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Decision Support

Range contracts: Risk sharing and beyond

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ABSTRACT

We introduce and study the *range* contract, which allows a buyer to procure from a supplier at a prescribed price any amount within a specified range. In return, the supplier is compensated up front for the width of the range with a *range fee*. This fee can be viewed as the buyer trading monetary value for reduced uncertainty. The range contract generalizes and unifies many common contracts, such as fixed-price, JIT, option, and quantity-flexibility contracts. The parameters that maximize the expected profit of the centralized supply chain are derived here and are shown to crucially depend on production flexibility. We also study here the buyer's expected profit-maximizing range endpoints as a function of the pricing parameters of the contract. Using the buyer's optimal range, we demonstrate how the supplier can set the contract's pricing parameters so as to maximize the supplier's expected profit for a uniform distribution of demand. We provide computational evidence, for uniformly distributed demand, that the range contract allows the optimal decentralized supply chain to attain significant reductions in standard deviation of profit in exchange for moderate reductions in expected value of profit. We further demonstrate computationally that both the buyer and supplier can benefit simultaneously, attaining higher risk-adjusted profits than the centralized supply chain.

1. Introduction

We introduce the *range* contract, which allows a buyer and supplier to share demand risk in a new way. In a range contract a range is available to the buyer who can order any quantity in the range, but pays in advance for the flexibility offered, as measured by the width of the range interval. The width of the range and the payment for it compensates the supplier for her flexibility. Many common contracts, such as fixed-price, just-in-time, option, quantity-flexibility as well as combinations of contracts (e.g., pairing of fixed-price and option contracts) can be cast as range contracts. However, the range contract has properties that are not apparent in the other contracts.

A unique characteristic of the range contract is that, despite risk-neutral firm decision making, risk reduction properties are achieved. We provide computational evidence, for uniformly distributed demand, that the range contract allows the optimal decentralized supply chain to attain significant reductions in standard deviation of profit in exchange for moderate reductions in expected value of profit. For example, a decentralized supply chain can attain 94.5 percent of the centralized supply chain's expected profit, yet only 80 percent of its standard deviation of profit. We utilize the notion of an *optimal risk-adjusted profit*, which is the maximized expected profit of a firm divided by its corresponding standard deviation. We demonstrate that the range contract makes it possible that both the supplier and buyer have higher optimal risk-adjusted profits than the centralized supply chain. Therefore, the range contract allows a "win-win" situation where both firms benefit from decentralization. Since the centralized supply chain can always mimic a decentralized supply chain, a managerial implication of this computational evidence is that the range contract can be used by those centralized supply chains where the reduction of risk is a priority.

The form of the range contract proposed here is motivated by a prevalent high-tech market environment characterized by inflexible production and short-lifecycle products. Due to the short lifecycles, demand learning is difficult, resulting in poor quality forecasts with substantial variability. Inflexible production diminishes a manufacturer's (buyer's) ability to respond to demand surprises, resulting in lost sales and loss of any first mover advantage.

Range contracts are especially relevant to the semiconductor industry, where capacity is expensive and excess capacity is a luxury. The range contract is a generalization of an option contract, and option contracts have been applied successfully in the semiconductor industry. For example, a recent Bloomberg Businessweek article, King (2012) reported that Intel has saved $125 million during 2008–2012 due to option contracts. As another example, according to executives at AMD's Memory Group, "supply agreements are important to chip makers because they guarantee that the billions of dollars invested

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in new production facilities will actually be used." (Associated Press 2001). Range contracts can be interpreted as the next generation of option contracts in this industry. Therefore, the motivation to apply range contracts is already present at supplier and buyer firms in high technology industries.

Related industries, where capacity is expensive and limited, can also benefit from the range contract, especially in the context of new products with volatile demand. For example, Apple experienced a shortage of screens for the iPod Touch shortly after launching the product in autumn 2007. In this case, the manufacturer did not anticipate the enthusiastic response to the product and did not contract for a sufficient quantity of touch-screens. To make matters worse, there were no alternative suppliers that had production capacity and could provide the additional units, regardless of the price. Consequently, Apple experienced substantial backlogs, likely loss of goodwill, and loss of sales (although made up since then). In this case, the manufacturer absorbed the total risk of the demand variability, with severe repercussions. In February 2010 it appeared that for its new iPad, Apple had contracted for all the available supplies of 9.7 inch screens, creating a shortage of such screens in the market for its competitors, as reported by another Bloomberg Businessweek article (Guglielmo & Hesseldahl, 2010).

The range contract studied here originated in the first author's consulting project for a large server manufacturer in Silicon Valley. That manufacturer (buyer) procured supplies from semiconductor manufacturers for products with short lifecycles and a high obsolescence rate. This manufacturer historically utilized fixed-price contracts, on a quarterly basis, which resulted in component shortages and lost sales. For products that turned out to be successful, these shortages resulted in a loss of momentum in the product introduction, resulting in a loss of any first-mover advantage. In many of these cases, contractual reservation of extra capacity compensating suppliers for their flexibility, would have resulted in the suppliers providing the additional components required to meet the extra demand.

1.1. Comparison to contracts in related literature

There have been many extensions of the basic price-only contract, which include: multiple selling seasons (Anupindi & Bassok, 1998), effort-dependent demand (Corbett & DeCroix, 2001), demand updating (Cachon & Lariviere, 2005), and competing manufacturers (Cachon & Kok, 2010), to name just a few. What we are proposing is a different generalization of the price-only contract, in which the demand support is split between the buyer and supplier. The issue of the risk associated with price-only contracts has been noted and addressed in research on contracts. Tsay, Nahmias, and Agrawal (1999) and Cachon (2003) provide good reviews of the literature on supply chain coordination with contracts.

The paper most relevant to ours is Cachon and Lariviere (2001), which studies a similar contract, which consists of a combination of firm order commitments and options for subsequent orders. However, they consider an environment with capacitated production with a single unit cost, whereas we allow cheap and expensive modes of production. Furthermore, the focus of Cachon and Lariviere (2001) is heavily on compliance, whereas we focus on quantifying the risk reduction properties of our contract (despite the firms taking a risk-neutral perspective). A study of option contracts in the semiconductor industry can be found in Brown and Lee (1997), and an analysis of a spot market's influence on option contracts can be found in Wu, Kleindorfer, and Zhang (2002). Recall that option contracts are a special case of range contracts (see Section 1.3).

Tsay (1999) studies a *quantity flexibility contract*, generalized by the range contract, where the buyer's final order quantity must be within a given percentage of an initial forecast. Bassok and Anupindi (1997) consider a different generalization of the quantity flexibility contract that specifies that cumulative orders placed over multiple periods by a buyer be at least as large as a given contracted quantity; in return for the commitment by the buyer, the supplier discounts the unit purchase price and this discount applies to all units purchased, with no upper bound on the order quantity. More complex variations on quantity flexibility contracts are considered by Tsay and Lovejoy (1999) and by Plambeck and Taylor (2007).

There are only a few other models that present some form of risk sharing. In addition to the basic quantity flexible contract, there are the *buy-back contract* and the *revenue-sharing contract*. In a buy-back contract, the supplier charges the buyer a fixed amount per unit purchased, but makes a (lower) per-unit payment to the buyer for each unit remaining at the end of the season; if the supplier's net salvage value is less than the buyer's net salvage value, the buyer salvages the units and the supplier credits the buyer for those units. See Pasternack (1985) and Cachon (2003) for further details. In a revenue-sharing contract, the supplier charges a fixed amount per unit purchased by the buyer, but the buyer gives the supplier a percentage of his revenue; see Cachon and Lariviere (2005) for further details. Cachon (2003) points out that, in their basic form, revenue-sharing contracts are essentially equivalent to buy-back contracts. In these contracts, the supplier ends up producing a *fixed* amount, and then *monetary* compensation substitutes for risk sharing. In contrast, under the generic *range contracts* proposed here, the supplier is required to face actual demand variability.

The range contract that we propose, like the quantity-flexibility contract, has the feature that the supplier is compensated for her increased exposure to demand risk. Tsay et al. (1999) point out that with (other types of) quantity flexible contracts, "this exercise of flexibility implies reconsideration of a prior decision, even the simplest model requires at least two decisions on the part of the buyer for each purchase: there is an initial inventory decision, and then revisions conditional on whatever new information about demand becomes available." In contrast, an additional novelty about the range contract is that it does not require a reconsideration of the decision—the buyer and supplier make a decision only *once*.

Other authors have considered contracts over multiple periods. Bassok and Anupindi (2008) study the problem of procurement using a flexible contract in a rolling horizon model. They formulate the problem and propose two heuristic policies, derive a lower bound, and demonstrate the performance of these heuristics numerically. Ross and Zhu (2008) study the procurement policy for a non-storable product (e.g. electricity) using a flexible contract in which the purchase quantity in each period must be within some predefined range. They formulate the objective value as the total of gains and losses between the contract price and the spot price. The structure of a swing contract's optimal value is then studied. A contract form between a buyer and a supplier with a total order quantity commitment over a multiple-period horizon is studied in Chen and Krass (2001). Under the contract the buyer agrees to procure a certain total quantity over the predetermined period of time. Extra quantity could be purchased at a different price. Dynamic considerations and inventory issues are beyond the scope of our paper and could form the basis of an extension study of the range contract.

1.2. The structure of the range contract

We model the demand D as a continuous random variable with distribution F, with mean and standard deviation equal to μ and σ, respectively. We assume that the support of the demand distribution is $[\ell, u]$, where $0 \le \ell \le u$. The interaction between the supplier and buyer follows a Stackelberg game, a common modeling technique in the contracting literature (see, for example, the large variety of contract analyses in the survey of Cachon, 2003). Indeed, most contracts, including fixed-price, buy-back, quantity-flexibility, quantity-discount, and sales-rebate, have the supplier proposing the contract's pricing parameters and the buyer choosing the order quantities.

In the range contract, the supplier first announces the per-unit *range fee* α and the per-unit wholesale price c. The buyer responds with the range $[x_1, x_2]$, by which the buyer commits to buy a minimum of x_1 units, for a total of q units, for q in the interval $[x_1, x_2]$. If the demand realized exceeds x_2, then excess demand is not covered by the supplier. If demand is less than x_1, the buyer incurs overage costs of c times the difference between x_1 and the realized demand. For quantities q not exceeding x_2, the buyer pays the supplier $cq + \alpha(x_2 - x_1)$. That is, the supplier gets, in addition to the wholesale value of the quantity purchased, also a bonus in the form of the range fee, that grows with the length of the range $\alpha(x_2 - x_1)$.

A non-trivial range contract is one where $x_1 < x_2$ and $[x_1, x_2] \subset [\ell, u]$. We show that a variety of contractual pricing and firm interactions are possible for a range contract, and demonstrate how many existing contracts fall within our framework.

1.3. How range contracts generalize existing contracts

The range contract is a strict generalization of fixed-price, JIT, option, and quantity-flexibility contracts:

- For $[x_1, x_2] = [\ell, u]$ and $\alpha = 0$, the range contract is a JIT contract.
- For $x_1 = x_2$, the contract is a fixed-price contract.
- In a quantity-flexibility contract, a buyer provides an initial forecast q and is obligated to order at least $(1 - \gamma)q$ for some $\gamma \in [0, 1]$, at a unit wholesale price of c, and the supplier is obligated to provide capacity up to $(1 + \gamma)q$. This quantity-flexibility contract is a range contract with $x_1 = (1 - \gamma)q$, $x_2 = (1 + \gamma)q$, and $\alpha = 0$.
- A range contract with $x_1 = 0$ is equivalent to an option contract with reserved capacity of x_2, unit reservation price α and unit exercise price c.

The range contract can also be viewed as a combination of two contracts: a price-only contract with fulfillment level x_1 and unit wholesale price c, and an option contract with reservation price α, exercise price c, and reserved capacity $x_2 - x_1$.

1.4. Supplier production costs and flexibility

In contrast to much of the contracting literature, we study the impact of varying production costs on firm and supply chain performance. The supplier's unit-production cost is typically not fixed. The supplier who is given a fixed order quantity can usually streamline her production operations so as to achieve an efficient production line with low unit-production costs. But a supplier facing uncertainty, as in a JIT context, would not be able (in general) to produce at the minimum possible unit-production cost. We study these two modes of production and let the most efficient unit production cost be p and the less efficient unit production cost under uncertainty be $p_1 > p$.

The supplier faces demand uncertainty on $[x_1, x_2]$, but is required to meet demand, as dictated by the contractual terms. The supplier will determine a production quantity $Q \in [x_1, x_2]$ in advance, where p is the unit production cost for these first Q units. Then, if realized demand exceeds this initial production amount, the supplier must produce again to meet demand, where p_1 is the unit production cost for quantities that exceed Q in the range $[x_1, x_2]$. The Newsvendor model is applied to find the appropriate value of Q.

We quantify the *supplier's flexibility* as the ratio of the unit production cost under a guaranteed order quantity p to the unit production cost under uncertainty $p_1 > p$: The supplier's flexibility is defined as $p/p_1 \in [0, 1]$. The closer the ratio is to 1, the higher the flexibility of the supplier since she is able to operate under uncertainty at a cost that is close to that under no uncertainty. We will show that the supplier's flexibility plays an important role under the range contract, as a key factor in determining the supplier's optimal choice of per-unit range fee.

We also show in Section 4 that the solution of a centralized (vertically integrated) supply chain can be viewed as having a range structure influenced by the supplier's flexibility. The centralized system faces uncertain demand. It can streamline its production for a guaranteed amount at a cheaper unit production cost p. If demand realized exceeds the amount produced, then additional production can take place at the higher unit production cost $p_1 > p$, up to an upper bound. The solution to this system has a range interpretation, where the guaranteed production amount is x_1 and the upper bound on production is x_2.

We show that the optimal lower bound x_1 depends on the flexibility of the firm p/p_1. For a fully flexible firm ($p_1 = p$), $x_1 = \ell$. This implies that the less flexibility a firm has, the higher the value of guaranteed minimum production x_1, and the overage cost is larger. As for the upper end of the interval x_2, we show that $x_2 = u$. This means that in a vertically integrated supply chain, the optimal range has all demands satisfied internally.

1.5. Contributions

The main contributions here are:

- Introducing the range contract, and showing it generalizes and unifies fixed-price, JIT, option, and quantity-flexibility contracts.
- Solving for the buyer's optimal range $[x_1^*, x_2^*]$ that maximizes his profits for a given c and α, spot price s, and unit revenue r. We also provide an extension for a convex range fee function.
- Solving for the supplier's optimal per-unit range fee α^* and unit wholesale price c^* that maximize her profit, as a function of the buyer's optimal range $[x_1^*, x_2^*]$, for uniformly distributed demand.
- Providing insights on how the selection of the supplier's optimal per-unit range fee α^* depends on her production costs p and p_1, and the spot price s.
- Deriving the structure of the optimal range in a vertically integrated supply chain, and showing that the minimum production quantity grows as the firm's production flexibility increases.
- Showing computationally for a uniform distribution that the range contract allows the optimal decentralized supply chain to attain significant reductions in standard deviation of profit in exchange for moderate reductions in expected value of profit.
- Showing computationally for a uniform distribution that the range contract allows both firms to obtain larger risk-adjusted profits than the centralized supply chain, a "win-win" situation.

Paper outline: In Section 2 we study the range contract and show how the range is determined by the buyer given values of α and c for a general distribution of demand. Section 3 describes the analysis of how the supplier chooses her optimal per-unit range fee and unit wholesale price, for a uniform distribution of demand, and provides a sensitivity analysis for different parameters. Section 4 gives the optimal range for the vertically integrated system. Section 5 analyzes and contrasts the mean-standard deviation profit tradeoffs for the individual firms and centralized supply chain. Section 6 provides concluding remarks.

2. The range maximizing the buyer's expected profit: determining x_1^* and x_2^*

In the next two sections we consider the range contract, where firm interactions are modeled as a Stackelberg game where the supplier is the leader. Therefore, we model the case where each firm maximizes their expected profit. While this approach is risk neutral with respect to the variability of profit, the contractual interaction between firms creates a variability reduction effect. We discuss this in full detail in Section 5.

Under our range contract, c is the unit wholesale price and α is the per-unit range fee, assumed given in this subsection; the next subsection analyzes how the supplier chooses this per-unit range fee and wholesale price optimally. The buyer, facing uncertain demand D, sells to the final customer at a unit revenue of r. For $D < x_1$, the buyer must pay the supplier for x_1 units, but only receives revenue on D units. For $D \in [x_1, x_2]$, the buyer receives a unit profit of $(r - c)$ on all D units. For $D > x_2$, the buyer is able to purchase additional units in the spot market, at a unit cost of s, where $c < s < r$. Regardless of the demand realized, the buyer must pay the supplier the range fee $\alpha(x_2 - x_1)$. Therefore, the buyer's profit random variable is

$$\pi_B(x_1, x_2, \alpha) = -\alpha(x_2 - x_1) + \begin{cases} rD - cx_1, & D < x_1 \\ (r-c)D, & x_1 \le D \le x_2 \\ rD - cx_2 - s(D - x_2), & D > x_2. \end{cases} \quad (1)$$

The decisions available to the buyer are to dictate the range limits x_1 and x_2. The buyer determines his optimal range by maximizing his expected profit:

$$\max_{x_1, x_2} \quad E[\pi_B(x_1, x_2, \alpha)] \quad \text{s.t.} \quad \ell \le x_1 \le x_2 \le u. \quad (2)$$

The next theorem characterizes the buyer's optimal behavior as the solution to Problem (2), as a function of the pricing parameters c and α of the range contract. Subsequently, we provide a generalization for convex range cost functions.

Theorem 1. *The buyer's optimality conditions are*

$$F(x_1^*) = \frac{\alpha}{c} \quad \text{and} \quad F(x_2^*) = 1 - \frac{\alpha}{s - c}.$$

In addition, for $x_1^* \le x_2^*$, $\alpha \le c(1 - \frac{c}{s})$ must hold.

Proof. The expected profit can be written as

$$E[\pi_B(x_1, x_2)] = \int_\ell^{x_1} (rD - cx_1) f(D) dD + \int_{x_1}^{x_2} (r - c) D f(D) dD + \int_{x_2}^{u} (rD - cx_2 - s(D - x_2)) f(D) dD - \alpha(x_2 - x_1).$$

The derivatives with respect to x_1 and x_2 are

$$\frac{\partial E[\pi_B]}{\partial x_1} = (r - c) x_1 f(x_1) - c \int_\ell^{x_1} f(D) dD - (r - c) x_1 f(x_1) + \alpha = \alpha - cF(x_1)$$

and

$$\frac{\partial E[\pi_B]}{\partial x_2} = (r - c) x_2 f(x_2) - (r - c) x_2 f(x_2) + (s - c) \int_{x_2}^{u} f(D) dD - \alpha = (s - c)(1 - F(x_2)) - \alpha,$$

respectively. The cross derivatives vanish and the second derivatives are:

$$\frac{\partial E[\pi_B]^2}{\partial x_1^2} = -cf(x_1)$$

$$\frac{\partial E[\pi_B]^2}{\partial x_2^2} = -(s - c) f(x_2).$$

These second derivatives are non-positive as $s > c$, the profit function is concave, and the first order conditions maximize the profit. Finally, $F(x_1) \le F(x_2)$ implies the *necessary condition*:

$\alpha \le c(1 - c/s)$. □

Remark 1. Note that if there is no spot market from which to purchase additional products, lost sales can be modeled by modifying the third case $(D > x_2)$ of Eq. (1) to $(r - c)x_2$. The second optimality condition of Theorem 1 would be modified slightly to $F(x_2^*) = 1 - \frac{\alpha}{r-c}$. Subsequent results hold exactly with the substitution $s = r$.

2.1. Convex range cost

Suppose now that the cost of the range is not linear in the length of the contractual range, $x_2 - x_1$, but rather is a convex function of this length. Thus $\alpha(\cdot)$ is a convex function of the range, $\alpha(x_2 - x_1)$, which is continuous and twice differentiable in x_1 and x_2. Using similar proof techniques, we have a generalization of Theorem 1.

Theorem 2. *The buyer's optimality conditions are*

$$F(x_1^*) = \frac{\alpha'(x_2^* - x_1^*)}{c} \quad \text{and} \quad F(x_2^*) = 1 - \frac{\alpha'(x_2^* - x_1^*)}{s - c}$$

In addition, for $x_1^* \le x_2^*$, $\alpha'(x_2^* - x_1^*) \le c(1 - \frac{c}{s})$ must hold.

3. Maximizing the supplier's expected profit: determining α^* and c^*

In the range contract, the parameter c is the contractual unit wholesale price, and the supplier determines the per-unit range fee α. We also discuss the two modes of supplier production, and study their effect on the performance of the range contract.

3.1. Supplier production modes

The supplier, knowing she will face demand in the interval $[x_1, x_2]$, where the endpoints satisfy the optimality conditions of Theorem 1, will use a Newsvendor model to determine a production quantity $Q \in [x_1, x_2]$, for which she will streamline production and achieve cost per unit p. If demand D is below Q, the supplier incurs costs on $Q - D$ unsold units. In the presence of salvage values v per unit, this cost will be $p - v$ per unsold unit, where we assume $v < p$. Recalling that the supplier *must* meet demand in the interval $[x_1, x_2]$, if demand realized D is above Q, the supplier must produce $D - Q$ units at the higher unit production cost p_1. This results in additional per-unit costs on the extra units of $p_1 - p$. Under the range contract, the supplier effectively receives an order quantity of x_1 if $D < x_1$, an order quantity of D if $D \in [x_1, x_2]$, and an order quantity of x_2 if $D > x_2$. Therefore, the supplier's distribution of orders is

$$\tilde{F}_{[x_1, x_2]}(D) = \begin{cases} \int_\ell^{x_1} f(x) dx, & D = x_1 \\ F(D), & D \in [x_1, x_2] \\ \int_{x_2}^{u} f(x) dx, & D = x_2, \end{cases} \quad (3)$$

and the supplier's optimal Newsvendor quantity Q satisfies

$$\tilde{F}_{[x_1, x_2]}(Q) = \frac{(p_1 - p)}{(p_1 - p) + (p - v)} = 1 - \frac{p - v}{p_1 - v}. \quad (4)$$

Note that this result suggests that, in the presence of positive salvage values, the definition of production flexibility should be modified to $(p - v)/(p_1 - v)$. In other words, having the opportunity to salvage "improves" production flexibility. For simplicity, in the sequel we let the salvage value $v = 0$. Consequently, since $Q \in [x_1, x_2]$ and applying Eqs. (3) and (4) with $v = 0$, the Newsvendor quantity Q must satisfy:

$$Q = \max\left\{x_1, \min\left\{x_2, F^{-1}\left(1 - \frac{p}{p_1}\right)\right\}\right\}$$

3.2. The supplier's profit random variable

We next formulate the supplier's profit random variable, which combines the revenues from the buyer and the supplier's internal production costs. For demand $D < Q$, the supplier incurs production costs of pQ, whereas for demand $D > Q$, the incurred production costs are $pQ + p_1(D - Q)$. For $D < x_1$, the supplier receives a unit revenue of c from the buyer on x_1 units. For $D \in [x_1, x_2]$, the supplier receives revenue on D units, and for $D > x_2$, the supplier only receives revenue on the first x_2 units. Under all demand scenarios, the supplier receives the range fee premium of $\alpha(x_2 - x_1)$. The supplier's profit random variable is

$$\pi_S(x_1, x_2, \alpha) = \alpha(x_2 - x_1) + \begin{cases} cx_1 - pQ, & D < x_1 \\ cD - pQ, & x_1 \le D \le Q \\ cD - pQ - p_1(D - Q), & Q \le D \le x_2 \\ cx_2 - pQ - p_1(x_2 - Q), & D > x_2. \end{cases} \quad (5)$$

To formulate the profit-maximization problem, recall that the supplier correctly anticipates that the buyer will choose $x_1 = F^{-1}(\alpha/c)$ and $x_2 = F^{-1}(1 - \alpha/(s - c))$, the buyer's profit maximizing range from Theorem 1. Consequently, the supplier will select α to maximize her own profit:

$$\begin{aligned} \max_{\alpha} \quad & E[\pi_S(x_1, x_2, \alpha)] \\ \text{s.t.} \quad & \alpha \le c\left(1 - \frac{c}{s}\right) \\ & F(x_1) = \alpha/c \\ & F(x_2) = 1 - \alpha/(s - c) \\ & \tilde{F}_{[x_1, x_2]}(Q) = 1 - \frac{p}{p_1}. \end{aligned} \quad (6)$$

The first constraint is to enforce $x_1 \le x_2$, as required by the range contract definition, which follows from Theorem 1. The last three constraints represent the buyer's optimal behavior and the supplier's Newsvendor production quantity.

In the remainder of this section we analytically solve Problem (6) for demand that is uniformly distributed on $[\ell, u]$. In particular, we study the supplier's optimal per-unit range fee, taking into account the buyer's optimal behavior. We also investigate the sensitivity of the supplier's optimal per-unit range fee to production costs, and the market spot price. Although the results are presented for a uniform demand distribution, we conjecture that similar structural relationships hold for other distributions as well; Taylor (2002) takes a similar approach of analyzing uniformly distributed demand, and conjecturing behavior for general demand distributions. Furthermore, our analysis techniques can be applied to a given distribution (e.g., poisson, gamma, etc.), as long as the density is positive on $[\ell, u]$. Unfortunately, we found a general analysis in terms of a generic distribution F to be intractable.

3.3. The supplier's optimal choice of α for $D \sim U[\ell, u]$

Here we study the supplier's selection of the optimal per-unit range fee α. We then study how this behavior depends on production costs, and the spot market price. We begin with a structural result that allows us to easily solve for the profit-maximizing per-unit range fee α.

Theorem 3. *The supplier's profit function is strictly concave in α.*

Proof. Note that $\frac{\partial x_1}{\partial \alpha} = \frac{1}{cf(x_1)}$ and $\frac{\partial x_2}{\partial \alpha} = -\frac{1}{(s-c)f(x_2)}$, which are derived using the derivative of the inverse of a function. Also, $\frac{\partial Q}{\partial \alpha} = 0$. The expected value of the supplier's profit can be written as

$$\begin{aligned} E[\pi_S(\alpha)] = & \int_l^{x_1} (cx_1 - pQ)f(D)dD + \int_{x_1}^{Q} (cD - pQ)f(D)dD \\ & + \int_Q^{x_2} (cD - pQ - p_1(D - Q))f(D)dD \\ & + \int_{x_2}^{u} (cx_2 - pQ - p_1(x_2 - Q))f(D)dD + \alpha(x_2 - x_1). \end{aligned} \quad (7)$$

Using the Leibniz integral rule, the derivative of the supplier's profit function (7) with respect to α is

$$\begin{aligned} \frac{\partial E[\pi_S(c, \alpha)]}{\partial \alpha} & = c\frac{\partial x_1}{\partial \alpha}F(x_1) + (c - p_1)\frac{\partial x_2}{\partial \alpha}(1 - F(x_2)) \\ & \quad + \alpha\left(\frac{\partial x_2}{\partial \alpha} - \frac{\partial x_1}{\partial \alpha}\right) + (x_2 - x_1) \\ & = \frac{\alpha}{cf(x_1)} - \frac{\alpha(c - p_1)}{(s - c)^2 f(x_2)} \\ & \quad - \alpha\left(\frac{1}{(s - c)f(x_2)} + \frac{1}{cf(x_1)}\right) + (x_2 - x_1). \end{aligned}$$

When demand is uniformly distributed, we note that $x_1 = \ell + (u - \ell)\frac{\alpha}{c}$, $x_2 = \ell + (u - \ell)(1 - \frac{\alpha}{s-c})$, and $f(x_1) = f(x_2) = 1/(u - \ell)$. In this case, the derivative of the profit becomes

$$\frac{\partial E[\pi_S](c, \alpha)}{\partial \alpha} = -(u - \ell)\left(\alpha\left(\frac{(c - p_1)}{(s - c)^2} + \frac{2}{s - c} + \frac{1}{c}\right) - 1\right). \quad (8)$$

Taking the second derivative, we get

$$\frac{\partial^2 E[\pi_S](c, \alpha)}{\partial \alpha^2} = -(u - \ell)\left(\frac{(c - p_1)}{(s - c)^2} + \frac{2}{s - c} + \frac{1}{c}\right),$$

which allows us to conclude that the profit function is strictly concave. □

Theorem 3 shows that the first order condition is sufficient for finding the maximizing value of α, and the next theorem determines the optimal value of the per-unit range fee. Let α^* denote the value

$$\alpha^* = \frac{c(s - c)^2}{s^2 - cp_1}. \quad (9)$$

Theorem 4. *The supplier's profit-maximizing value of α is α^*.*

Proof. By Theorem 3, the profit function is strictly concave, and we set the profit derivative (8) to zero for uniformly distributed demand to find the optimal per-unit range fee α^*, which is equal to

$$\alpha^* = \frac{c(s - c)^2}{s^2 - cp_1}.$$

Note that algebraic manipulations show that $\alpha^* < c(1 - c/s)$ (cf., the constraints of the supplier's problem), which proves that α^* is feasible. □

Theorem 4 provides the profit-maximizing value of α, as given in Eq. (9), which solves the supplier's profit-maximization problem. The structure of α^* provides insight into how the different problem parameters affect the optimal range fee. The next results focus on the effect of production flexibility, and how it affects the optimal per-unit range fee.

Note that α^* is increasing in p_1, which shows that diminished ability to respond to realized uncertain demand (i.e., a higher p_1) induces the supplier to require higher compensation for involvement in the contract. We can also use this observation to make a statement about production flexibility, which is measured by the ratio p/p_1. As the ratio increases, production flexibility increases, since the supplier is able to operate under uncertainty at a cost that is close to that under no uncertainty. Therefore, for fixed p, as flexibility increases (p_1 decreases), α^* decreases. Therefore, more flexible production allows a supplier to charge the buyer a lower per-unit range fee.

We also characterize α^*'s dependence on the spot market price s. The derivative $\frac{\partial \alpha^*}{\partial s} = \frac{2c^2(s-c)(s-p_1)}{(cp_1-s^2)^2} > 0$, which shows α^* is increasing in s. This shows that, holding all other parameters constant, increasing the spot price of the product allows the supplier to charge a higher per-unit range fee premium, since the attractiveness of the spot market for the buyer is decreased.

Finally, we characterize α^*'s dependence on the wholesale price c. The derivative $\frac{\partial \alpha^*}{\partial c} = \frac{(s-c)\left(s^2(s-c)-2c(s^2-cp_1)\right)}{(s^2-cp_1)^2}$. Unsurprisingly, the sign of the derivative depends on the parameters, and we are unable to make general statements regarding the relationship of α^* and c.

3.4. The supplier's optimal choice of c for $D \sim U[\ell, u]$

We next study how the supplier would choose the per-unit wholesale price c. While analytically intractable, even for uniformly distributed demand, we are able to show numerically that $\frac{\partial E[\pi_S(x_1^*, x_2^*, \alpha^*)]}{\partial c} > 0$, for a large variety of parameter values. In other words, the supplier's expected profit is increasing in c, and the supplier prefers to increase it to its maximum value of $c^* = s$. If this were possible, then $\alpha^* = 0$, $x_1^* = \ell$ and $x_2^* = u$, and the range contract simplifies to a JIT contract.

However, in many cases the wholesale price is set by the marketplace (e.g., commodities) or as the result of firm negotiations. Therefore, in Section 5, we present computational results as a function of c, so that we can identify what value the range contract will have for a given wholesale price.

4. Maximizing the centralized supply chain expected profit

The concept of a range $[x_1, x_2]$ is not only relevant in the context of contracts, but is also meaningful for the vertically integrated supply chain, where the buyer and supplier are part of a single firm. In this context, we identify an optimal range that maximizes the expected centralized supply chain profit. The centralized profit is defined by the summation of the buyer and supplier's profit random variables, from (1) and (5), respectively; we let the centralized range be denoted by $[y_1, y_2]$ to avoid confusion with the decentralized range $[x_1, x_2]$:

$$\pi_{SC}(y_1, y_2) = \pi_B + \pi_S = \begin{cases} rD - py_1, & D < y_1 \\ rD - py_1 - p_1(D - y_1), & y_1 \le D \le y_2 \\ rD - py_1 - p_1(y_2 - y_1) - s(D - y_2), & D > y_2. \end{cases} \tag{10}$$

The interpretation of the lower bound of this range is that the centralized supply chain will commit to and produce the amount y_1 regardless of the demand realization, at the cheaper unit production cost p. Here, for the centralized system, y_1 is analogous to Q, the optimal supplier Newsvendor quantity discussed in Section 3.1. The system will then produce in a JIT manner, up to the upper bound of the interval y_2, at the higher production cost p_1. For demand beyond the upper production limit of y_2, the system can procure extra units in the spot market with spot price s. The centralized supply chain optimal range solves the following problem:

$$\max_{\ell \le y_1 \le y_2 \le u} E[\pi_{SC}(y_1, y_2)] \tag{11}$$

The next theorem characterizes the centralized supply chain's optimal range as the solution to Problem (11).

Theorem 5. *The centralized supply chain's optimality conditions are*

$$F(y_1^*) = 1 - \frac{p}{p_1} \qquad \text{and} \qquad F(y_2^*) = 1.$$

Proof. Let $\pi_{SC} = \pi_B + \pi_S$ be the system's profit. Then, the expected profit can be written as

$$E[\pi_{SC}] = \int_{\ell}^{x_1} (rD - px_1) f(D) dD + \int_{x_1}^{x_2} (rD - px_1 - p_1(D - x_1)) f(D) dD + \int_{x_2}^{u} (rD - px_1 - p_1(x_2 - x_1) - s(D - x_2)) f(D) dD$$

Here the partial derivatives are:

$$\frac{\partial E[\pi_{SC}]}{\partial x_1} = (p_1 - p) - p_1 F(x_1), \text{ and} \tag{12}$$

$$\frac{\partial E[\pi_{SC}]}{\partial x_2} = (s - p_1)(1 - F(x_2)). \tag{13}$$

The cross derivatives both vanish and the second derivatives are both nonnegative:

$$\frac{\partial^2 E[\pi_{SC}]}{\partial x_1^2} = -p_1 f(x_1) < 0, \text{ and} \tag{14}$$

$$\frac{\partial^2 E[\pi_{SC}]}{\partial x_2^2} = -(s - p_1) f(x_2) < 0. \tag{15}$$

Therefore the function $E[\pi_{SC}](x_1, x_2)$ is convex. Setting both first derivatives to 0 we complete the proof. □

Recall that the production flexibility is p/p_1. This ratio is always less than or equal to 1, and the closer the ratio to 1, the higher the flexibility since the firm is able to operate under uncertainty at a cost that is close to that under no uncertainty. The optimal value of y_1^* in the vertically integrated system is affected by this flexibility. The more flexible the firm is, the lower is the value of y_1^*. For a fully flexible firm ($p_1 = p$), $y_1^* = \ell$ and the supply chain can afford to operate as a JIT system. For less flexible firms, the value of y_1^* is greater than ℓ, which means, in terms of the supply chain, that it is optimal to possibly overproduce at the cheaper unit production cost p, and risk the loss of unsold units, in order to capitalize on the efficient production range. Therefore, there are consequences to supply chains with less flexible production. As for the upper bound of the interval y_2^*, it is always equal to the maximum possible demand u. This is so unless the production flexibility is so low as to have the value of p_1 exceeding the unit revenue r, in which case there will be no production at the unit production cost p_1. Instead, there will be production of a fixed quantity $y_1^* = y_2^*$, which is guaranteed in advance and there is no production under uncertainty.

5. Trading off mean profit reductions for higher standard deviation of profit reductions: computational results

In this section, we demonstrate some unique characteristics of the range contract: Despite risk-neutral firm decision making, risk reduction properties are achieved, both at the (decentralized) supply chain level as well as at the individual firm level. In particular, we contrast the mean and standard deviation of the optimal decentralized supply chain profits with those of the optimal centralized supply chain. We utilize computational experiments for uniformly distributed demand $D \sim U[\ell, u]$.

The firms' optimal profit random variables are determined using the solutions to Problems (2) and (6), where the buyer and supplier maximize their respective expected profits. The solutions to Problem (2) for uniformly distributed demand are

$$x_1^* = F^{-1}\left(\frac{\alpha}{c}\right) = \ell + \left(\frac{\alpha}{c}\right)(u - \ell),$$

and

$$x_2^* = F^{-1}\left(1 - \frac{\alpha}{s - c}\right) = \ell + \left(1 - \frac{\alpha}{s - c}\right)(u - \ell),$$

which we derive from Theorem 1. The solution to Problem (6) for uniformly distributed demand is

$$\alpha^* = \frac{c(s - c)^2}{s^2 - cp_1},$$

as given in Theorem 4. We denote the *optimal buyer profit* by $\pi_B^* = \pi_B(x_1^*, x_2^*, \alpha^*)$ and the *optimal supplier profit* as $\pi_S^* = \pi_S(x_1^*, x_2^*, \alpha^*)$, where $\pi_B(x_1, x_2, \alpha)$ and $\pi_S(x_1, x_2, \alpha)$ are defined in Eqs. (1) and (5), respectively. We let the *optimal decentralized supply chain profit* be denoted as

$$\pi_{dSC}^* = \pi_B^* + \pi_S^*. \tag{16}$$

The centralized supply chain profit will be evaluated using the optimal range (y_1^*, y_2^*) that maximizes the expected profit in Problem (11):

$$y_1^* = F^{-1}\left(1 - \frac{p}{p_1}\right) = \ell + \left(1 - \frac{p}{p_1}\right)(u - \ell)$$

and

$$y_2^* = F^{-1}(1) = u,$$

as shown in Theorem 5. We denote the *optimal centralized supply chain profit* as

$$\pi_{SC}^* = \pi_{SC}(y_1^*, y_2^*), \tag{17}$$

where $\pi_{SC}(y_1, y_2)$ is defined in Eq. (10).

5.1. Small reductions in expected profit lead to large reductions in risk under the range contract

We present here evidence, based on uniformly distributed demand on $[10, 100]$, that the range contract induces an attractive tradeoff between the standard deviation of profit (risk) and expected profit for the optimal decentralized supply chain, with respect to the optimal centralized supply chain: For any percentage reduction in expected profit, the decentralized supply chain obtains a larger percentage reduction in the standard deviation of profit. We begin by defining the ratio of the optimal decentralized expected profit to that of the centralized supply chain:

$$R_\mu = \frac{E[\pi_{dSC}^*]}{E[\pi_{SC}^*]}. \tag{18}$$

Note that $R_\mu \le 1$ precisely measures the degree of double marginalization. We define a similar ratio to measure the relative standard deviation of profit in the two supply chains. Letting SD[X] denote the standard deviation of a random variable X, we define the ratio of the optimal decentralized standard deviation of profit to that of the centralized supply chain:

$$R_\sigma = \frac{\mathrm{SD}[\pi_{dSC}^*]}{\mathrm{SD}[\pi_{SC}^*]}. \tag{19}$$

If $R_\sigma < 1$, then the decentralized supply chain has less variability in profits than the centralized supply chain, a desired quality if the expected profits for the two supply chains are identical. We next present a computational study that provides insight into how R_μ and R_σ are related, for various parameters.

We consider uniformly distributed demand on $[\ell, u] = [10, 100]$. The spot price is $s = 90$ and the unit revenue is $r = 100$. The efficient unit production cost is $p = 10$ and we consider five unit production costs under uncertainty, namely $p_1 \in \{10, 30, 50, 70, 90\}$, to model different degrees of production flexibility. Note that when $p_1 = 10$, $p_1 = p$, and when $p_1 = 90$, $p_1 = s$; these are the minimum and maximum possible values for p_1. In our computational studies, the resulting behavior for $p_1 = 90$ is indistinguishable from that for $p_1 = 70$, and is omitted. In Fig. 1 we plot R_μ as a function of R_σ, where c is varied in $[10, 90]$. Finally, note that $(R_\sigma, R_\mu) = (1, 1)$ when $c = s$ in all plots, meaning that the expected profits and standard deviations of profits are identical for the decentralized and centralized supply chains.

Fig. 1 shows that the range contract introduces a tradeoff between the standard deviation of profit and the expected profit for the optimal decentralized supply chain. Note that for all curves in Fig. 1, the double marginalization inefficiency is at most 5.5 percent, since $R_\mu \ge 0.945$ everywhere. At the same time, the range contract can significantly reduce the standard deviation of the decentralized supply chain profits, with respect to the centralized supply chain. For example, when $p_1 = 30$, the ratio R_σ can reach 0.915, while the ratio R_μ only reaches 0.965; in other words, a reduction of 3.5 percent in

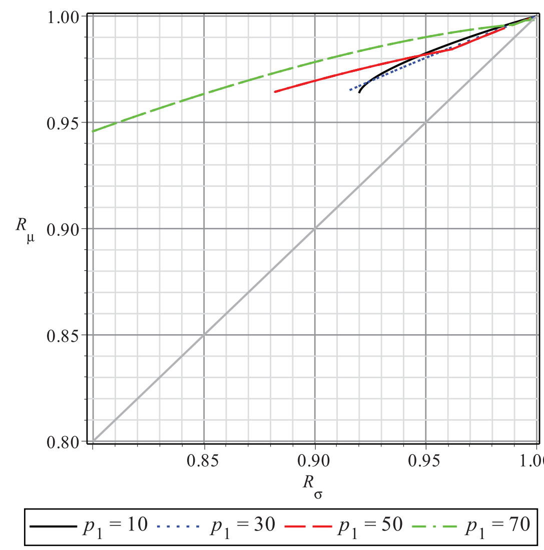

Fig. 1. Tradeoff curve of $R_\mu = \frac{E[\pi_{dSC}^*]}{E[\pi_{SC}^*]}$ versus $R_\sigma = \frac{\mathrm{SD}[\pi_{dSC}^*]}{\mathrm{SD}[\pi_{SC}^*]}$ for $D \sim U[10, 100]$, $p = 10$, $s = 90$, $r = 100$, $p_1 \in \{10, 30, 50, 90\}$ and $c \in [10, 90]$.

expected profit is accompanied by a reduction in the standard deviation of profit of 8.5 percent. For other values of p_1, the tradeoff can be even more appealing: For $p_1 = 70$, the standard deviation of the decentralized supply chain can be reduced by 20 percent ($R_\sigma = 0.80$) from the centralized supply chain's standard deviation, while still only sacrificing approximately 5.5 percent of the expected profit. In all cases, the plots remain above the diagonal, signifying that any percent reduction in expected profit results in at least as much reduction in the standard deviation of profit (and usually much more). We also observe that as the difference between p_1 and p increases, the performance of the decentralized supply chain *improves*, with respect to the centralized supply chain (the curves shift to the left). Therefore, under a range contract, the decentralized supply chain benefits from production inefficiencies, from a risk-adjusted point of view. Since the centralized supply chain can always mimic a decentralized supply chain, the range contract can be used by those centralized supply chains where the reduction of risk is a priority.

Of course, if these benefits are attained by sacrificing one firm's performance, the practical appeal of the range contract vanishes. Therefore, we next study the individual firms and show that both can *simultaneously* benefit from range contract.

5.2. Both supplier and buyer can benefit under the range contract

We define the buyer's *optimal risk-adjusted unit profit* as

$$\pi_B^A = \frac{E[\pi_B^*]}{\mathrm{SD}[\pi_B^*]}. \tag{20}$$

This ratio measures the amount of expected profit that is obtained per unit of standard deviation of profit, and the higher the ratio, the better. Note that this ratio is simply the inverse of the coefficient of variation of the buyer's profit. We similarly define the supplier's *optimal risk-adjusted unit profit* as

$$\pi_S^A = \frac{E[\pi_S^*]}{\mathrm{SD}[\pi_S^*]} \tag{21}$$

and the centralized supply chain's *optimal risk-adjusted unit profit* as

$$\pi_{SC}^A = \frac{E[\pi_{SC}^*]}{\mathrm{SD}[\pi_{SC}^*]} \tag{22}$$

We show that there exists an interval of wholesale prices c where the buyer and supplier benefit in terms of optimal risk-adjusted unit

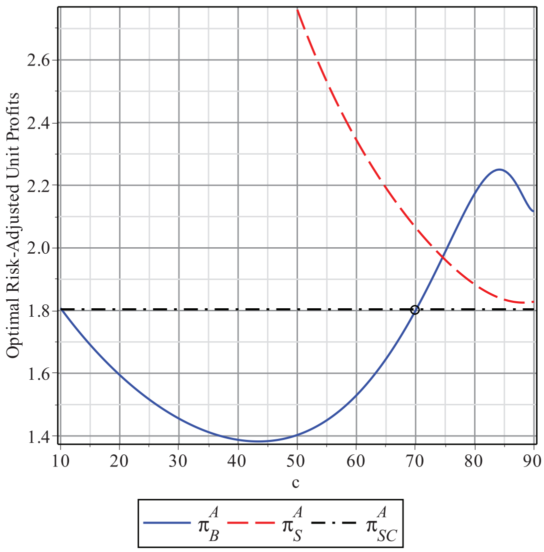

Fig. 2. Optimal risk-adjusted unit profits π_B^A, π_S^A and π_{SC}^A, for $D \sim U[10, 100]$, $p = 10$, $p_1 = 50$, $s = 90$, $r = 100$.

profits for $D \sim U[10, 100]$. In Fig. 2 we plot π_B^A, π_S^A and π_{SC}^A as a function of c for the parameters listed above and selecting $p_1 = 50$ (the other values of p_1 gave very similar results). This figure shows that in the interval of approximately $c \in [70, 90]$, *both* firms benefit from the range contract, in the form of larger optimal risk-adjusted unit profits as compared to the centralized supply chain. Therefore, from a risk-adjusted point of view, both firms can benefit from the range contract.

6. Conclusions

We introduce a new class of supply chain contracts, called range contracts, that generalize fixed-price, JIT, option, and quantity-flexibility contracts. Range contracts enable a buyer, who faces random demand, and a supplier, with varying modes of production, to share the cost of the demand variability in a new way. It is demonstrated here that any non-trivial range contract will reduce the collective decentralized supply chain risk. The structure of the range contract that minimizes this risk is characterized here. We also study a Stackelberg game where the optimal strategies for each firm to maximize their respective expected profits under this contract are derived. We also study the concept of a range in the context of a vertically integrated system. We introduce the idea of production flexibility and show that it has a direct impact on the profit-maximizing range in the vertically integrated supply chain.

We demonstrate computationally that under the range contract a fractional decrease in expected profit results in a larger fractional decrease in the standard deviation of profit. We also show computationally that, under the range contract, it is possible for both the supplier and buyer to have larger risk-adjusted unit profits than the centralized supply chain, creating a “win-win” scenario.

The set up proposed here can be extended under certain circumstances. For instance, when the goods procured are non-perishable and can be stored in inventory, the supplier may offer quantity discounts. This extension of the model would be to allow for quantity discounts within the range specified in advance. It would be interesting to study, under this setup, the optimal strategies for the buyer, supplier and the behavior of the integrated supply chain. In this situation where goods can be stored in inventory, it is of interest to study the extension of the range contract to a dynamic setting.

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