. Therefore, it is possible that P_1 is more efficient than P_2, but $P_1(q) > P_2(q)$ for all $q \in [\ell, u]$. Therefore, production efficiency and production costs are drastically different concepts, especially in terms of how they affect the supplier's optimal wholesale price and buyer's optimal order quantity. Indeed, efficiency alone affects the determination of these values. In the next two subsections, we study how production efficiency and total costs affect the profits of the buyer and supplier.

3.2. *The impact of production efficiency on the buyer's profit*

We investigate here the impact of increased supplier's production efficiency on the buyer's profit and show that his profits *always* increase.

Theorem 3. *Increased production efficiency always increases the buyer's optimal profit.*

Proof. Let D be the random variable representing demand and $f(x)$ its density. Let the buyer's maximized profit, as a function of the wholesale price w and the optimal order quantity $q(w) = F^{-1}(1 - w/r)$, be

$$\begin{aligned}\Pi(w) &= r\,E[\min\{q(w), D\}] - wq(w)\\ &= r\left(\int_0^{q(w)} xf(x)dx + \int_{q(w)}^{\infty} q(w)f(x)dx\right) - wq(w).\end{aligned}$$

Using the Leibniz integral rule, the derivative of the buyer's optimal profit is

$$\begin{aligned}\frac{\partial \Pi(w)}{\partial w} &= r\left(\frac{\partial q(w)}{\partial w}q(w)f(q(w)) - \frac{\partial q(w)}{\partial w}q(w)f(q(w))\right.\\ &\quad \left.+ \int_{q(w)}^{\infty}\frac{\partial q(w)}{\partial w}f(x)dx\right) - w\frac{\partial q(w)}{\partial w} - q(w)\\ &= r\frac{\partial q(w)}{\partial w}(1 - F(q(w))) - w\frac{\partial q(w)}{\partial w} - q(w).\end{aligned}$$

Substituting $F(q(w)) = 1 - w/r$, leads to

$$\frac{\partial \Pi(w)}{\partial w} = w\frac{\partial q(w)}{\partial w} - w\frac{\partial q(w)}{\partial w} - q(w) = -q(w) < 0.$$

Since, from Theorem 2, increased production efficiency drives down the optimal wholesale price w, and $\partial \Pi(w)/\partial w < 0$, the buyer's optimal profit increases with increased efficiency. Note that the increased production efficiency drives down the optimal wholesale price irrespective of the fixed production costs (Corollary 1); therefore, the buyer's increase in profit is independent of the fixed production costs. ■

An important managerial implication of Theorem 3 is that a buyer is motivated to seek out more-efficient suppliers, as the dynamics of their interaction always allow the buyer to benefit monetarily from increased supplier's production efficiency.

3.3. *The impact of production efficiency on the supplier's profit*

In this section we study how the supplier's profit is affected by increasing production efficiency. We again utilize the following notation for two production cost functions:

$$P_1(q) = c_1 + \int_0^q p_1(x)dx \quad \text{and}$$
$$P_2(q) = c_2 + \int_0^q p_2(x)dx,$$

where $p_1(q) < p_2(q)$ for all $q \in [\ell, u]$; in other words, $P_1(q)$ is more efficient than $P_2(q)$. The fixed costs c_1 and c_2 can take any values, allowing us to differentiate between the effect of production efficiency and total costs. The next theorems show how the supplier's profit depends on these characteristics.

Theorem 4. *If $c_1 = c_2$, then increased supplier production efficiency increases the supplier's optimal profit.*

Proof. Let $P_1(q) = c + \int_0^q p_1(x)dx$ and $P_2(q) = c + \int_0^q p_2(x)dx$ be two production cost functions with identical fixed costs ($c_1 = c_2 = c$), where the former is more efficient; i.e., $p_1(q) < p_2(q)$. Consequently, $P_1(q) < P_2(q)$, for all q. Let $\Pi_1(q) = w(q)q - P_1(q)$ and $\Pi_2(q) = w(q)q - P_2(q)$ denote the supplier's profit functions under the first and second cost functions, respectively. Clearly, $\Pi_1(q) > \Pi_2(q)$, for all q. Let q_1^* and q_2^* denote the optimal ordering quantities under each production function, respectively; from Theorem 2, we know that $q_1^* > q_2^*$. Therefore, we have that

$$\Pi_2(q_2^*) < \Pi_1(q_2^*)$$
$$< \Pi_1(q_1^*), \qquad (\text{since } q_1^* \text{ maximizes } \Pi_1(q))$$

which shows that the supplier's profit strictly increases with higher production efficiency, assuming zero fixed production costs. ■

For convenience in analyzing the case with differing fixed costs, we provide the following definition, where $\Pi_1(q_1^*)$ and $\Pi_2(q_2^*)$ are defined in the proof of Theorem 4.

Definition 5. Let $\Delta = \Pi_1(q_1^*) - \Pi_2(q_2^*) > 0$ denote the profit increment, due to higher efficiency, under the identical fixed cost case of Theorem 4.

Theorem 5.

1. *If $c_1 \le c_2$, then increased supplier production efficiency increases the supplier's optimal profit.*
2. *Otherwise,*
   (i) *If $c_1 - c_2 < \Delta$, then increased supplier production efficiency increases the supplier's profit.*
   (ii) *If $c_1 - c_2 > \Delta$, then increased supplier production efficiency decreases the supplier's profit.*
   (iii) *If $c_1 - c_2 = \Delta$, then increased supplier production efficiency maintains the supplier's profit.*

Proof. Consider the production cost functions with positive fixed costs c_1 and c_2, namely, $P_1(q) = c_1 + \int_0^q p_1(x)dx$ and $P_2(q) = c_2 + \int_0^q p_2(x)dx$. Improved efficiency—i.e., $p_1(q) < p_2(q)$, $\forall q$—increases the supplier's profit if and only if

$$w(q_1^*)q_1^* - c_1 - \int_0^{q_1^*} p_1(x)dx > w(q_2^*)q_2^* - c_2 - \int_0^{q_2^*} p_2(x)dx.$$

Since q_1^*, q_2^* and Δ do not depend on c_1 and c_2 (cf. Theorem 2), it follows that this is equivalent to $c_1 - c_2 < \Delta$. If increased efficiency requires a higher fixed cost, $c_1 > c_2$, then there is a finite bound Δ on the increase $c_1 - c_2$ that allows an improvement in the supplier's profit. ■

Theorems 4 and 5 show that an improved production efficiency increases the supplier's profit, except for the case where an increased efficiency is associated with high fixed costs. These theorems allow a supplier to determine whether an investment (increased fixed cost) in increased efficiency (reduced marginal cost) is justifiable, by looking at the resulting change in profit.

Since the buyer always benefits from increased supplier efficiency, this makes the more-efficient suppliers in the market more competitive compared with the less-efficient suppliers. Furthermore, Theorems 3 and 5 show that, as long as the increased efficiency is associated with a limited increase in fixed production costs, both firms will benefit from increased supplier efficiency.

3.4. *The impact of production efficiency on the supply chain profit*

In this section we show that the decentralized supply chain's profit is increasing in the supplier's production efficiency.

Theorem 6. *The decentralized supply chain's optimal profit is increasing in the supplier's production efficiency.*

Proof. Let $\Pi_{SC}(q) = r \min\{q, D\} - P(q) = r(\int_0^q xf(x)dx + \int_q^\infty qf(x)dx) - P(q)$ denote the supply chain's expected profit. The derivative:

$$\frac{\partial \Pi_{SC}(q)}{\partial q} = r\left(qf(q) - qf(q) + \int_q^\infty qf(x)dx\right)$$
$$- p(q) = r(1 - F(q)) - p(q).$$

From the first-order optimality condition (2) in the proof of Theorem 1, we have that $r(1 - F(q)) = rqf(q) + p(q)$, which implies that $\partial \Pi_{SC}(q)/\partial q = rqf(q) > 0$. Since

Theorem 2 showed that increased production efficiency results in a higher order quantity, we conclude that higher efficiency results in higher supply chain profits. ■

Theorem 6 shows that, regardless of how the individual firm profits behave, the total supply chain profit always increases when the supplier's production efficiency increases. However, Theorem 5, which shows that fixed production costs can reduce the supplier's profit despite increased efficiency (case 2, part ii), implies that a supplier might not be willing to invest in more-efficient production. Theorem 6 shows that a "win-win" scenario can still be achieved. For instance, a two-part tariff, where the buyer provides the supplier with a lump sum payment, in addition to the revenues from the price-only contract, can motivate the supplier to invest in a higher production efficiency, thus increasing the entire supply chain profit and increasing both firms' profits.

4. Demand uncertainty and its impact on supply chain behavior

In this section, we study the effect of demand uncertainty on the firm's optimal decisions and resulting profits, in the presence of general production cost functions. We first partially characterize the effect of uncertainty on the optimal supplier and buyer's profits. We then demonstrate that increasing demand uncertainty induces a higher optimal ordering quantity from the buyer, which generalizes the variability results of Lariviere and Porteus (2001). To prove our results, we define a class of demand random variables D_i that normalize to a given canonical random variable Z.

Definition 6. A random variable Z is said to be *canonical* if its mean $\mu_Z = 0$ and its standard deviation $\sigma_Z = 1$.

Definition 7. The *class of demand random variables generated by* Z with distribution $H(z)$ and density $h(z)$, $\mathcal{D}_Z = \{D_i : i = 1, 2, \ldots\}$, is the collection of all random variables D_i with mean $\mu_i \in \mathbb{R}$, standard deviation $\sigma_i > 0$, and distribution F_i such that $(D_i - \mu_i)/\sigma_i = Z$ and $F_i(x) = H(x - \mu_i)/\sigma_i$ for all i.

Example 3. For $Z \sim N(0, 1)$, a standard normal random variable, with mean equal to zero and standard deviation equal to one, all normal random variables are the class generated by Z. ◀

4.1. *The impact of demand uncertainty on the supplier's profit*

Recall that the supplier's profit can be written as a function of the order quantity q, since the buyer applies the Newsvendor solution $q(w) = F^{-1}(1 - w/r) \Leftrightarrow w(q) = r(1 - F(q))$:

$$\Pi_s(q) = w(q)q - P(q) = r(1 - F(q))q - P(q).$$

The following theorem may be viewed as a standard Newsvendor result. However, it is new in the sense that it focuses on the profit function, rather than the optimal order quantities.

Theorem 7. *For a given production cost function $P(q)$, canonical random variable Z generating $\mathcal{D}_Z$, $D \in \mathcal{D}_Z$, and optimal ordering quantity $q^* = F^{-1}(1 - w^*/r)$, then*

1. *The supplier's profit $\Pi_s(q^*)$ is increasing in the mean demand μ.*
2. *If $q^* > \mu$, the supplier's profit $\Pi_s(q^*)$ is increasing in the standard deviation of demand σ.*
3. *If $q^* < \mu$, the supplier's profit $\Pi_s(q^*)$ is decreasing in the standard deviation of demand σ.*

This theorem complements the well-known Newsvendor result that states that the order quantity increases with the variance when the critical ratio is greater than 0.5 and otherwise decreases with the variance (for distributions having the mean equal to the mode).

Proof. We use the change of variable $z = (q^* - \mu)/\sigma$ and, noting that $F(q^*) = H((q^* - \mu)/\sigma)$, deduce that $q^* = \mu + \sigma H^{-1}(1 - w^*/r)$, since $q^* = F^{-1}(1 - w^*/r)$. The supplier's profit function can now be written as

$$\begin{aligned}\Pi_s(q^*) &= w(q^*)q^* - P(q^*)\\ &= r(1 - F(q^*))q^* - P(q^*)\\ &= r\left(1 - H\left(\frac{q^* - \mu}{\sigma}\right)\right)q^* - P(q^*).\end{aligned}$$

Fixing σ and taking the derivative of $\Pi_s(q^*)$ with respect to μ, we obtain

$$\frac{\partial \Pi_s(q^*)}{\partial \mu} = \frac{rq^*}{\sigma} h\left(\frac{q^* - \mu}{\sigma}\right) > 0.$$

Similarly, fixing μ and taking the derivative of $\Pi_s(q^*)$ with respect to σ, we obtain

$$\frac{\partial \Pi_s(q^*)}{\partial \sigma} = rh\left(\frac{q^* - \mu}{\sigma}\right)\left(\frac{q^* - \mu}{\sigma^2}\right)q^*.$$

This completes the proof. ■

Theorem 7 allows us to partially characterize the dependence of the supplier's profit on the coefficient of variation ρ. For example, for any q, fixing σ and increasing μ (i.e., decreasing ρ) will move the profit function up, which implies that the optimal profit also increases. Similarly, fixing μ and increasing σ (i.e., increasing ρ) will move the profit function down for all $q < \mu$ and will move the profit function up for $q > \mu$. If the optimal order quantity $q(w) = F^{-1}(1 - w/r)$ is greater than μ, the optimal profit increases. However, if $q(w) < \mu$, then we are unable to ascertain whether the optimal profit increases or decreases.

4.2. *The impact of demand uncertainty on the buyer's profit*

We next show that the buyer's optimal profit usually decreases when the coefficient of variation increases; similar results for linear production costs were shown in Lariviere and Porteus (2001). We include it here to complement Theorem 7 on the effect of uncertainty on the supplier's profit.

Theorem 8. *For a given production cost function $P(q)$ and canonical random variable Z generating $\mathcal{D}_Z$, if $D_1, D_2 \in \mathcal{D}_Z$, then*

1. *If $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$, then the buyer's expected profit is greater under D_1.*
2. *If $\sigma_1 = \sigma_2$ and $\mu_1 < \mu_2$, then the buyer's expected profit is greater under D_2.*

Proof. Let $D \in \mathcal{D}_Z$ be the random variable representing demand and $f(x)$ its density. Let the buyer's maximized expected profit, as a function of the optimal order quantity q^* and the optimal wholesale price $w(q^*) = r(1 - F(q^*))$, be

$$\Pi_b(q^*) = r\,E[\min\{q^*, D\}] - w(q^*)q^*.$$

Using the definition of an expectation, we get

$$\begin{aligned}
\Pi_b(q^*) & \\
&= r\left(\int_{-\infty}^{q^*} xf(x)dx + \int_{q^*}^{\infty} q^* f(x)dx\right) - w(q^*)q^* \\
&= r\left(\int_{-\infty}^{q^*} xf(x)dx + \int_{q^*}^{\infty} q^* f(x)dx - (1 - F(q^*))q^*\right) \\
&= r\left(\int_{-\infty}^{q^*} xf(x)dx\right),
\end{aligned}$$

where the second equality is from $w(q^*) = r(1 - F(q^*))$.

We next use the change of variable $z = (x - \mu)/\sigma$ and, noting that $F(q^*) = H((q^* - \mu)/\sigma)$, deduce that $q^* = \mu + \sigma H^{-1}(1 - w/r)$, since $q^* = F^{-1}(1 - w/r)$. The buyer's optimal profit can now be written as

$$\begin{aligned}
\Pi_b(q^*) &= r\int_{-\infty}^{(q^*-\mu)/\sigma} (\mu + \sigma z)h(z)dz \\
&= r\int_{-\infty}^{H^{-1}(1-w/r)} (\mu + \sigma z)h(z)dz.
\end{aligned}$$

Finally, applying the Leibniz integral rule, we can calculate the derivatives

$$\frac{\partial \Pi_b(q^*)}{\partial \mu} = r\int_{-\infty}^{H^{-1}(1-w/r)} h(z)dz > 0,$$

and

$$\begin{aligned}
\frac{\partial \Pi_b(q^*)}{\partial \sigma} &= r\int_{-\infty}^{H^{-1}(1-w/r)} zh(z)dz \\
&= r\int_{0}^{H^{-1}(1-w/r)} zh(z)dz + r\int_{-\infty}^{0} zh(z)dz \\
&= r\int_{0}^{H^{-1}(1-w/r)} zh(z)dz - r\int_{0}^{\infty} zh(z)dz \\
&< 0.
\end{aligned}$$

Therefore, fixing σ and decreasing μ (i.e., increasing ρ) results in a decrease in the buyer's optimal profit. Alternatively, fixing μ and increasing σ (i.e., increasing ρ) also results in an decrease in the optimal profit. ■

4.3. *The effect of demand uncertainty on the supplier's behavior*

Here, we study the effect of demand uncertainty, quantified via the coefficient of variation of the demand distribution $\rho = \sigma/\mu$, on the optimal order quantity and wholesale price in Theorem 1. We require the following definition of stochastic ordering.

Definition 8. The random variable D_2 with distribution F_2 is stochastically larger than random variable D_1 with distribution F_1 if $F_2(q) \leq F_1(q)$, for all q.

The next theorem proves that, as the buyer's demand uncertainty increases, the supplier induces a higher ordering quantity from the buyer.

Theorem 9. *For a given production cost function $P(q)$ and canonical random variable Z generating $\mathcal{D}_Z$, if $D_1, D_2 \in \mathcal{D}_Z$ with D_2 stochastically larger than D_1 and $\rho_2 > \rho_1$, then*

1. *If $p(q)$ is increasing or decreasing slower than marginal revenues, then $q_2^* > q_1^*$.*
2. *If $p(q)$ is decreasing faster than marginal revenues, then $q_2^* < q_1^*$.*

Proof. We adopt and expand on the proof of Theorem 3 in Lariviere and Porteus (2001). Let F_i, $i = 1, 2$ be the distributions of D_i with means μ_i, standard deviations σ_i, and densities f_i, where $\rho_2 > \rho_1$. Let $S(q) = F_2^{-1}(F_1(q))/q$.

We claim that the derivative $\partial S(q)/\partial q$ is positive; to see this, we use the identities $F_i(q) = H(q - \mu_i)/\sigma_i$ for $q \in [\ell, u]$ and $F_i^{-1}(\alpha) = \mu_i + \sigma_i H^{-1}(\alpha)$ for $\alpha \in [0, 1]$, to write

$$\begin{aligned}
S(q) &= \left(\mu_2 + \sigma_2 H^{-1}\left(H\left(\frac{q - \mu_1}{\sigma_1}\right)\right)\right) \Big/ q \\
&= \sigma_2\left(\frac{1}{\sigma_1} + \frac{1}{q}\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)\right),
\end{aligned}$$

which gives

$$\frac{\partial S(q)}{\partial q} = \frac{\sigma_2(\rho_2 - \rho_1)}{\rho_2\rho_1 q^2} > 0.$$

We next express $\partial S(q)/\partial q$ differently, by taking the derivative without referring to the canonical distribution H:

$$\frac{\partial S(q)}{\partial q} = \frac{\partial F_2^{-1}(F_1(q))/q}{\partial q} = \frac{q f_1(q)/f_2(F_2^{-1}(F_1(q))) - F_2^{-1}(F_1(q))}{q^2}. \quad (4)$$

Since we had previously determined $\partial S(q)/\partial q > 0$, we know the numerator of Equation (4) is positive, which can be rearranged to give

$$q f_1(q) > F_2^{-1}(F_1(q)) f_2(F_2^{-1}(F_1(q))). \quad (5)$$

Dividing both sides of Inequality (5) by $1 - F_1(q) = 1 - F_2(F_2^{-1}(F_1(q)))$, we obtain

$$\frac{q f_1(q)}{1 - F_1(q)} > \frac{F_2^{-1}(F_1(q)) f_2(F_2^{-1}(F_1(q)))}{1 - F_2(F_2^{-1}(F_1(q)))}. \quad (6)$$

Let $\alpha = F_1(q)$; since $q \in [\ell, u]$, $\alpha \in [0, 1]$. Inequality (6) can then be rewritten as

$$g_1\big(F_1^{-1}(\alpha)\big) > g_2\big(F_2^{-1}(\alpha)\big), \ \forall \alpha \in [0, 1]. \quad (7)$$

Since D_2 is stochastically larger than D_1 (i.e., $F_2(q) \le F_1(q)$ $\forall q \in [\ell, u]$), then $F_1^{-1}(\alpha) \le F_2^{-1}(\alpha)$ $\forall \alpha \in [0, 1]$. Due to the assumption of IGFR, g_2 is increasing and $g_2(F_1^{-1}(\alpha)) \le g_2(F_2^{-1}(\alpha))$, which, combined with Inequality (7) gives $g_1(F_1^{-1}(\alpha)) > g_2(F_1^{-1}(\alpha))$. Substituting $q = F_1^{-1}(\alpha)$ gives $g_1(q) > g_2(q)$ for all $q \in [\ell, u]$. Since $F_1(q) \ge F_2(q)$ for all $q \in [\ell, u]$, we obtain the inequality

$$(1 - F_1(q))(1 - g_1(q)) < (1 - F_2(q))(1 - g_2(q)), \quad \forall q \in [\ell, u]. \quad (8)$$

We can write the optimality condition of Theorem 1, namely, Equation (1), as

$$\underbrace{(1 - F(q))(1 - g(q))}_{LHS(q)} = \underbrace{p(q)/r}_{RHS(q)}. \quad (9)$$

We have shown in Equation (8) that for $\rho_2 > \rho_1$ and all $q \in [\ell, u]$, $LHS(q, \rho_1) < LHS(q, \rho_2)$. Therefore, the function $LHS(q, \rho)$ is increasing in ρ for all $q \in [\ell, u]$ and is decreasing in q for a given ρ. In contrast, $RHS(q)$ is independent of F and therefore independent of ρ. Let q_i^* be the optimal solution when D_i is the demand random variable: $LHS(q_i^*, \rho_i) = RHS(q_i^*)$. Consequently, $LHS(q_1^*, \rho_2) > LHS(q_1^*, \rho_1) = RHS(q_1^*)$. If $RHS(q)$ is increasing, then $q_2^* > q_1^*$. If $RHS(q)$ is decreasing, but more slowly than $LHS(q)$, then $q_2^* > q_1^*$. Finally, if $RHS(q)$ is decreasing faster than $LHS(q)$, then $q_2^* < q_1^*$. Figure 1 demonstrates this argument graphically, for the special case where the functions $LHS(q, \rho)$ and $RHS(q)$ are linear, and $RHS(q)$ increasing. ■

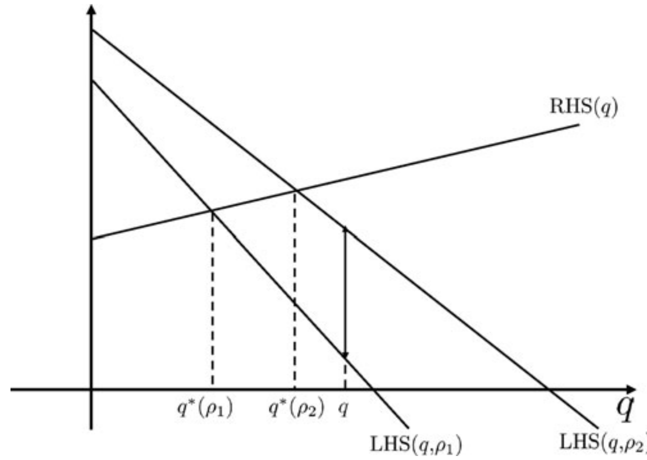

Fig. 1. Illustration of proof of Theorem 9 for $\rho_2 > \rho_1$.

5. The interaction of production efficiency and demand uncertainty

We consider here the interaction of production efficiency and demand uncertainty. It is demonstrated here that as efficiency increases, the agility of the supplier's response increases, as reflected in more-pronounced changes in the production quantities. In order to formalize this, we introduce a new concept of *agility*. Let $q^*(\rho, P(q))$ denote the optimal order quantity for a demand distribution with coefficient of variation ρ and production cost function $P(q)$.

Definition 9. For a given production function $P(q)$, a canonical random variable Z, and the class of random variables $\mathcal{D}_Z$ generated from Z, the *quantity agility of uncertainty* is

$$q^*(\rho_2, P(q)) - q^*(\rho_1, P(q)),$$

for $D_1, D_2 \in \mathcal{D}_H$ with $\rho_1 < \rho_2$.

Recall from Definition 4 in Section 3 that increased efficiency is associated with a smaller marginal production cost function $p(q)$. The next theorem shows that the uncertainty agility rises as the production efficiency rises. Subsequently, after the proof of Theorem 10, we argue that this increased agility is beneficial for the supply chain as it emphasizes the benefit of increased production efficiency.

Theorem 10. *Under the following conditions:*

1. *$D_1, D_2 \in \mathcal{D}_Z$, generated by a canonical random variable Z;*
2. *$\rho_2 > \rho_1$;*
3. *densities $f_2(q) \le f_1(q)$ for all q, implying that D_2 is stochastically larger than D_1;*
4. *$g_1(q)$ increasing faster than $g_2(q)$;*
5. *increasing concave marginal production cost functions,*

as the supplier's production efficiency increases, the quantity agility of uncertainty also increases; i.e., $q^*(\rho_2, P(q)) - q^*(\rho_1, P(q))$ is non-decreasing in the efficiency of $P(q)$.

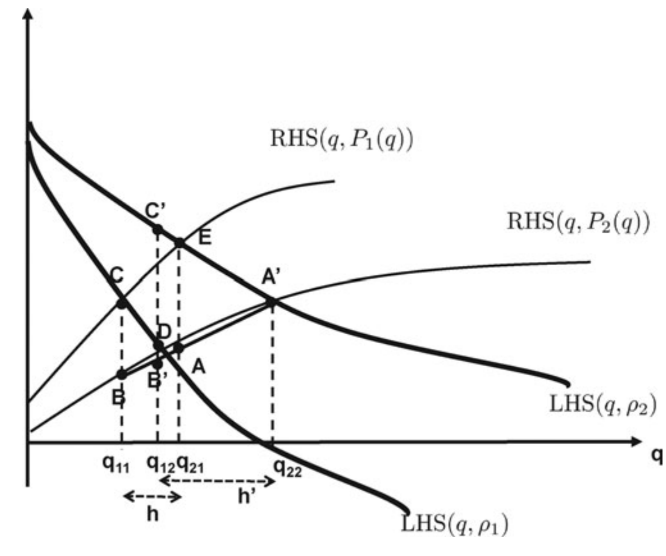

Fig. 2. Illustration of the geometry of the proof of Theorem 10.

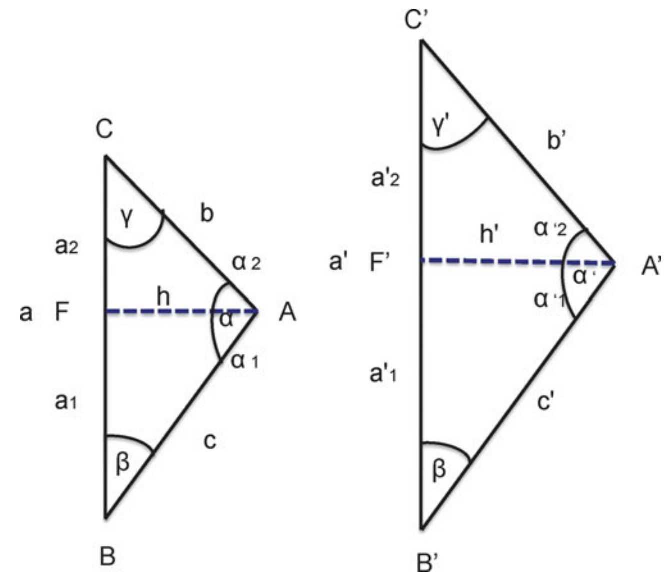

Fig. 3. Detail of the geometry of the proof of Theorem 10.

Proof. Let $P_2(q)$ be more efficient than $P_1(q)$. Continuing the logic given in Equation (9), Fig. 2 illustrates the functions $LHS(q)$ for the two coefficients of variation $\rho_1 < \rho_2$ and the functions $RHS(q)$ for the two different production functions $P_1(q)$ and $P_2(q)$. Let point C be the intersection of $RHS(q, P_1(q))$ and $LHS(q, \rho_1)$ and the value of q at C is q_{11} (with the first subscript referring to the LHS function and the second subscript to the RHS function). Similarly, point D is the intersection of $RHS(q, P_2(q))$ and $LHS(q, \rho_1)$ with q value equal to q_{12} . Point E is the intersection of $RHS(q, P_1(q))$ and $LHS(q, \rho_2)$, at $q = q_{21}$, and point A' is the intersection of $RHS(q, P_2(q))$ and $LHS(q, \rho_2)$ with $q = q_{22}$.

Let point $B = (q_{11}, RHS(q_{11}, P_2(q_{11}))$ be the projection of point C onto $RHS(q, P_2(q))$. Let $C' = (q_{12}, LHS(q_{12}, \rho_2))$ be the projection of point D onto $LHS(q, \rho_2)$. Let the line segment connecting A' and B be denoted by $[A', B]$. Point B' is the projection of D onto this line segment (for $q = q_{12}$). Lastly, point A is the projection of E onto the line segment $[A', B]$ at $q = q_{21}$. The statement of the theorem is equivalent to showing that the horizontal distance h' between q_{22} and q_{12} is greater than the horizontal distance h between q_{21} and q_{11}. The line segment $[A, F]$ is the height of the triangle ABC of length h and $[A', F']$ is the height of the triangle $A'B'C'$ of length h'.

We display in Fig. 3 the relevant geometry of Fig. 2: Points A, B, and C are connected with line segments to create one triangle ABC, and points A', B', and C' are connected with line segments to create a second triangle $A'B'C'$. Let $a = |BC|$, $b = |AC|$, and $c = |AB|$ denote the lengths of the sides of triangle ABC; a', b', and c' are defined similarly.

Since D_2 is stochastically larger than D_1, Theorem 9 tells us that $LHS(q, \rho_2) - LHS(q, \rho_1) > 0$ for all $q \in [\ell, u]$. Recall that $LHS(q, \rho_i) = (1 - F_i(q))(1 - g_i(q))$, with F_i as the distribution of D_i. Since $g_1(q)$ is increasing faster than $g_2(q)$ and $F_1(q)$ is increasing faster than $F_2(q)$ (due to the assumption that $f_1(q) \geq f_2(q)$ for all q), we conclude that $LHS(q, \rho_1)$ is *decreasing* faster than $LHS(q, \rho_2)$, which implies that

$$90^\circ \geq \gamma' > \gamma. \tag{10}$$

Note that $\gamma' \leq 90^\circ$ since in the proof of Theorem 9, $LHS(q, \rho)$ is decreasing in q for any ρ. Since $\gamma < \gamma'$ and the line segments $[B, C]$ and $[B', C']$ are vertical, it follows that

$$\alpha' < \alpha. \tag{11}$$

We next argue that $a' > a$. Since $RHS(q, P_1(q))$ is increasing faster than $RHS(q, P_2(q))$, due to the greater efficiency of P_2, the difference $RHS(q, P_1(q)) - RHS(q, P_2(q))$ is increasing, and we conclude that $|C'D|$ is larger than $|CB|$. Furthermore, since $RHS(q, P_2(q))$ is concave, due to the concavity of $p_2(q)$, $|B'C'|$ is larger than $|C'D|$, and we conclude that the length of $[C, B]$ is at most that of $[C', B']$ and thus

$$a < a'. \tag{12}$$

Note that the triangles AFB and $A'F'B'$ are similar right triangles with angle $\alpha_1 = 90^\circ - \beta = \alpha'_1$. We prove next that $a_1 = |BF| < a'_1 = |B'F'|$. Since the triangles are similar it will follow that $h < h'$ as stated. For a contradiction, suppose not, and $a'_1 \leq a_1$; then $c' \leq c$ since the triangles are similar. Applying the Law of Sines to triangle ABC, we get $c/\sin(\gamma) = b/\sin(\beta)$, or $c = b\sin(\gamma)/\sin(\beta)$. Similarly, for $A'B'C'$, we get $c' = b'\sin(\gamma')/\sin(\beta)$. Since $c' \leq c$, $1 \geq c'/c = b' \cdot \sin(\gamma')/b \cdot \sin(\gamma)$, and therefore $b' \times \sin(\gamma') \leq b \times \sin(\gamma)$. This, with Equation (10), and γ and

γ' are both less than 90°, implies that

$$\frac{b'}{b} \leq \frac{\sin(\gamma)}{\sin(\gamma')} < 1.$$

From first principles, $a_2 = b\sin(\alpha_2)$ and $a_2' = b'\sin(\alpha_2')$. The fact that $\alpha_1 = \alpha_1'$ and Equation (11) imply that $\alpha_2 > \alpha_2'$; since triangle ACF is a right triangle, $\alpha_2 < 90^\circ$ and $\sin(\alpha_2')/\sin(\alpha_2) < 1$. Therefore,

$$\frac{a_2'}{a_2} = \frac{b'}{b} \times \frac{\sin(\alpha_2')}{\sin(\alpha_2)} < 1.$$

Therefore, $a_2' < a_2$ and with the assumption that $a_1' \leq a_1$ we get $a' = a_1' + a_2' < a_1 + a_2 = a$, which contradicts Equation (12). Therefore, $a_1 < a_1'$ and since the triangles AFB and $A'F'B'$ are similar, it follows that all sides of $A'F'B'$ are larger than the respective sides of AFB. In particular, $h' > h$, as stated. ■

Remark 1. Note that the less-efficient supplier, with marginal production cost function $p_1(q)$, is not required to have $p_1(q)$ concave in order for the statement of Theorem 10 to hold. Furthermore, the marginal production cost functions are not required to be increasing for the theorem to hold; if they are decreasing, but slower than marginal revenues, then the theorem still holds (c.f., Theorem 9).

Theorem 10 allows us to conclude that, under the stated conditions, the effect of decreased uncertainty is more pronounced for efficient production functions than for inefficient ones. In other words, the inefficient production functions force lower ordering quantities by the buyer, which are less affected by uncertainty. In contrast, suppliers with more-efficient production are better able to respond effectively to increased uncertainty in a supply chain.

6. Conclusions

In this article we investigate, for the first time, the effect of general production cost functions on the behavior of a supplier and a buyer, who interact via a price-only contract. We show the surprising result that a buyer will *always* benefit from increased supplier production efficiency. We demonstrate that, as long as fixed production costs do not grow excessively, the supplier benefits from increased efficiency as well. We analyze the impact of increased demand uncertainty on order quantities and the profit implications for the parties involved. The interaction of demand uncertainty and production efficiency is also studied, and it is shown that increased production efficiency results in the optimal contract purchasing quantities being more sensitive to risk. This new concept of risk agility allows us to conclude that increased production efficiency allows a firm to be more responsive to changes in risk.

Future directions of this work could consider an extension of our price-only contracts studied here to other supply chain contracts, such as buy-back and revenue-sharing contracts, as per the effect of general production cost functions, corresponding efficiency, and uncertainty agility. It would also be interesting to investigate how general production cost functions affect the entire supply chain, in particular the system inefficiency as represented by double marginalization.

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Dorit S. Hochbaum is a Chancellor full professor at the University of California at Berkeley in Industrial Engineering and Operations Research (IEOR). Professor Hochbaum holds a Ph.D from the Wharton school of Business at the University of Pennsylvania. Her research interests are in the design and analysis of efficient algorithms, approximation algorithms, network flow–related problems, efficient utilization of resources, and discrete and continuous optimization. Her recent application areas of research relate to developing efficient methodologies for image segmentation, clustering, customer segmentation, prediction, ranking, group decision making and data mining. She has also investigated inverse problems, with applications varying from medical prognosis and error correction to financial risk assessment and prediction. Professor Hochbaum is the author of over 150 papers that appeared in the operations research, management science, and theoretical computer science literature. Professor Hochbaum was named in 2004 an honorary doctorate of Sciences of the University of Copenhagen for her work on approximation algorithms. In 2005 Professor Hochbaum was awarded the title of INFORMS fellow. She is the winner of the 2011 INFORMS Computing Society prize for her work on algorithms for image segmentation. In 2014 she was selected as a Fellow of the Society for Industrial and Applied Mathematics (SIAM).

Michael R. Wagner is an assistant professor of operations management at the Michael G. Foster School of Business at the University of Washington. His research interests are in decision making under uncertainty (stochastic, online, and robust optimization, as well as hybrids thereof). In particular, he investigates the value of information in a variety of application domains, such as supply chain management, inventory control, resource allocation, and logistics.