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Designing practical coordinating contracts in decentralized projects

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Abstract

Managing decentralized projects (DPs) effectively is a critical issue today as projects have become increasingly complex, costly, and strategically important (especially IT and new product development projects). In this article, we consider a DP that is composed of n serial stages with stochastic durations; the project is planned, organized, and funded by a client organization that contracts the work at each stage to independent contractors. Following previous research and practice, we assume that the client and contractors incur time-dependent costs (including indirect/overhead costs), resource-related direct costs, and incentive payments. We initially propose a general linear time-based incentive contract and show that a simplified version of this contract can indeed coordinate a DP when discounting is not considered. The proposed contract sets the penalty due date to the start of each contractor's stage, and the optimal penalty cost rate is set equal to the client's overhead/indirect cost; it actually implies that the contractor is required to compensate for the client's indirect/overhead cost for the duration of the stage. This greatly simplifies the design and implementation of an optimal (coordinating) contract in a DP as the coordinating contract does not require the contractor's private cost information. When discounting is present, we show that a nonlinear contract coordinates the project and provide numerical evidence for when the linear time-based incentive contract is a good approximation.

KEYWORDS

coordinating contract, decentralized project, incentive payments, project management, time-based contracts

1 | INTRODUCTION

The importance of strategic projects in both the public and private sectors has increased greatly in recent years. Many of these projects are characterized by high costs, complexity, and risk, as well as potential returns. New product development (NPD) projects in the pharmaceutical industry illustrate the significant risks associated with NPD projects, where the average cost of successfully developing a new oncology drug was estimated at \$1.861 M USD with a median duration of 13.1 years and an overall probability of success of 3.4% (DiMasi et al., 2016; Wong et al., 2019). Many other new product development (NPD) and IT projects face similar conditions.

In addition to their complexity and risk, many of these projects are also decentralized; that is, there is a project owner or client who owns the intellectual property rights and defines, plans, and funds the project although much of the work is performed by independent contractors. The client receives a payoff when the project is successfully completed (the estimated median revenue for a new approved oncology drug was \$1.7B USD). Taneri and De Meyer (2017) reported that over half of all approvals for new drug applications in 2015 by the US Food and Drug Administration (FDA) were given to decentralized organizations or "alliances" consisting of innovators and partners. In addition, many NPD projects can be viewed as a series of sequential stages; even

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when a project is characterized by a general precedence network, it can frequently be subdivided into sequential groups of tasks or stages that are separated by review points or "stage gates" to improve project monitoring and control (Santiago & Vakili, 2005). In many NPD projects, the project naturally defines a sequential series of tasks or stages. For example, NPD development projects in the pharmaceutical industry are frequently characterized by a series of sequential phases or stages (Wong et al., 2019). Taneri and De Meyer (2017) noted that many drug development efforts were completed by sequential alliances where one party exerts "most of the efforts at any point in time".

When projects are decentralized, a major challenge facing project owners is how to align the goals of the owner (or client) and contractors who act in their own self-interest (although general economic conditions frequently play a significant role in these projects as they influence costs as well as the range of alternative choices faced by contractors). An increasing number of researchers have started to recognize the importance of defining optimal contracts to align conflicting goals between the client and contractors (Chen, Klastorin, & Wagner, 2015; Chen & Lee, 2017; Kwon, Lippman, & Tang, 2010). This article extends this stream of work in several directions by including more general cost functions for both the client and contractors, relaxing assumptions about the distribution of the stage durations and risk-neutral contractors, and showing that clients do not need contractors' private information to define optimal contracts.

In this article, we consider time-based incentive contracts that include penalty costs if a project is completed after a given due date and a bonus payment if the project is completed prior to (other) stated due dates. These types of contracts are also known as incentive/disincentive or I/D contracts. While there are many types of I/D contracts, most incentive contracts used today are variations of the time-based contract analyzed in this article (Bubshait, 2003).

Following previous research and observed practice, we assume that the project stages are stochastic and a function of the work rate that is determined by the contractor in response to the fixed fees, penalty rates, bonus rates, and due dates set by the client. Initially, we make mild assumptions about the distributions that describe stage durations. When we need to examine the optimal contracts in more complex environments (i.e., include discounting), we assume the stage duration distribution is exponential which is consistent with previous research (Buss & Rosenblatt, 1997).

One of the key issues in specifying contracts in decentralized organizations is the assumption of perfect information; most previous studies assume that cost parameters for both the client and contractors are known by all parties (Kwon, Lippman, McCardle, & Tang, 2010). In this article, we initially assume that the client has such information about the contractors' cost parameters but then we relax this assumption and show how a client can design and implement an optimal contract without having access to the contractors' private information.

To develop a benchmark for analyzing a decentralized project (DP), we consider a centralized project where the client controls all work in the project. Specifically, we derive closed form solutions for the optimal work rates at each stage that maximize the client's (and project's) expected profit. The results we derive for a centralized project serve as the baseline for our analysis of DPs and provide a means for defining a coordinating contract in a DP. A coordinating contract in a DP is a contract that results in a Nash equilibrium when no stakeholder in the project has any incentive to deviate from the project's optimal actions.[1]

When discounting is negligible and contractors are risk neutral, the project can be decomposed into n independent single-stage projects. Our results in this case indicate that a simple linear incentive contract (LIC), with all due dates set to the beginning of each stage, coordinates a DP and maximizes a risk-neutral client's expected profit when the optimal penalty cost rate is set to the client's indirect/overhead cost. The simple LIC that we advocate can be interpreted as follows: in addition to receiving a fixed payment from the client, the contractor is required to compensate for the client's indirect/overhead cost for the duration of the stage (as the "due dates" are set to the beginning of the stage and the penalty cost rate is equal to the client's indirect/overhead cost). Indeed, this type of Incentive/Disincentive (I/D) contract has been commonly used in highway transportation and other infrastructure projects. Although there might exist other optimal (coordinating) contract based on other penalty and bonus due dates, the contract that we advocate has a distinct advantage that it does not require the contractor's private information. That is, in this article, we show how a client can set the optimal parameters in a contract for each stage that coordinates the DP without knowing a contractor's private information. Furthermore, we show that a linear time-based incentive contract coordinates the project whether the contractors are risk neutral or risk averse.

When discounting is considered, we show that a nonlinear incentive contract (retaining the feature of the simple linear time-based incentive contract that all due dates are equal to the starting time of each stage) will coordinate a DP under the assumption that stage durations are exponential. Using numerical studies, we show that the linear time-based incentive contract used when there is no discounting provides a good approximation to the nonlinear coordinating contract when the discount rate is relatively small; furthermore, as noted above, the LIC has the additional advantage of being easy to implement as it does not require the client to have the contractors' private information.

[1]Our definition follows work in supply chain management; for example, see Cachon (2003)

1.1 | Literature review

Contracts in decentralized organizations have been widely studied by economists, mostly at a "rather high level of abstraction somewhat removed from the realm of practical application" (Weitzman, 1980). Other researchers have provided anecdotal and limited empirical evidence that incentive contracts offer superior performance to non-incentive contracts with respect to maximizing the client's expected profit (Meng & Gallagher, 2012). Gutierrez and Paul (2000) discussed the problem of partitioning a project into stages that are allocated to independent contractors to minimize project risks. Their model minimized the expected project makespan; when there were multiple optimal solutions, they selected the partition that minimized the variance of project makespan. In a related paper, Paul and Gutierrez (2005) used the concept of stochastic ordering to analyze the case when a client wants to select a single contractor from a pool of N possible contractors. They show that a fixed price contract minimizes the expected cost to the client when the contractors are risk neutral (that may not hold when the contractors are risk averse). Following Paul and Gutierrez, we use stochastic ordering and consider both risk neutral and risk-averse players; however, we show that a simple incentive contract dominates a fixed price contract.

Bayiz and Corbett (2005) analyzed the case when a LIC was used to coordinate the relative efforts of all contractors when a contractor's work effort could not be observed by the client. Their research suggested that contracts that increase payments to contractors if their relative tasks are completed before a given due date are weakly superior to fixed-price contracts in terms of a shorter expected project makespan and higher expected profits for the client. Kwon, Lippman, and Tang (2010) showed that time-based and cost-sharing contracts can achieve optimal channel coordination when there is a single contractor (assuming that task durations are exponentially distributed and the cost of a contractor is a quadratic function of the work rate). In contrast, our work extends to a serial project with multiple contractors while relaxing the assumption of exponentially distributed durations. In a related work, Kwon, Lippman, and Tang (2011) analyzed the decision to outsource some or all of the stages in a project (with two stages) when the client pays each contractor a fixed amount that is negotiated at time zero. Tang, Zhang, and Zhou (2015) analyze a DP where the work is outsourced to a single contractor who is selected from two candidate contractors using a reverse auction. They consider two contracts. The first contract is based on the client's specifying the fixed price and due date used to define any penalty and/or bonus payments. In the second contract, each contractor submits a bid specifying a fixed payment and due date, and the client selects the contractor that maximizes his expected payoff.

Chen and Lee (2017) showed that the delivery-schedule-based contracts are able to coordinate the decentralized supply chain in a project management context. In their work, however, they assumed that payments, penalty rates, and bonus rates are exogenously given; their main focus is on the client's optimal decision about the targeted material delivery schedule, as well as the contractors' optimal decisions about their production schedules. Kwon et al. (2010b) studied payment timing options (no delayed or delayed payments) in a stochastic project when all contractors worked concurrently; under the delayed payment contract, the client received payment when all tasks were completed. They focused on maximizing expected profit for a risk-neutral client and did not consider coordinating contracts in their paper. Conversely, Chen et al. (2015) studied payment timing options in a stochastic serial project that maximized the client's expected profit; following the Kwon, Lippman, McCardle, and Tang (2010) paper, they did not consider coordinating contracts. The results found by Chen et al. (2015) differed significantly from those in Kwon, Lippman, McCardle, and Tang (2010) suggesting that the design of the project network is an important factor when considering payment options. In Kwon, Lippman, McCardle, and Tang (2010) and Chen et al. (2015), the authors also assumed that all information was available to both the client and contractors; in this article, we relax this assumption. Dawande, Janakiraman, and Qi (2019) studied incentive contracts in both parallel and serial projects under the assumption of perfect information and exponential stage durations. While we limit our analysis to serial stochastic projects, we include more general cost functions for both the client and contractors that includes indirect/overhead costs for both the client and contractors and contractor-dependent cost function parameters, relax the assumption of exponential stage durations (when the discount rate is negligible), and consider risk-averse as well as risk-neutral clients.

1.2 | Scope and paper contributions

This article is organized as follows. In section 2, we define a general time-based incentive contract that includes most incentive contracts in practice today. This general incentive contract allows multiple possible due dates for penalty costs as well as multiple possible due dates for bonus payments. To derive a benchmark for analyzing a DP, we analyze a centralized project where the client controls all stages of the project.

Initially, in section 3, we consider the case when discounting can be ignored[2] and provide closed-form solutions for the optimal work rates at each stage in this case. Our results indicate that an incentive contract, when due dates are set equal to the starting time of each stage and the optimal penalty cost rate is set equal to the client's indirect/overhead cost, coordinates the project and maximizes a risk-neutral client's expected profit. This type of contract also provides the significant advantage that a client can set the parameters of the coordinating contract without knowing a contractor's private

[2]Discounting can be ignored when the risk free interest rate is small or the project is relatively short-term.

information. It is important to note that we derive these results using only mild assumptions about the first moment of the distribution of task durations.

In section 4, we relax the assumption in section 3 that discounting is negligible and define a nonlinear incentive contract that coordinates a DP when stage durations are exponentially distributed. We present an efficient procedure that allows a client to set the parameters of the optimal contract for each contractor/stage.

Given that the contract defined in section 4 is nonlinear and requires the contractors' private information to coordinate a DP, it may be difficult to implement in practice. In section 5, we numerically demonstrate that this nonlinear contract can be approximated by a LIC (based on a Taylor series approximation) and will coordinate a DP for risk-neutral contractors when the discount rate is small. Our results in this section are based on extensive numerical analyses and indicate that the LIC performs better for smaller discount rates. Furthermore, our numerical results suggest that the use of incentive contracts is increasingly important as discount rates increase.

In section 6, we extend our results to the case when the client or contractors are risk averse. Using stochastic dominance, we show that the linear contract defined in section 3 will also coordinate a DP for a risk-averse client or contractors for any strictly concave utility function when there is no discounting. In the final section, we summarize our results and discuss the managerial and practical implications of our findings.

2 | STACKELBERG MODEL DEFINED

We assume that a project consists of n sequential stages; without loss of generality, we assume that the stages are sequentially indexed $i = 1, \ldots, n$ from the start to the completion of the project. Following previous research (e.g., Buss & Rosenblatt, 1997; Chen et al., 2015; Kamien & Schwartz, 1972; Kwon, Lippman, McCardle, & Tang, 2010), we assume that the client receives a fixed payment Q when the project is completed. This amount may represent the expected value of future profits earned by a new product or the social welfare accrued by a completed public infrastructure project. We initially assume that the discount rate is sufficiently low that we can ignore discounting costs, although we relax this assumption later in the paper.

Following previous project management literature (Klastorin, 2011), we assume that there are three types of project-related costs. First, there are time-dependent costs that include a fixed overhead/indirect cost per time unit that reflects administrative costs (e.g., managerial and security costs), utility expenses, construction cranes, and so on. We let C_o denote the overhead/indirect cost rate incurred by the client during the project duration and K_i denotes the overhead/indirect cost rate incurred by the ith contractor during her respective stage. Second, there are direct resource costs at each stage that can be approximated by $k_i {r_i}^2$ where r_i is the work rate at stage i and k_i is the resource cost parameter that reflects the complexity and difficulty of the ith stage (Chen et al., 2015; Kwon, Lippman, McCardle, & Tang, 2010). In a DP, the work rate r_i is a decision variable that is set by each ith contractor to maximize their respective expected profit. Incentive costs define the third cost category; these will be discussed later.

The duration of stage i is denoted by t_i, where t_i is a non-negative random variable that is stochastically non-increasing in the work rate $r_i > 0$ (i.e., a larger work rate r_i will lead to a higher probability of completing stage i in a shorter time span). We assume that t_i are independent; initially, we only assume that the distributions are defined such that $E[t_i] = a_i r_i^{-1}$ for a given $a_i > 0$. In sections 4 and 5, we restrict our analysis to the case when $t_i \sim \exp(r_i)$; this assumption has been previously used by numerous researchers, including Buss and Rosenblatt (1997), Tavares (2002), and Klastorin and Mitchell (2007).

We consider a general incentive contract that is defined by a payment q_i that is negotiated and paid by the client at the start of the project and any penalty or bonus payments made at the completion of each stage when the stage's duration has been realized. Assuming a discounting rate $\alpha \geq 0$, the payment made by the client to each ith contractor is defined as

$$\rho_i(q_i, D_i^j, \hat{D}_i^j, P_i^j, B_i^j) = q_i - e^{-\alpha \sum_{k=1}^{i} t_k} \times \left(\sum_{j=1}^{P_{\max}} f[P_i^j, (t_i - D_i^j)^+] - \sum_{j=1}^{B_{\max}} g[B_i^j, (\hat{D}_i^j - t_i)^+] \right) \quad (1)$$

where $f[P_i^j, (t_i - D_i^j)^+]$ and $g[B_i^j, (\hat{D}_i^j - t_i)^+]$ define the penalty and bonus payment functions, respectively. The decision variables and parameters that we use to define and analyze variations of this contract are given below.

Contractor i's decision variable

$r_i > 0$ work rate for stage i.

Client's decision variables

$D_i^{P_{\max}} \geq \cdots \geq D_i^1 \geq \hat{D}_i^{B_{\max}} \geq \cdots \geq \hat{D}_i^1 \geq 0$ where D_i^j are the due dates for penalty payments and $\hat{D}_i^j$ are due dates for bonus payments,

P_i^j is the penalty cost per time period tardy if stage i is completed after due date D_i^j (for $j = 1, 2, \ldots, P_{\max}$),

B_i^j is the bonus per time period paid by client to contractor i if stage i is completed prior to due date $\hat{D}_i^j$ (for $j = 1, 2, \ldots, B_{\max}$), and

q_i is the payment made to the ith contractor at the beginning of the project.

Parameters

Q is the payment to the client when project is completed,

C_o is the indirect/overhead cost rate incurred by the client during project makespan,

K_i is the indirect/overhead cost rate incurred by contractor i during stage i, α is the non-negative discount rate,
k_i is the variable resource cost parameter at stage i,

θ_i is the minimum acceptable expected profit for contractor i, and

t_i is the duration of stage i (non-negative random variable).

To avoid trivial and unrealistic cases, we assume that $C_o > 0$, $K_i > 0$, and $k_i > 0$ for all $i = 1, \ldots, n$. The parameter θ_i defines the minimum expected profit that the ith contractor will require to participate in this project; θ_i represents the current economic conditions and alternative investments available to the contractor at the time when the incentive contract is negotiated. In a weak economy, θ_i would be small and possibly zero if a contractor has few alternatives and needs the project to remain economically viable. These participation constraints are analogous to "individual rationality (IR)" constraints in the principle-agent problem literature (Hurwicz, 1972).

Furthermore, it is clear from (1) that the problem decomposes into n independent stages when $\alpha = 0$ and contractors are risk neutral. Not only does this simplify the analysis and allows us to define an optimal (coordinating) contract where the client is not required to know contractors' private cost information, we will also show that the coordinating contract defined in this case also holds when the client is risk averse and provides a good approximation for cases when $\alpha > 0$.

3 | LINEAR TIME-BASED INCENTIVE CONTRACTS WITHOUT DISCOUNTING

Linear time-based incentive contracts are widely used in practice (Tang et al., 2015); assuming $\alpha = 0$, we can modify the general contract defined by (1) by letting $f[P_i^j, (t_i - D_i^j)^+] = P_i^j[t_i - D_i^j]^+$ and $g[B_i^j, (\widehat{D}_i^j - t_i)^+] = B_i^j[\widehat{D}_i^j - t_i]^+$. In this case, the client pays each contractor the amount:

$$\rho_i = q_i - \sum_j \{P_i^j[t_i - D_i^j]^+ - B_i^j[\widehat{D}_i^j - t_i]^+\}. \tag{2}$$

As previously noted, we assume that the earliest due date for penalties must equal or exceed the latest due date for bonuses; that is, $D_i^1 \geq \widehat{D}_i^{B_{\max}}$ (for all $i = 1, \ldots, n$), to avoid the case when the contractor is both paying a penalty and receiving a bonus. Furthermore, it should be noted that the aggregate penalties paid by any ith contractor will be monotonically non-decreasing with the duration of stage i. A similar statement can be made for the bonuses.

To develop a benchmark for analyzing DPs, we initially consider a centralized project where the client controls all stages of the project and sets the work rates r_i at each ith stage. The results from the centralized project allow us to define a coordinating contract for DPs.

3.1 | Centralized project

In a centralized project (CP), the client wants to maximize his expected profit that is defined by the payment Q received when the project is completed minus the indirect/overhead costs $\sum_i (C_o + K_i)t_i$ and direct resource costs $\sum_i k_i r_i^2 t_i$ where t_i is the realized duration of each stage.

Letting Π_{CP} denote the client's profit in a centralized project and assuming a discount rate $\alpha = 0$,

$$\Pi_{\text{CP}} = Q - \sum_{i=1}^{n} (C_o + K_i + k_i r_i^2) t_i. \tag{3}$$

Based on our assumption that $E[t_i] = a_i r_i^{-1}$ for all $a_i > 0$,

$$E[\Pi_{CP}] = Q - \sum_{i=1}^{n} \left[\left(\frac{C_o + K_i}{r_i} \right) + k_i r_i \right] a_i. \tag{4}$$

Using first-order conditions (FOCs), the client would set the optimal work rates as follows:

$$r_i^* = \sqrt{\frac{C_o + K_i}{k_i}} \tag{5}$$

that uniquely maximize $E[\Pi_{CP}]$ due to the strict concavity of the objective. Using (4) and (5), the maximum expected profit for the centralized project (and client) is

$$E^*[\Pi_{CP}] = Q - 2\sum_{i=1}^{n} a_i \sqrt{k_i (C_o + K_i)}. \tag{6}$$

3.2 | Decentralized project: Linear incentive contract

In this case, the client outsources the n stages of the project to independent contractors and sets the contract terms with each contractor at the start of each stage (or project). Given the LIC defined by (2), each ith contractor subsequently decides if she will participate and, if so, determines the work rate that maximizes her expected profit. In this case, the problem of defining an optimal LIC decomposes into n independent subproblems for each contractor where each subproblem is a Stackelberg game between the client and contractor. In these games, a client needs to know each contractor's best response to any proposed contract that typically requires knowing the contractors' cost parameters. However, it is important to note that a client generally does not have access to this private information.

To avoid this problem, the client can modify the contract defined by (2) by setting all due dates relating to penalty fees and bonus payments equal to the starting time of each stage (i.e., set $D_i^j = \widehat{D}_i^j = 0$ for all i,j). Under the terms of this simplified contract, each ith contractor is paid an amount equal to

$$\rho_i = q_i - P_i t_i. \tag{7}$$

We will show that the contract defined by (7) is a coordinating contract that maximizes the expected profit of the project and does not require the client to have the contractors' private information. Although there could be other coordinating contracts with the due dates not being equal to the starting time of each stage (that is, set $D_i^j \neq 0$ for some i, j), those contracts require that the client has access to the contractors' private cost information (i.e., values of k_i and K_i) to coordinate the project; such contracts are harder to implement while bring no additional benefit to either the client or the contractors compared to the contract defined by (7), as the latter is an optimal (coordinating) contract.

We refer to the contract defined by (7) as a LIC. In a LIC, the penalty charges begin to accrue as soon as the ith stage begins (i.e., mathematically we set due penalty due date equal to the starting time of that stage). These contracts have been widely used in transportation (highway) infrastructure projects where they are sometimes referred to as "lane rental" contracts (since a contractor must "rent" a lane to close it). According to the Washington State Department of Transportation (2015), "the intent is to minimize the impacts of a project on the traveling public … [by creating] a monetary incentive for the contractor to be innovative and minimize the duration of lane closures" (WSDOT 2015).

Given $E[t_i] = a_i r_i^{-1}$, the expected profit for the i^{th} contractor is

$$E[\pi_i] = q_i - (P_i + K_i + k_i r_i^2)\frac{a_i}{r_i}. \tag{8}$$

Given that other projects may be available to contractors, we require $E[\pi_i] \geq \theta_i$ where θ_i defines the minimum expected profit that the ith contractor will require to participate in this project. Applying FOCs, the optimal work rate for the ith contractor who participates in this project would be $r_i^* = \sqrt{\frac{P_i+K_i}{k_i}}$; second-order conditions (SOCs) confirm that this solution is unique and minimizes the contractors' expected cost. This simple result leads to the following proposition.

Proposition 1

a. *If the client offers the ith contractor a LIC defined by (7) with* $D_i^j = \widehat{D}_i^j = 0$ for all i, j; $P_i = C_o$, *and any* $q_i^* \geq 0$ *that ensures contractor participation, the contract will coordinate the entire project.*
b. *In addition, the split of the total profit among the client and the contractors is subject to negotiation over the fixed payment* q_i *paid at the beginning of each stage. In particular, the contract maximizes the client's expected profit if the client sets* $q_i^* = 2a_i\sqrt{k_i(C_o + K_i)} + \theta_i$, *where the expected profit for the contractors and client, respectively, are as follows:*

$$E^*[\pi_i] = \theta_i$$

$$E^*[\Pi_c] = Q - 2\sum_{i=1}^{n} a_i\sqrt{k_i(C_o + K_i)} - \sum_{i=1}^{n}\theta_i.$$

Proof See Appendix. ■

It is important to note that, after setting $P_i^* = C_o$ (that does not require the client to have contractors' private information), specific values of q_i only determine how the project's profits are allocated between the client and contractors; any value of q_i will result in a coordinating contract that maximizes the project's expected profit. In practice, the final values of q_i are determined through incremental negotiations and represent the relative bargaining power between the client and contractors. In contrast, when the client has access to the contractors' private information (e.g., k_i, K_i, and θ_i), the client would optimally set each contractor's fixed payment equal to q_i^*, which maximizes the client's expected profit while leads to the minimal expected profits of the contractors.

3.3 | Decentralized project: Fixed price contracts

The contract defined by (7) becomes a fixed price contract when $P_i = 0$; that is, a contractor receives a payment q_i regardless of the time needed to complete her stage. In this case, the optimal work rate for the ith contractor is

$$r_i^* = \sqrt{\frac{K_i}{k_i}}$$

and it can be seen that $q_i^* = 2a_i\sqrt{k_i K_i} + \theta_i$. The client's expected optimal profit, in this case, is then

$$E[\Pi_C^{FP}] = Q - \sum_i \theta_i - \sum_i a_i\left[\frac{(2K_i + C_o)}{\sqrt{\frac{K_i}{k_i}}}\right] \tag{9}$$

(the contractors' expected profits do not change). Corollary 1 implies that an incentive contract always outperforms a fixed price contract when $C_o > 0$; furthermore, the benefit increases with the value of C_o. Even though the fixed-price contract is a special case of the incentive contract, this result is not intuitively obvious given the nature of the Stackelberg game where the client and contractors have competing objectives.

Corollary 1 *The client's maximum expected profit under an incentive contract defined by Proposition 1 is greater than or equal to the client's expected profit defined by (9) using a fixed price contract. Furthermore, the expected makespan of the project using the incentive contract defined by Proposition 1 is less than the expected makespan when a fixed price contract is used.*

Proof See Appendix. ■

4 | INCENTIVE CONTRACTS WITH DISCOUNTED CASH FLOWS

When discounting is considered (i.e., $\alpha > 0$), we can define a coordinating contract in a decentralized serial project when the density of the duration of each stage is $f(t) = r_i e^{-r_i t}$ (i.e., exponentially distributed durations) by setting $D_i^j = \hat{D}_i^j = 0$ for all i,j and letting $f[P_i^j, (t_i - D_i^j)^+] = e^{P_i t_i}$ in (1). While the assumption that durations are exponential is somewhat more restrictive than our previous assumption about the duration distribution, we note that exponential task durations have been used in numerous previous studies (Buss & Rosenblatt, 1997; Klastorin & Mitchell, 2007 and Tavares, 2002). Given the nature of the penalty cost function, we refer to this contract as an EXIN (*Ex*ponential *In*centive) contract. To show that the EXIN contract is a coordinating contract, we initially analyze a centralized project using the same assumptions.

4.1 | CP when α > 0

The project's (and client's) collective discounted profit in this case is

$$\Pi_{CP}(\hat{r}_1, \dots, \hat{r}_n) = Qe^{-\alpha \sum_{i=1}^n t_i} - \sum_{i=1}^n \times \left[e^{-\alpha \sum_{j=1}^{i-1} t_i} \int_0^{t_i} (C_o + K_i + k_i \hat{r}_i^2) e^{-\alpha x} dx \right] \tag{10}$$

where $\hat{r}_i$ denotes the work rates when $\alpha > 0$. The expected discounted profit in a centralized project, denoted by $E[\Pi_{CP}(\hat{r}_1, \dots, \hat{r}_n)]$, is defined as follows:

$$E[\Pi_{CP}(\hat{r}_1, \dots, \hat{r}_n)] = Q \prod_{i=1}^n \frac{\hat{r}_i}{\alpha + \hat{r}_i} - \sum_{i=1}^n \times \left(\frac{C_o + K_i + k_i \hat{r}_i^2}{\alpha + \hat{r}_i} \right) \prod_{j=1}^{i-1} \frac{\hat{r}_j}{\alpha + \hat{r}_j}. \tag{11}$$

Following our previous analysis of a centralized project, a risk-neutral client wants to set the work rates $(\hat{r}_1^*, \dots, \hat{r}_n^*)$ in each stage to maximize the expected profit defined by (11). The optimal work rates $(\hat{r}_1^*, \dots, \hat{r}_n^*)$ in this case are defined in Proposition 2.

Proposition 2 *When* $\Pi_{CP}(\hat{r}_1, \dots, \hat{r}_n)$ *is defined by (10) and* $t_i \sim \exp(\hat{r}_i)$, $E[\Pi_{CP}(\hat{r}_1^*, \dots, \hat{r}_n^*)] \geq E[\Pi_{CP}(\hat{r}_1, \dots, \hat{r}_n)]$ *for all* $\hat{r}_1, \dots, \hat{r}_n > 0$ *when* $\hat{r}_i^* = \sqrt{\alpha^2 + \frac{\hat{Q}_i\, \alpha + C_o + K_i}{k_i}} - \alpha$ *where* $\hat{Q}_i = Q \prod_{j=i+1}^n \frac{\hat{r}_j^*}{\alpha + \hat{r}_j^*} - \sum_{j=i+1}^n \frac{[C_o + K_j + k_j (\hat{r}_j^*)^2]}{\alpha + \hat{r}_j^*} \prod_{m=i+1}^{j-1} \frac{\hat{r}_m^*}{\alpha + \hat{r}_m^*}$ for $i = n-1, \dots, 1$ and $\hat{Q}_n = Q$.

Proof See Appendix. ∎

4.2 | DP when α> 0

Assume that the client offers each ith contractor a payment q_i at the beginning of the project and each contractor reacts by setting an appropriate work rate $\hat{r}_i$. When the client uses an EXIN contract, a contractor's discounted profit is now defined as

$$\pi_i(\hat{r}_i) = \left[(q_i - e^{P_i t_i}) e^{-\alpha t_i} - \int_0^{t_i} (K_i + k_i \hat{r}_i^2) e^{-\alpha x} dx \right] e^{-\alpha \sum_{j=1}^{i-1} t_j}. \tag{12}$$

Since $f(t) = \hat{r}_i\, e^{-\hat{r}_i t}$, the expected profit for contractor i who is willing to participate in the project is equal to

$$E[\pi_i(\hat{r}_i)] = \left[\frac{q_i \hat{r}_i - K_i - k_i \hat{r}_i^2}{\alpha + \hat{r}_i} - \frac{\hat{r}_i}{\alpha - P_i + \hat{r}_i} \right] \prod_{j=1}^{i-1} \frac{\hat{r}_j}{\alpha + \hat{r}_j}. \tag{13}$$

A risk-neutral contractor's objective is to maximize her expected discounted profit defined by (13). The unique work rate that maximizes her expected profit is defined by Proposition 3.

Proposition 3 Given $q_i > 0, P_i \geq 0$, then work rates $\hat{r}_i^*$ uniquely maximize $E[\pi_i(\hat{r}_i)]$ as defined by (13), where $\hat{r}_i^*$ are the unique solutions to

$$\hat{r}_i^* = \sqrt{\frac{[k_i(\hat{r}_i^*)^2 + 2k_i \hat{r}_i^* \alpha - (q_i \alpha + K_i)](\alpha - P_i + \hat{r}_i^*)^2}{(P_i - \alpha)}} - \alpha > 0. \tag{14}$$

Proof See Appendix. ∎

Proposition 3 provides an implicit expression for each contractor i's optimal work rate $\hat{r}_i^*$ for any values of $q_i > 0$ and $P_i \geq 0$. Moreover, we show in the proof that this work rate always exists $(\hat{r}_i^*(q_i, \ P_i) > 0)$ and is unique. Using the results of Proposition 3, the expected discounted contractor profit can be simplified as follows:

$$E[\pi_i(\hat{r}_i^*)] = \left[\frac{k_i(\hat{r}_i^*)^2 - K_i}{\alpha} - \frac{P_i (\hat{r}_i^*)^2}{\alpha(\alpha - P_i + \hat{r}_i^*)^2} \right] \prod_{j=1}^{i-1} \frac{\hat{r}_j^*}{\alpha + \hat{r}_j^*}. \tag{15}$$

Our results assume that contractors are willing to participate in the project. As we noted in section 2, this may not be the case when contractors have better alternative investment opportunities. As a result, we imposed participation constraints for a risk-neutral contractor as a lower bound on the expected profit that we denoted by the opportunity cost θ_i; that is, a risk-neutral contractor would only participate in the project if $E[\pi_i(\hat{r}_i^*)] \geq \theta_i$ where $E[\pi_i(\hat{r}_i^*)]$ is defined by (15) and the $\hat{r}_i^*$ values are defined in Proposition 3. This assumption is reasonable given that contractors often disclose their opportunity costs during negotiations. However, private contractor cost information, such as their overhead cost rates, are typically not disclosed during negotiations. Our model does not require the client to know this private contractor information.

4.3 | Client response to an EXIN contract

Using an EXIN contract, a client's discounted profit is defined as

$$\Pi_C = Qe^{-\alpha\sum_{i=1}^{n} t_i} - \sum_{i=1}^{n}\left[(q_i - e^{P_i t_i})e^{-\alpha\sum_{j=1}^{i} t_j}\right] - \sum_{i=1}^{n}\left[e^{-\alpha\sum_{j=1}^{i-1} t_i}\int_0^{t_i} C_o e^{-\alpha x}dx\right].$$

A risk-neutral client who wants to maximize his expected profit would find the optimal values q_i^* and P_i^* by solving Problem *P1*:

$$\max_{q_i,P_i} E[\Pi_C] = Q\prod_{i=1}^{n}\frac{\hat{r}_i^*}{\alpha+\hat{r}_i^*} - \sum_{i=1}^{n}\left[\left(\frac{q_i\hat{r}_i^*}{\alpha+\hat{r}_i^*} - \frac{\hat{r}_i^*}{\alpha-P_i+\hat{r}_i^*}\right)\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*}\right] - \sum_{i=1}^{n}\left[\frac{C_o}{\alpha+\hat{r}_i^*}\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*}\right] \quad (P1)$$

$$s.t. \left[\frac{k_i(\hat{r}_i^*)^2 - K_i}{\alpha} - \frac{P_i(\hat{r}_i^*)^2}{\alpha(\alpha-P_i+\hat{r}_i^*)^2}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*} \geq \theta_i \text{ for all} i = 1, \dots, n,$$

$$q_i > 0$$

$$P_i \geq 0$$

where $\hat{r}_i^*$ are defined by (14) and Proposition 3. Recognizing that the client's expected profit defined in problem *P1* is similar to the client's expected profit in the centralized case defined by (11), we can develop an efficient procedure to solve problem *P1*. This procedure, that we denote as the X-procedure, calculates the optimal work rate at each stage and then finds the optimal penalty P_i^* and initial payment q_i^* for each contractor who is paid an amount $q_i^* - e^{P_i^* t_i}$ based on their realized stage makespan t_i. This procedure is described below; the proof that the X-procedure finds the optimal solution is given in the Appendix.

4.4 | X-Procedure

1. Set $\hat{r}_i^{**} = \sqrt{\alpha^2 + \frac{\hat{Q}_i\alpha + C_o + K_i}{k_i}} - \alpha$ where $\hat{Q}_i = Q\prod_{j=i+1}^{n}\frac{r_j^*}{\alpha+r_j^*} - \sum_{j=i+1}^{n}\frac{[C_o+K_j+k_j(r_j^*)^2]}{\alpha+r_j^*}\prod_{m=i+1}^{j-1}\frac{r_m^*}{\alpha+r_m^*}$ for $i = n-1, \dots, 1$ and $\hat{Q}_n = Q$.
2. Set $P_i^* = \frac{\xi_i\alpha(\alpha+\hat{r}_i^{**})}{\xi_i\alpha+(\hat{r}_i^{**})^2}$, where $\xi_i = \frac{k_i(\hat{r}_i^{**})^2-K_i}{\alpha} - \frac{\theta_i}{\prod_{j=1}^{i-1}\frac{\hat{r}_j^{**}}{\alpha+\hat{r}_j^{**}}}$ for $i = n, \dots, 1$.
3. Set $q_i^* = \frac{k_i(\hat{r}_i^{**})^2-K_i+2k_i\hat{r}_i^{**}\alpha}{\alpha} - \frac{(P_i^*-\alpha)(\alpha+\hat{r}_i^{**})^2}{\alpha(\alpha-P_i^*+\hat{r}_i^{**})^2}$ for $i = n, \dots, 1$.

Proposition 4 shows that the EXIN contract coordinates a DP. This proposition indicates that an EXIN contract is optimal for a risk-neutral client since the client's expected payout is equal to the optimal centralized project profit minus the sum of the opportunity costs θ_i paid to the contractors.

Proposition 4 *An EXIN contract, where the client pays each contractor an amount equal to* $q_i^* - e^{P_i^* t_i}$ *when* q_i^* *and* P_i^* *are calculated by the X-Procedure, coordinates a decentralized serial stochastic project when stage durations* $t_i \sim \exp(r_i)$. *The client obtains an expected profit equal to* $E[\Pi_{CP}(\hat{r}_1^*, \dots, \hat{r}_n^*)] - \sum_{i=1}^{n}\theta_i$ *and each ith contractor obtains an expected profit equal to their opportunity cost* θ_i.

Proof See Appendix. ■

Note that our results generalize the findings of Dawande et al. (2019), for a serial project. In particular, our model includes non-zero indirect cost rates for both the client and contractors, direct resource cost factors (k_i) that are stage-dependent and positive opportunity costs. If we adopt the more restriction assumptions made by Dawande et al., our centralized work rates are identical to those given by Dawande et al., 2019.[3] To show this equivalence, we note that the work rate at stage i is given as follows (by Proposition 2):

$$r_i = \sqrt{\alpha^2 + \frac{Q_i\alpha + C_o + K_i}{k_i}} - \alpha$$

where

$$Q_i = Q\prod_{j=i+1}^{n}\frac{r_j}{\alpha+r_j} - \sum_{j=i+1}^{n}\frac{C_o+K_j+k_j(r_j)^2}{\alpha+r_j}\prod_{m=i+1}^{j-1}\frac{r_m}{\alpha+r_m},$$

$$i = n-1, \dots, 1 \text{ and } Q_n = Q.$$

Following Dawande et al., we set $C_o = K_i = 0$, and define r_i as λ_i, Q_i as V_{i+1}, and k_i as κ. In this case, our optimal work rate simplifies to

$$\lambda_i = \sqrt{\alpha^2 + \frac{V_{i+1}\alpha}{\kappa}} - \alpha,$$

that agrees with Proposition 6 in Dawande et al. However, relabeling Q_i as V_{i+1} must be proved, since the definition of V_i is different in Dawande et al.: $V_i = V_{i+1} - 2\kappa\lambda_i$. The proof of equivalence is given in Proposition 5.

Proposition 5 If $C_o = K_i = 0$ and $k_i = \kappa$, then $Q_i = V_{i+1}$ and $r_i = \lambda_i$ for $i = 1, \dots, n$.

Proof See Appendix. ■

[3] Note that hat's and *'s are omitted for convenience.

5 | USING A LIC TO IMPLEMENT AN EXIN CONTRACT

It is likely that an EXIN contract will be difficult to implement in practice given the nonlinear form of the contract and the fact that implementing the contract requires the client to have contractors' private information (e.g., values of k_i and K_i). The LIC proposed in section 3 avoids these difficulties; we note that the LIC can be derived from the definition of the EXIN contract using the linear term in a Taylor's series approximation. However, the question remains if the LIC is a good approximation for an EXIN contract.

When $\alpha > 0$, the client's discounted profit in a decentralized serial project using a LIC is equal to

$$\Pi_{DP}(q_i, P_i) = Qe^{-\alpha T} - \sum_{i=1}^{n} e^{-\alpha \sum_{j=1}^{i-1} t_j} \times \left[(q_i - P_i t_i) e^{-\alpha t_i} + C_o \int_0^{t_i} e^{-\alpha t} dt \right]. \quad (16)$$

In this case, we were unable to find closed-form solutions; as a result, we used extensive numerical analysis to test the impact of implementing a LIC when there is a positive discount rate. To find solutions, we used a numerical procedure that solved for a contractor's optimal work rate as a function of q, P, and α. This procedure formed the basis of a search algorithm that found the values of $q_i^*(\alpha)$ and $P_i^*(\alpha)$ that maximized the client's expected profit for a given discount rate α.

Our results were consistent over a variety of cost parameters and discount rates. To illustrate our findings, consider the results for a risk-neutral client and risk-neutral contractors in a two-stage project with parameters $Q = 350$, $C_o = 20$, $k_1 = k_2 = 20$, $K_1 = K_2 = 5$. When $\alpha = 0$, the client would set $q_i*(0) = \$44.72$, $P*(0) = C_o = \$20$, and earn an expected profit of \$215.84. As the discount rate α increases, the client's optimal expected profit decreases as indicated in Figure 1 (in general, the values of $q^*(\alpha)$ and $P^*(\alpha)$ increase as $\alpha > 0$). If a client continues to use $q^*(0)$ and $P^*(0)$ when $\alpha > 0$, the error introduced (in terms of the client's expected profit) increases with increasing values of α to a maximum of 14.37% (when $\alpha = 0.15$).

Our results also indicate that for small values of α (e.g., $\alpha \leq 0.05$), a client who uses the closed-form results derived when $\alpha = 0$ will earn a nearly optimal expected profit without requiring extensive numerical analysis. This relationship is indicated in Figure 1.

The results in Figure 1 also indicate a significant advantage to the client of an incentive contract over a fixed price contract (when $p = 0$). When there is no discounting, the client's expected profit is approximately 27% lower when using a fixed price contract; this loss in expected profit increases to 67% when α = 0.15. Clearly, it is in the client's best interest to use an incentive contract, especially as the discount rate increases.

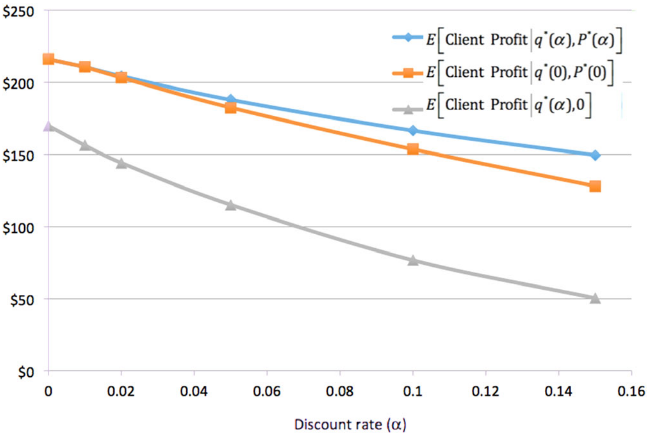

FIGURE 1 Client's discounted expected profit using incentive and fixed-price contracts [Colour figure can be viewed at wileyonlinelibrary.com]

6 | RISK AVERSE CLIENT OR CONTRACTORS

The results in this article assumed that the client and contractors are risk neutral. When stage durations are exponential, however, we can show that our results apply when the client or contractors are risk averse. For example, in the case of a centralized project when $\alpha = 0$, we showed in section 3.1 that a client's expected profit is maximized when he sets work rates $r_i^* = \sqrt{\frac{C_o + K_i}{k_i}}$. This result also maximizes the utility of a risk-averse client for any strictly concave utility function as indicated in Corollary 2.

Corollary 2 *When the client's profit* $\Pi_{CP}(r_1, \ldots, r_n)$ *is defined by (3) and* $f(t) = r_i e^{-r_i t}$, $E[u(\Pi_{CP}(r_1^*, \ldots, r_n^*))] \geq E[u(\Pi_{CP}(r_1, \ldots, r_n))]$ *for all* $r_1, \ldots, r_n > 0$ *and any monotone increasing and strictly concave utility function* u *if* $r_i^* = \sqrt{\frac{C_o + K_i}{k_i}}$ *for all* $i = 1, \ldots, n$.

Proof See Appendix. ■

When $\alpha > 0$, extensive numerical tests indicate that these results do not hold in general as the optimal solution depends on the specific form of the utility function.

For a DP, we showed in section 3.2 that when contractors set their optimal work rates equal to $r_i^* = \sqrt{\frac{P_i + K_i}{k_i}}$, a LIC defines a coordinating contract for risk-neutral contractors. In

Corollary 3, we show that this result also holds if the contractors are risk averse as defined by any strictly concave utility function.

Corollary 3 *When the contractors' realized profits are defined by* $\pi_i(r_i) = q_i - (K_i + P_i + k_i r_i^2)t_i$, *then* $E\{u[\pi_i(r_i^*)]\} \geq E\{u[\pi_i(r_i)]\}$ *for all* $r_i > 0$ *and any monotone increasing and strictly concave utility function* u, *when the contractors' work rates are defined by* $r_i^* = \sqrt{\frac{C_o + K_i}{k_i}}$.

Proof See Appendix. ■

7 | CONCLUSIONS AND EXTENSIONS

In this article, we analyzed a DP consisting of a series of stochastic stages where a client organization funds, directs, and manages the project but outsources the work at each stage to an independent contractor who maximizes her expected profit. These types of projects occur frequently and represent many strategic projects, including IT and new product development projects.

Given that the client and contractors have conflicting goals, the type of contract used by the client is critically important. In this article, we focused on general incentive contracts and showed that a LIC with the due dates set to zero will coordinate a DP when discounting can be ignored and a nonlinear incentive contract (the EXIN contract) with due dates set to zero will coordinate a DP when discount rates are positive. In addition to incentive contracts, our analysis included fixed-price contracts that are special cases of the incentive contracts when the penalty cost rates are set to zero.

To evaluate these contracts, we analyzed the project performance for a centralized project and showed how a client would set his optimal work rate to maximize the expected profit or expected utility in this case. We used the results for a centralized project as a benchmark for analyzing DPs.

When using the EXIN contract, we developed an efficient procedure to find the optimal values of q_i and P_i that coordinate the project (i.e., maximize the overall profit of the project). Recognizing that an EXIN contract may be difficult to implement in practice, we proposed using a LIC when $\alpha > 0$ as a LIC has two advantages over an EXIN contract. The first advantage is that LIC is easy to implement (set $P_i^* = C_o$) and has a simple linear penalty function. The second advantage is that a LIC does not require the client to have contractors' private knowledge. Our numerical analysis indicated that a LIC is a reasonably good approximation of an EXIN contract, especially for small discount rates.

We are continuing to analyze various types of contracts in DPs, including "cost plus" contracts when a client pays a contractor her audited direct costs in addition to a fixed fee (or percentage of the costs) for overhead/indirect costs. Cost-plus contracts are widely used in practice and provide a means to allocate risks from contractors to the client. A better understanding of these—and related—contracts could have a significant impact on the ultimate cost and success of many complex and costly projects.

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APPENDIX

Proof of Proposition 1 If the client offers the ith contractor, a linear incentive contract defined by (7) with all the due dates set to zero, the expected profit of the ith contractor has been given by (8). Applying the FOC of (8), the contractor's optimal choice of the work rate will be $r_i^* = \sqrt{(P_i + K_i)/k_i}$. In contrast, in the centralized project, the client would set the optimal work rates at $r_i^* = \sqrt{(C_o + K_i)/k_i}$. It is clear that if the client in the decentralized project sets $P_i = C_o$, then the contractor will choose the same optimal work rate as what the client would do in the centralized project. Therefore, with all due dates being zero and $P_i = C_o$, the contract will coordinate the entire project, provided that the contract ensures participation of the contractor. Note that the client can use this contract to coordinate the decentralized project without knowing the contractors' private information (e.g., k_i and K_i).

While part (a) of this proposition has established the coordinating contract, which maximizes the total expected profit of the entire project, the division of the total expected profit between the client and the contractors depends on the values of q_i. As we noted, the values of q_i can be determined by the relative negotiation powers of those parties; a higher q_i implies a higher negotiation power of the ith contractor.

In particular, if ideally the client knows the contractor's private information on θ_i or the client has the absolute negotiation power, then at $q_i^* = 2a_i\sqrt{k_i(C_o + K_i)} + \theta_i$, the expected profit of the ith contractor will be θ_i. To see this result, note that under the coordinating contract,

$$E^*[\pi_i] = q_i^* - \frac{\left(P_i + K_i + k_i r_i^{*2}\right) a_i}{r_i^*} = q_i^* - \frac{\left(C_o + K_i + k_i r_i^{*2}\right) a_i}{r_i^*} = \left(2a_i\sqrt{k_i(C_o + K_i)} + \theta_i\right) - \frac{2(C_o + K_i)a_i}{\sqrt{\frac{C_o + K_i}{k_i}}} = \theta_i.$$

Therefore, the coordinating contract leads to the maximum expected profit for the entire decentralized project, and at $q_i^* = 2a_i\sqrt{k_i(C_o + K_i)} + \theta_i$, each contractor receives only the minimum acceptable expected profit, θ_i. As a result, the client's expected profit is maximized. Specifically, the client's maximum expected profit must be equal to the maximum expected profit of the entire project less the sum of the contractors' minimum acceptable expected profits. That is,

$$E^*[\Pi_c] = E^*[\Pi_{CP}] - \sum_{i=1}^{n}\theta_i = Q - 2\sum_{i=1}^{n} a_i\sqrt{k_i(C_o + K_i)} - \sum_{i=1}^{n}\theta_i.$$

Q.E.D. ■

Proof of Corollary 1 The expected client profit using the incentive contract $\rho_i = q_i - P_i t_i$ was shown by Proposition 1 to be equal to

$$E^*[\Pi_c] = Q - 2\sum_{i=1}^{n} a_i\sqrt{k_i(C_o + K_i)} - \sum_{i=1}^{n}\theta_i$$

For $k_i > 0$, we want to show that

$$E^*[\Pi_c] - E\left[\Pi_c^{FP}\right] \geq 0$$

$$Q - 2\sum_{i=1}^{n} a_i\sqrt{k_i(C_o + K_i)} - \sum_{i=1}^{n}\theta_i \geq Q - \sum_i \theta_i - \sum_i a_i\left[\frac{(2K_i + C_o)}{\sqrt{\frac{K_i}{k_i}}}\right]$$

That is, we want to show that

$$2\sum_{i=1}^{n} a_i\sqrt{k_i(C_o + K_i)} \leq \sum_i a_i\left[\frac{(2K_i + C_o)}{\sqrt{\frac{K_i}{k_i}}}\right]$$

which is true for any $C_o \geq 0$.

The expected make span for a project using a fixed price contract is

$$E^{\mathrm{FP}}\left[\sum_{i=1}^{n} t_i\right] = \sum_{i=1}^{n}\frac{a_i}{\sqrt{\frac{K_i}{k_i}}}$$

while the expected make span for a project using an incentive contract is

$$E^{\mathrm{IC}}\left[\sum_{i=1}^{n} t_i\right] = \sum_{i=1}^{n} \frac{a_i}{\sqrt{\frac{P_i+K_i}{k_i}}}$$

Thus, $E^{FP}\left[\sum_{i=1}^{n} t_i\right] > E^{\mathrm{IC}}\left[\sum_{i=1}^{n} t_i\right]$ for any $P_i, k_i > 0$.

Q.E.D. ∎

Proof of Proposition 2 From (11), the expected discounted profit at time 0 is (the hat's omitted for clarity)

$$E\left[\Pi_{\mathrm{CP}}\right] = Q\prod_{i=1}^{n}\frac{r_i}{\alpha + r_i} - \sum_{i=1}^{n}\frac{\left(C_o + K_i + k_i {r_i}^2\right)}{\alpha + r_i}\prod_{j=1}^{i-1}\frac{r_j}{\alpha + r_j}.$$

We optimize from the last stage n. We isolate the terms in $E[\Pi_{\mathrm{CP}}]$ that only includes r_n as follows

$$E\left[\Pi_{\mathrm{CP}}(n)\right] = \left(Q\frac{r_{n-1}}{\alpha + r_{n-1}}\frac{r_n^*}{\alpha + r_n^*} - \frac{\left(C_o + K_{n-1} + k_{n-1}{r_{n-1}}^2\right)}{\alpha + r_{n-1}} - \frac{\left(C_o + K_n + k_n (r_n^*)^2\right)}{\alpha + r_n^*}\frac{r_{n-1}}{\alpha + r_{n-1}}\right)\prod_{j=1}^{n-2}\frac{r_j}{\alpha + r_j}$$

$$= \left(\underbrace{\left(Q\frac{r_n^*}{\alpha + r_n^*} - \frac{\left(C_o + K_n + k_n (r_n^*)^2\right)}{\alpha + r_n^*}\right)}_{\text{constant}}\frac{r_{n-1}}{\alpha + r_{n-1}} - \frac{\left(C_o + K_{n-1} + k_{n-1}{r_{n-1}}^2\right)}{\alpha + r_{n-1}}\right)\prod_{j=1}^{n-2}\frac{r_j}{\alpha + r_j}.$$

$$\frac{\partial E\left[\Pi_{\mathrm{CP}}\right]}{\partial r_n} = \left(\frac{-k_n {r_n}^2 - 2k_n r_n a + (Qa + C_o + K_n)}{(\alpha + r_n)^2}\right)\prod_{j=1}^{n-1}\frac{r_j}{\alpha + r_j}.$$

$$\frac{\partial^2 E\left[\Pi_{\mathrm{CP}}\right]}{\partial {r_n}^2} = \left(-\frac{2\left(Qa + ka^2 + C_o + K_n\right)}{(a + r)^3}\right)\prod_{j=1}^{n-1}\frac{r_j}{\alpha + r_j} < 0.$$

Setting $\frac{\partial E[\Pi_{\mathrm{CP}}]}{\partial r_n} = 0$, we have $r_n^* = \sqrt{\alpha^2 + \frac{Q\alpha + C_o + K_n}{k_n}} - \alpha$ and the second order condition confirms that r_n^* maximizes $E[\Pi_{\mathrm{CP}}]$. Note that $r_n^*\,(Q, \alpha, C_o, K_n, k_n)$ is a function of the project overall parameters and specific cost parameters only related to stage n. Next, we solve for stage

$n - 1$. We isolate the terms in $E[\Pi_{\mathrm{CP}}]$ that only includes r_{n-1} as follows:

$$E\left[\Pi_{\mathrm{CP}}(n-1)\right] = \left(Q\frac{r_{n-1}}{\alpha + r_{n-1}}\frac{r_n^*}{\alpha + r_n^*} - \frac{\left(C_o + K_{n-1} + k_{n-1}{r_{n-1}}^2\right)}{\alpha + r_{n-1}} - \frac{\left(C_o + K_n + k_n (r_n^*)^2\right)}{\alpha + r_n^*}\frac{r_{n-1}}{\alpha + r_{n-1}}\right)\prod_{j=1}^{n-2}\frac{r_j}{\alpha + r_j}$$

$$= \left(\underbrace{\left(Q\frac{r_n^*}{\alpha + r_n^*} - \frac{\left(C_o + K_n + k_n (r_n^*)^2\right)}{\alpha + r_n^*}\right)}_{\text{constant}}\frac{r_{n-1}}{\alpha + r_{n-1}} - \frac{\left(C_o + K_{n-1} + k_{n-1}{r_{n-1}}^2\right)}{\alpha + r_{n-1}}\right)\prod_{j=1}^{n-2}\frac{r_j}{\alpha + r_j}.$$

Clearly, $r_{n-1}^* = \sqrt{\alpha^2 + \frac{\widehat{Q}_{n-1}\alpha + C_o + K_{n-1}}{k_{n-1}}} - \alpha$, where $\widehat{Q}_{n-1} = Q\frac{r_n^*}{\alpha + r_n^*} - \frac{\left(C_o + K_n + k_n (r_n^*)^2\right)}{\alpha + r_n^*}$. Then, through induction, we can show that $r_i^* = \sqrt{\alpha^2 + \frac{\widehat{Q}_i\alpha + C_o + K_i}{k_i}} - \alpha$, where

$$\widehat{Q}_i = Q\prod_{j=i+1}^{n}\frac{r_j^*}{\alpha + r_j^*} - \sum_{j=i+1}^{n}\frac{\left(C_o + K_j + k_j\left(r_j^*\right)^2\right)}{\alpha + r_j^*}\prod_{m=i+1}^{j-1}\frac{r_m^*}{\alpha + r_m^*} \quad \text{for } i = 1, \ldots, n-1.$$

Q.E.D. ∎

Proof of Proposition 3 Here, we assume $P_i > \alpha$; note that P_i is the client's decision variable and can be viewed as a constant in the subcontractor i's problem. In Proposition 4, we will then show that, at the equilibrium, $P_i > \alpha$ is always true $\left(P_i^* > \alpha\right)$.

Subcontractor i's optimization problem is

$$\max_{\hat{r}_i} E[\pi_i] = \left[\frac{q_i\hat{r}_i - K_i - k_i\hat{r}_i^2}{\alpha+\hat{r}_i} - \frac{\hat{r}_i}{\alpha - P_i + \hat{r}_i}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j}{\alpha+\hat{r}_j}.$$

Since $\frac{\partial^2 E[\pi_i]}{\partial \hat{r}_i^2} = \left[\frac{-k_i\alpha^2 - 2\alpha q_i - 2K_i}{(\alpha+\hat{r}_i)^3} - \frac{2(P_i-\alpha)}{(\alpha-P_i+\hat{r}_i)^3}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j}{\alpha+\hat{r}_j} < 0, (P_i > \alpha)$ to find the maximizer r_i^*, we set $\frac{\partial E[\pi_i]}{\partial \hat{r}_i} = \left[\frac{(q_i - 2k_i\hat{r}_i)\alpha + K_i - k_i\hat{r}_i^2}{(\alpha+\hat{r}_i)^2} + \frac{P_i-\alpha}{(\alpha-P_i+\hat{r}_i)^2}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j}{\alpha+\hat{r}_j} = 0$. Which implies $\frac{(q_i - 2k_i\hat{r}_i^*)\alpha + K_i - k_i(\hat{r}_i^*)^2}{(\alpha+\hat{r}_i^*)^2} + \frac{P_i-\alpha}{(\alpha-P_i+\hat{r}_i^*)^2} = 0$, $\hat{r}_i^* = \sqrt{\frac{\left[k_i(\hat{r}_i^*)^2 + 2k_i\hat{r}_i^*\alpha - (q_i\alpha + K_i)\right](\alpha - P_i + \hat{r}_i^*)^2}{(P_i-\alpha)}} - \alpha > 0$. Moreover, $\lim_{\hat{r}_i\to 0}\frac{\partial E[\pi_i]}{\partial \hat{r}_i} = \left[\frac{q_i\alpha + K_i}{\alpha^2} + \frac{1}{(\alpha-P_i)^2}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j}{\alpha+\hat{r}_j} > 0$ and $\lim_{\hat{r}_i\to\infty}\frac{\partial E[\pi_i]}{\partial \hat{r}_i} = [-k_i]\prod_{j=1}^{i-1}\frac{\hat{r}_j}{\alpha+\hat{r}_j} < 0$ we can conclude that $\hat{r}_i^* > 0$ exists and is unique. Q.E.D. ■

Proof of Proposition 4 From Proposition 3, we can rearrange $\hat{r}_i^* = \sqrt{\frac{\left[k_i(\hat{r}_i^*)^2 + 2k_i\hat{r}_i^*\alpha - (q_i\alpha + K_i)\right](\alpha - P_i + \hat{r}_i^*)^2}{(P_i-\alpha)}} - \alpha$ to obtain $p_i = \frac{k_i(\hat{r}_i^*)^2 - K_i + 2k_i\hat{r}_i^*\alpha}{\alpha} - \frac{(P_i-\alpha)(\alpha+\hat{r}_i^*)^2}{\alpha(\alpha-P_i+\hat{r}_i^*)^2}$. Next substitute p_i in the objective of *P1*; we have that

$$\max_{\hat{r}_i^*, P_i} E[\Pi_C] = Q\prod_{i=1}^{n}\frac{\hat{r}_i^*}{\alpha+\hat{r}_i^*} - \sum_{i=1}^{n}\left[\left(\frac{\left(k_i(\hat{r}_i^*)^2 - K_i + 2k_i\hat{r}_i^*\alpha\right)\hat{r}_i^*}{\alpha(\alpha+\hat{r}_i^*)} - \frac{P_i(\hat{r}_i^*)^2}{\alpha(\alpha-P_i+\hat{r}_i^*)^2}\right)\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*}\right]$$
$$-\sum_{i=1}^{n}\left[\frac{C_o}{\alpha+\hat{r}_i^*}\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*}\right]$$
$$s.t$$
$$\left[\frac{k_i(\hat{r}_i^*)^2 - K_i}{\alpha} - \frac{P_i(\hat{r}_i^*)^2}{\alpha(\alpha-P_i+\hat{r}_i^*)^2}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*} \geq \theta_i \qquad \forall i. \tag{P2}$$

We first assume that the constraints are all binding, $\left[\frac{k_i(\hat{r}_i^*)^2 - K_i}{\alpha} - \frac{P_i(\hat{r}_i^*)^2}{\alpha(\alpha-P_i+\hat{r}_i^*)^2}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*} = \theta_i \quad \forall i$, and then show that this is always the case at equilibrium. The binding constraints imply that $\frac{P_i(\hat{r}_i^*)^2}{\alpha(\alpha-P_i+\hat{r}_i^*)^2}\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*} = \frac{k_i(\hat{r}_i^*)^2 - K_i}{\alpha}\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*} - \theta_i \quad \forall i$. Substitute this expression into the objective of P2 we have

$$\max_{\hat{r}_i^*} E[\Pi_C] = Q\prod_{i=1}^{n}\frac{\hat{r}_i^*}{\alpha+\hat{r}_i^*} - \sum_{i=1}^{n}\left[\left(\frac{\left(k_i(\hat{r}_i^*)^2 - K_i + 2k_i\hat{r}_i^*\alpha\right)\hat{r}_i^*}{\alpha(\alpha+\hat{r}_i^*)} - \frac{k_i(\hat{r}_i^*)^2 - K_i}{\alpha}\right)\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*} + \theta_i\right]$$
$$-\sum_{i=1}^{n}\left[\frac{C_o}{\alpha+\hat{r}_i^*}\prod_{j=1}^{i-1}\frac{\hat{r}_j^*}{\alpha+\hat{r}_j^*}\right],$$

which can be simplified as

$$\max_{\hat{r}_i^*} E[\Pi_C] = \underbrace{Q\prod_{i=1}^{n}\frac{r_i}{\alpha+r_i} - \sum_{i=1}^{n}\left[\left(\frac{C_o + K_i + k_i r_i^2}{\alpha+r_i}\right)\prod_{j=1}^{i-1}\frac{r_j}{\alpha+r_j}\right]}_{\text{Centralized profit } E[\Pi_{CP}]} - \sum_{i=1}^{n}\theta_i. \tag{P3}$$

Since $\sum_{i=1}^{n}\theta_i$ is a constant, the solution to P3 is the same as the centralized solution, namely, $\hat{r}_i^{**} = r_i^*$. Previously, we have assumed that the constraints in P2 are binding; here, we provide the justification. The maximum expected profit for the whole system is at most $E\left[\Pi_{CP}\left(r_1^*, \ldots, r_n^*\right)\right]$, since the subcontractors, in total, want at least $\sum_{i=1}^{n}\theta_i$, the maximum that the client could obtain is $E\left[\Pi_C^{\text{CP}}\left(r_1^*, \ldots, r_n^*\right)\right] - \sum_{i=1}^{n}\theta_i$. This is exactly what the client has obtained at equilibrium when the constraints are binding. Note that $\hat{r}_i^* = \sqrt{\frac{\left[k_i\left(\hat{r}_i^*\right)^2 + 2k_i\hat{r}_i^*\alpha - \left(q_i\alpha + K_i\right)\right]\left(\alpha - P_i + \hat{r}_i^*\right)^2}{\left(P_i - \alpha\right)}} - \alpha$ were obtained from Proposition 3, where we have assumed $P_i > \alpha \quad \forall i$. Since this assumption also leads the client to receiving the maximum expected profit, hence we can conclude that at the equilibrium $P_i^* > \alpha \quad \forall i$. Thus far, we have shown that the solution to P1 is equivalent to the solution that maximizes (11). Hence, the first step is to set $\hat{r}_i^{**} = \sqrt{\alpha^2 + \frac{\hat{Q}_i\alpha + C_o + K_i}{k_i}} - \alpha$ where $\hat{Q}_i = Q\prod_{j=i+1}^{n}\frac{\hat{r}_j^{**}}{\alpha + \hat{r}_j^{**}} - \sum_{j=i+1}^{n}\frac{\left(C_o + K_j + k_j\left(\hat{r}_j^{**}\right)^2\right)}{\alpha + \hat{r}_j^{**}}\prod_{m=i+1}^{j-1}\frac{\hat{r}_m^{**}}{\alpha + \hat{r}_m^{**}}$ for $i = n, \ldots, 1$. Since $\left[\frac{k_i\left(\hat{r}_i^{**}\right)^2 - K_i}{\alpha} - \frac{P_i\left(\hat{r}_i^{**}\right)^2}{\alpha\left(\alpha - P_i + \hat{r}_i^{**}\right)^2}\right]\prod_{j=1}^{i-1}\frac{\hat{r}_j^{**}}{\alpha + \hat{r}_j^{**}} = \theta_i \quad \forall i$. Then, it is easy to show that

$$P_i^* = \frac{2\alpha\xi_i\left(\alpha + \hat{r}_i^{**}\right) + \left(\hat{r}_i^{**}\right)^2 + \hat{r}_i^{**}\sqrt{4\alpha\xi_i\left(\alpha + \hat{r}_i^{**}\right) + \left(\hat{r}_i^{**}\right)^2}}{2\alpha\xi_i},$$

$$\text{where } \xi_i = \frac{k_i\left(\hat{r}_i^{**}\right)^2 - K_i}{\alpha} - \frac{\theta_i}{\prod_{j=1}^{i-1}\frac{\hat{r}_j^{**}}{\alpha + \hat{r}_j^{**}}} \quad \text{for } i = n, \ldots, 1.$$

Finally, we can then conclude that $q_i^* = \frac{k_i\left(\hat{r}_i^{**}\right)^2 - K_i + 2k_i\hat{r}_i^{**}\alpha}{\alpha} - \frac{\left(P_i^* - \alpha\right)\left(\alpha + \hat{r}_i^{**}\right)^2}{\alpha\left(\alpha - P_i^* + \hat{r}_i^{**}\right)^2}$, for $i = n, \ldots, 1$.
Q.E.D. ■

Proof of Corollary 2 Since the duration of tasks are independent of each other and Q is constant, stochastically maximizing $\Pi_{CP}(r_1, \ldots, r_n) = Q - \sum_{i=1}^{n}\left[\left(C_o + K_i + k_i r_i^2\right)t_i\right]$ is equivalent to stochastically minimizing the costs at each stage, namely, $\max_{r_i > 0} Pb\left[\left(C_o + K_i + k_i r_i^2\right)t_i \leq y\right]$ for all $y \geq 0$. Recall that t_i is exponentially distributed; thus, this problem is equivalent to $\max_{r_i > 0}\left[1 - e^{-\frac{r_i}{C_o + K_i + k_i r_i^2}y}\right]$. Note that $1 - e^{-\frac{r_i}{C_o + K_i + k_i r_i^2}y}$ is maximized when the exponent $\frac{r_i}{C_o + K_i + k_i r_i^2}y$ is maximized. From $\frac{\partial}{\partial r_i}\frac{r_i}{C_o + K_i + k_i r_i^2}y = \frac{y\left(C_o + K_i - k_i r_i^2\right)}{\left(C_o + K_i + k_i r_i^2\right)^2}$, we can see that $\frac{\partial}{\partial r_i}\frac{r_i}{C_o + K_i + k_i r_i^2}y > 0 \quad \forall 0 < r_i < \sqrt{\frac{C_o + K_i}{k_i}}$ and $\frac{\partial}{\partial r_i}\frac{r_i}{C_o + K_i + k_i r_i^2}y < 0 \quad \forall r_i > \sqrt{\frac{C_o + K_i}{k_i}}$. This result implies that $\frac{r_i}{C_o + K_i + k_i r_i^2}y$ is unimodal in r_i and has a maximum at $r_i{}^* = \sqrt{\frac{C_o + K_i}{k_i}}$, which does not depend on y. Thus, the cost of each stage is stochastically minimized when $r_i{}^* = \sqrt{\frac{C_o + K_i}{k_i}}$. By Theorem 1.A.3 (Shaked and Shanthikumar 2007), the total cost $\sum_{i=1}^{n}\left(C_o + K_i + k_i r_i^2\right)t_i$ is stochastically minimized when $r_i{}^* = \sqrt{\frac{C_o + K_i}{k_i}}$ for $i = 1, \ldots, n$. Since Q is constant, we have $Pb\left(Q - \sum_{i=1}^{n}\left(C_o + K_i + k_i\left(r_i^*\right)^2\right)t_i \leq y\right) \leq Pb\left(Q - \sum_{i=1}^{n}\left(C_o + K_i + k_i\left(r_i\right)^2\right)t_i \leq y\right)$ for $y \geq 0$ and $r_i \geq 0 \quad \forall i = 1, \ldots, n$.

Thus, $\Pi_{\text{CP}}(r_1{}^*, \ldots, r_n{}^*) \geq_{\text{FSD}} \Pi_{\text{CP}}(r_1, \ldots, r_n), \quad \forall r_i > 0$, where $\geq_{\text{FSD}}$ indicates first order stochastic dominance.
Q.E.D. ■

Proof of Corollary 3 The objective is to find a $\hat{r}_i^*$ such that $\pi_i\left(\hat{r}_i^*\right) \geq_{\text{FSD}} \pi_i\left(\hat{r}_i\right)$ for all $\hat{r}_i > 0$. Since q_i is constant, it is equivalent to $\max_{\hat{r}_i \geq 0} Pb\left[\left(P_i + K_i + k_i\hat{r}_i^2\right)t_i \leq y\right]$ for all y. Since t_i is exponentially distributed, it is equivalent to $\max_{\hat{r}_i \geq 0}\left[1 - e^{-\frac{\hat{r}_i}{P_i + K_i + k_i\hat{r}_i^2}y}\right]$. Similar to the proof of Corollary 2, the exponent $\frac{\hat{r}_i}{P_i + K_i + k_i\hat{r}_i^2}y$ is uniquely maximized when $\hat{r}_i^* = \sqrt{\frac{P_i + K_i}{k_i}}$, which does not depend on y.
Q.E.D. ■

Proof of Proposition 5 The proof is by induction. The base case is $i = n$, which is easy to see since, right after Equation (11) in Dawande et al., $V_{n+1} = R$, which is our Q, which equals Q_n; thus, $Q_n = V_{n+1}$, which implies $r_n = \lambda_n$.

We next assume that $r_{i+1} = \lambda_{i+1}$ and $Q_{i+1} = V_{i+2}$, and prove that $Q_i = V_{i+1}$, which implies that $r_i = \lambda_i$. Note that our definition of Q_i can be recast into a recursive form:

$$Q_i = Q\prod_{j=i+1}^{n}\frac{r_j}{\alpha + r_j} - \sum_{j=i+1}^{n}\frac{\kappa r_j^2}{\alpha + r_j}\prod_{m=i+1}^{j-1}\frac{r_m}{\alpha + r_m}$$

$$= \left(\frac{r_{i+1}}{\alpha + r_{i+1}}\right)\left(Q\prod_{j=i+2}^{n}\frac{r_j}{\alpha + r_j} - \sum_{j=i+2}^{n}\frac{\kappa r_j^2}{\alpha + r_j}\prod_{m=i+2}^{j-1}\frac{r_m}{\alpha + r_m}\right) - \frac{\kappa r_{i+1}^2}{\alpha + r_{i+1}}\underbrace{\prod_{m=i+1}^{i}\frac{r_m}{\alpha + r_m}}_{=1}$$

$$= \left(\frac{r_{i+1}}{\alpha + r_{i+1}}\right)(Q_{i+1} - \kappa r_{i+1})$$

$$= \left(\frac{\lambda_{i+1}}{\alpha + \lambda_{i+1}}\right)(V_{i+2} - \kappa\lambda_{i+1}),$$

where the last equality is by the induction hypothesis. Note that, since λ_{i+1} is optimal, the last expression is equal to Equation (11) in Dawande et al. (with their index n equaling our $i+1$); therefore, $\left(\frac{\lambda_{i+1}}{\alpha+\lambda_{i+1}}\right)(V_{i+2} - \kappa\lambda_{i+1}) = V_{i+1}$, and we conclude that $Q_i = V_{i+1}$, which implies that $r_i = \lambda_i$, completing the proof.

Q.E.D. ■