

Chapter 23

Electrical Potential

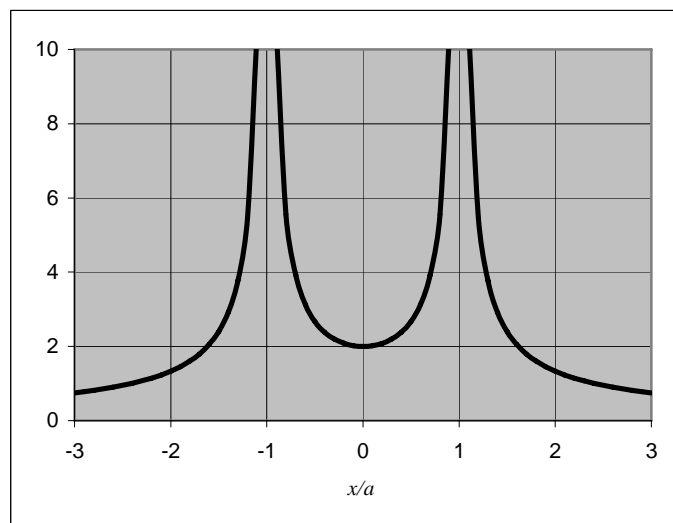
31 •• [SSM] Two identical positively charged point particles are fixed on the x -axis at $x = +a$ and $x = -a$. (a) Write an expression for the electric potential $V(x)$ as a function of x for all points on the x -axis. (b) Sketch $V(x)$ versus x for all points on the x axis.

Picture the Problem For the two charges, $r = |x - a|$ and $|x + a|$ respectively and the electric potential at x is the algebraic sum of the potentials at that point due to the charges at $x = +a$ and $x = -a$.

(a) Express $V(x)$ as the sum of the potentials due to the charges at $x = +a$ and $x = -a$:

$$V = kq \left(\frac{1}{|x - a|} + \frac{1}{|x + a|} \right)$$

(b) The following graph of V as a function of x/a was plotted using a spreadsheet program:



35 • A uniform electric field is in the $-x$ direction. Points a and b are on the x -axis, with a at $x = 2.00$ m and b at $x = 6.00$ m. (a) Is the potential difference $V_b - V_a$ positive or negative? (b) If $|V_b - V_a|$ is 100 kV, what is the magnitude of the electric field?

Picture the Problem We can use the relationship $E_x = -(dV/dx)$ to decide the sign of $V_b - V_a$ and $E = \Delta V/\Delta x$ to find E .

(a) Because $E_x = -(dV/dx)$, V is greater for larger values of x . So:

$$V_b - V_a \text{ is positive.}$$

(b) Express E in terms of $V_b - V_a$ and the separation of points a and b :

$$E_x = \frac{\Delta V}{\Delta x} = \frac{V_b - V_a}{\Delta x}$$

Substitute numerical values and evaluate E_x :

$$E_x = \frac{100 \text{ kV}}{6.00 \text{ m} - 2.00 \text{ m}} = \boxed{25.0 \text{ kV/m}}$$

37 •• The electric field on the x axis due to a point charge fixed at the origin is given by $\vec{E} = (b/x^2)\hat{i}$, where $b = 6.00 \text{ kV}\cdot\text{m}$ and $x \neq 0$. (a) Find the magnitude and sign of the point charge. (b) Find the potential difference between the points on the x -axis at $x = 1.00 \text{ m}$ and $x = 2.00 \text{ m}$. Which of these points is at the higher potential.

Picture the Problem We can integrate $E_x = -dV_x/dx$ to obtain $V(x)$ and then use this function to find the electric potential difference between the given points.

(a) We know that this field is due to a point charge because it varies inversely with the square of the distance from the point charge. Because \vec{E}_x is positive, the sign of the charge must be positive.

Because the given electric field is that due to a point charge, it follows that:

$$kq = 6.00 \frac{\text{kV}}{\text{m}} \Rightarrow q = \frac{6.00 \frac{\text{kV}}{\text{m}}}{k}$$

Substitute the numerical value of k and evaluate q :

$$q = \frac{6.00 \frac{\text{kV}}{\text{m}}}{8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}} = \boxed{668 \text{ nC}}$$

(b) The potential difference between the points on the x -axis at $x = 1.00 \text{ m}$ and $x = 2.00 \text{ m}$ is given by:

$$\Delta V = V_1 - V_2 \quad (1)$$

From $E_x = -\frac{dV_x}{dx}$ we have:

$$\int_{\infty}^x dV = -\int_{\infty}^x E_x dx = -kq \int_{\infty}^x x^{-2} dx$$

or

$$V(x) - V(\infty) = \frac{kq}{x}$$

Letting $V(\infty) = 0$ yields:

$$V(x) = \frac{kq}{x} = \frac{6.00 \text{ kV} \cdot \text{m}}{x}$$

Substituting in equation (1) yields:

$$\begin{aligned} V_1 - V_2 &= \frac{6.00 \text{ kV} \cdot \text{m}}{1.00 \text{ m}} - \frac{6.00 \text{ kV} \cdot \text{m}}{2.00 \text{ m}} \\ &= \boxed{3.00 \text{ kV}} \end{aligned}$$

Because $V_1 = V_2 + 3.00 \text{ kV}$, the point at $x = 2.00 \text{ m}$ is at the higher potential.

40 • A charge of $+10.0 \mu\text{C}$ is uniformly distributed on a thin spherical shell of radius 12.0 cm . (Assume the potential is zero very far from all charges.)

- (a) What is the magnitude of the electric field just outside and just inside the shell? (b) What is the magnitude of the electric potential just outside and just inside the shell? (c) What is the electric potential at the center of the shell? (d) What is the magnitude of the electric field at the center of the shell?

Picture the Problem We can construct Gaussian surfaces just inside and just outside the spherical shell and apply Gauss's law to find the electric field at these locations. We can use the expressions for the electric potential inside and outside a spherical shell to find the potential at these locations.

(a) Apply Gauss's law to a spherical Gaussian surface of radius $r < 12.0$ cm:

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0} = 0$$

because the charge resides on the outer surface of the spherical surface. Hence

$$\vec{E}(r < 12.0 \text{ cm}) = \boxed{0}$$

Apply Gauss's law to a spherical Gaussian surface of radius $r > 12.0$ cm:

$$E(4\pi r^2) = \frac{q}{\epsilon_0}$$

and

$$E(r > 12.0 \text{ cm}) = \frac{q}{4\pi r^2 \epsilon_0} = \frac{kq}{r^2}$$

Substitute numerical values and evaluate $E(r > 12.0 \text{ cm})$:

$$E(r > 12.0 \text{ cm}) = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10.0 \mu\text{C})}{(0.120 \text{ m})^2} = \boxed{6.24 \text{ MV/m}}$$

(b) Express and evaluate the potential just inside the spherical shell:

$$V(r \leq R) = \frac{kq}{R} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10.0 \mu\text{C})}{0.120 \text{ m}} = \boxed{749 \text{ kV}}$$

Express and evaluate the potential just outside the spherical shell:

$$V(r \geq R) = \frac{kq}{r} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10.0 \mu\text{C})}{0.120 \text{ m}} = \boxed{749 \text{ kV}}$$

(c) The electric potential inside a uniformly charged spherical shell is constant and given by:

$$V(r \leq R) = \frac{kq}{R} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(10.0 \mu\text{C})}{0.120 \text{ m}} = \boxed{749 \text{ kV}}$$

(d) In Part (a) we showed that: $\vec{E}(r < 12.0 \text{ cm}) = \boxed{0}$

43 • Find the maximum surface charge density σ_{max} that can exist on the surface of any conductor before dielectric breakdown of the air occurs.

Picture the Problem We can solve the equation giving the electric field at the surface of a conductor for the greatest surface charge density that can exist before dielectric breakdown of the air occurs.

Relate the electric field at the surface of a conductor to the surface charge density:

$$E = \frac{\sigma}{\epsilon_0}$$

Solve for σ under dielectric breakdown of the air conditions:

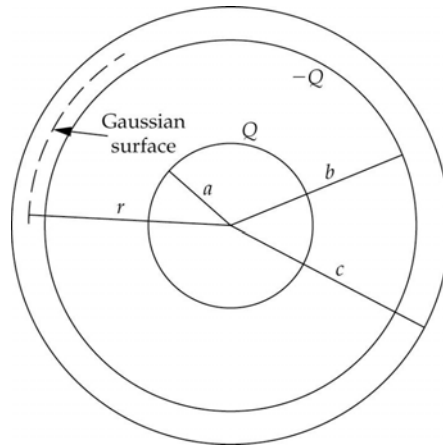
$$\sigma_{\text{max}} = \epsilon_0 E_{\text{breakdown}}$$

Substitute numerical values and evaluate σ_{\max} :

$$\begin{aligned}\sigma_{\max} &= (8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3 \text{ MV/m}) \\ &\approx \boxed{3 \times 10^{-5} \text{ C/m}^2}\end{aligned}$$

44 •• A conducting spherical shell of inner radius b and outer radius c is concentric with a small metal sphere of radius $a < b$. The metal sphere has a positive charge Q . The total charge on the conducting spherical shell is $-Q$. (Assume the potential is zero very far from all charges.) (a) What is the electric potential of the spherical shell? (b) What is the electric potential of the metal sphere?

Picture the Problem The diagram is a cross-sectional view showing the charges on the sphere and the spherical conducting shell. A portion of the Gaussian surface over which we'll integrate E in order to find V in the region $r > b$ is also shown. For $a < r < b$, the sphere acts like point charge Q and the potential of the metal sphere is the sum of the potential due to a point charge at its center and the potential at its surface due to the charge on the inner surface of the spherical shell.



(a) Express $V_{r>b}$:

$$V_{r>b} = -\int E_{r>b} dr$$

Apply Gauss's law for $r > b$:

$$\oint_S \vec{E}_r \cdot \hat{n} dA = \frac{Q_{\text{enclosed}}}{\epsilon_0} = 0$$

and $E_{r>b} = 0$ because $Q_{\text{enclosed}} = 0$ for $r > b$.

Substitute for $E_{r>b}$ to obtain:

$$V_{r>b} = -\int (0) dr = \boxed{0}$$

(b) Express the potential of the metal sphere:

$$V_a = V_{Q \text{ at its center}} + V_{\text{surface}}$$

Express the potential at the surface of the metal sphere:

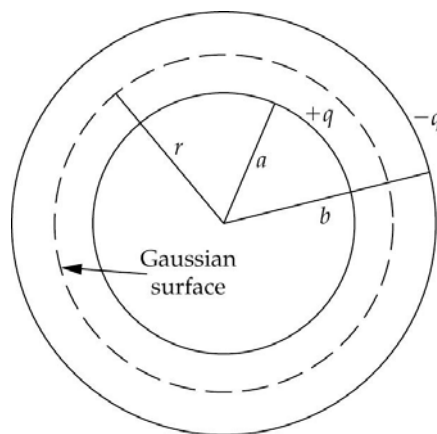
$$V_{\text{surface}} = \frac{k(-Q)}{b} = -\frac{kQ}{b}$$

Substitute and simplify to obtain:

$$V_a = \frac{kQ}{a} - \frac{kQ}{b} = \boxed{kQ \left(\frac{1}{a} - \frac{1}{b} \right)}$$

45 •• [SSM] Two coaxial conducting cylindrical shells have equal and opposite charges. The inner shell has charge $+q$ and an outer radius a , and the outer shell has charge $-q$ and an inner radius b . The length of each cylindrical shell is L , and L is very long compared with b . Find the potential difference, $V_a - V_b$ between the shells.

Picture the Problem The diagram is a cross-sectional view showing the charges on the inner and outer conducting shells. A portion of the Gaussian surface over which we'll integrate E in order to find V in the region $a < r < b$ is also shown. Once we've determined how E varies with r , we can find $V_a - V_b$ from $V_b - V_a = -\int E_r dr$.



Express the potential difference $V_b - V_a$:

$$V_b - V_a = -\int E_r dr \Rightarrow V_a - V_b = \int E_r dr$$

Apply Gauss's law to cylindrical Gaussian surface of radius r and length L :

$$\oint_S \vec{E} \cdot \hat{n} dA = E_r (2\pi r L) = \frac{q}{\epsilon_0}$$

Solving for E_r yields:

$$E_r = \frac{q}{2\pi \epsilon_0 r L}$$

Substitute for E_r and integrate from $r = a$ to b :

$$V_a - V_b = \frac{q}{2\pi \epsilon_0 L} \int_a^b \frac{dr}{r} = \boxed{\frac{2kq}{L} \ln \left(\frac{b}{a} \right)}$$

48 •• The electric potential at the surface of a uniformly charged sphere is 450 V. At a point outside the sphere at a (radial) distance of 20.0 cm from its

surface, the electric potential is 150 V. (The potential is zero very far from the sphere.) What is the radius of the sphere, and what is the charge of the sphere?

Picture the Problem Let R be the radius of the sphere and Q its charge. We can express the potential at the two locations given and solve the resulting equations simultaneously for R and Q .

Relate the potential of the sphere at its surface to its radius:

$$\frac{kQ}{R} = 450 \text{ V} \quad (1)$$

Express the potential at a distance of 20.0 cm from its surface:

$$\frac{kQ}{R + 0.200 \text{ m}} = 150 \text{ V} \quad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{\frac{kQ}{R}}{\frac{kQ}{R + 0.200 \text{ m}}} = \frac{450 \text{ V}}{150 \text{ V}}$$

or

$$\frac{R + 0.200 \text{ m}}{R} = 3 \Rightarrow R = \boxed{10.0 \text{ cm}}$$

Solving equation (1) for Q yields:

$$Q = (450 \text{ V}) \frac{R}{k}$$

Substitute numerical values and evaluate Q :

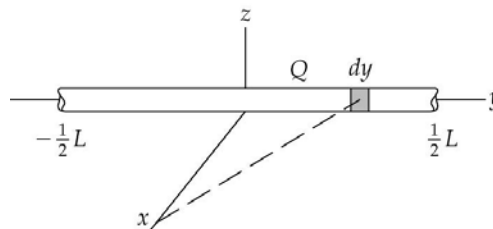
$$Q = (450 \text{ V}) \frac{(0.100 \text{ m})}{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)}$$

$$= \boxed{5.01 \text{ nC}}$$

51 •• [SSM] A rod of length L has a total charge Q uniformly distributed along its length. The rod lies along the y -axis with its center at the origin. (a) Find an expression for the electric potential as a function of position along the x -axis. (b) Show that the result obtained in Part (a) reduces to $V = kQ/|x|$ for $|x| \gg L$.

Explain why this result is expected.

Picture the Problem Let the charge per unit length be $\lambda = Q/L$ and dy be a line element with charge λdy . We can express the potential dV at any point on the x axis due to the charge element λdy and integrate to find $V(x, 0)$.



(a) Express the element of potential dV due to the line element dy :

$$dV = \frac{k\lambda}{r} dy$$

where $r = \sqrt{x^2 + y^2}$ and $\lambda = \frac{Q}{L}$.

Substituting for r and λ yields:

$$dV = \frac{kQ}{L} \frac{dy}{\sqrt{x^2 + y^2}}$$

Use a table of integrals to integrate dV from $y = -L/2$ to $y = L/2$:

$$\begin{aligned} V(x,0) &= \frac{kQ}{L} \int_{-L/2}^{L/2} \frac{dy}{\sqrt{x^2 + y^2}} \\ &= \boxed{\frac{kQ}{L} \ln \left(\frac{\sqrt{x^2 + \frac{1}{4}L^2} + \frac{1}{2}L}{\sqrt{x^2 + \frac{1}{4}L^2} - \frac{1}{2}L} \right)} \end{aligned}$$

(b) Factor x from the numerator and denominator within the parentheses to obtain:

$$V(x,0) = \frac{kQ}{L} \ln \left(\frac{\sqrt{1 + \frac{L^2}{4x^2}} + \frac{L}{2x}}{\sqrt{1 + \frac{L^2}{4x^2}} - \frac{L}{2x}} \right)$$

Use $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ to obtain:

$$V(x,0) = \frac{kQ}{L} \left\{ \ln \left(\sqrt{1 + \frac{L^2}{4x^2}} + \frac{L}{2x} \right) - \ln \left(\sqrt{1 + \frac{L^2}{4x^2}} - \frac{L}{2x} \right) \right\}$$

Let $\varepsilon = \frac{L^2}{4x^2}$ and use $(1 + \varepsilon)^{1/2} = 1 + \frac{1}{2}\varepsilon - \frac{1}{8}\varepsilon^2 + \dots$ to expand $\sqrt{1 + \frac{L^2}{4x^2}}$:

$$\left(1 + \frac{L^2}{4x^2}\right)^{1/2} = 1 + \frac{1}{2} \frac{L^2}{4x^2} - \frac{1}{8} \left(\frac{L^2}{4x^2}\right)^2 + \dots \approx 1 \text{ for } |x| \gg L.$$

Substitute for $\left(1 + \frac{L^2}{4x^2}\right)^{1/2}$ to obtain:

$$V(x,0) = \frac{kQ}{L} \left\{ \ln \left(1 + \frac{L}{2x} \right) - \ln \left(1 - \frac{L}{2x} \right) \right\}$$

Let $\delta = \frac{L}{2x}$ and use $\ln(1 + \delta) = \delta - \frac{1}{2}\delta^2 + \dots$ to expand $\ln\left(1 \pm \frac{L}{2x}\right)$:

$$\ln\left(1 + \frac{L}{2x}\right) \approx \frac{L}{2x} - \frac{L^2}{4x^2} \quad \text{and} \quad \ln\left(1 - \frac{L}{2x}\right) \approx -\frac{L}{2x} - \frac{L^2}{4x^2} \quad \text{for } x \gg L.$$

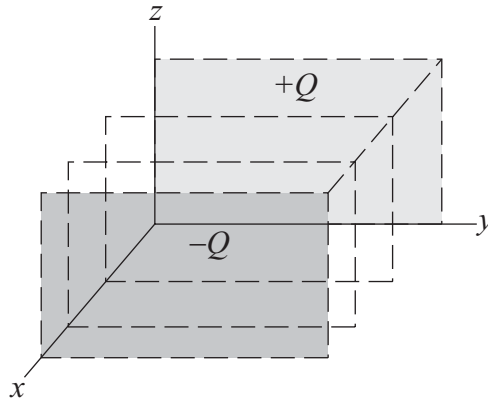
Substitute for $\ln\left(1 + \frac{L}{2x}\right)$ and $\ln\left(1 - \frac{L}{2x}\right)$ and simplify to obtain:

$$V(x,0) = \frac{kQ}{L} \left\{ \frac{L}{2x} - \frac{L^2}{4x^2} - \left(-\frac{L}{2x} - \frac{L^2}{4x^2} \right) \right\} = \boxed{\frac{kQ}{x}}$$

Because, for $|x| \gg L$, the charge carried by the rod is far enough away from the point of interest to look like a point charge, this result is what we would expect.

61 •• [SSM] Consider two parallel uniformly charged infinite planes that are equal but oppositely charged. (a) What is (are) the shape(s) of the equipotentials in the region between them? Explain your answer. (b) What is (are) the shape(s) of the equipotentials in the regions not between them? Explain your answer.

Picture the Problem The two parallel planes, with their opposite charges, are shown in the pictorial representation.



(a) Because the electric field between the charged plates is uniform and perpendicular to the plates, the equipotential surfaces are planes parallel to the charged planes.

(b) The regions to either side of the two charged planes are equipotential regions, so any surface in either of these regions is an equipotential surface.

67 •• [SSM] (a) How much charge is on the surface of an isolated spherical conductor that has a 10.0-cm radius and is charged to 2.00 kV? (b) What is the electrostatic potential energy of this conductor? (Assume the potential is zero far from the sphere.)

Picture the Problem The potential of an isolated spherical conductor is given by $V = kQ/r$, where Q is its charge and r its radius, and its electrostatic potential energy by $U = \frac{1}{2}QV$. We can combine these relationships to find the sphere's electrostatic potential energy.

(a) The potential of the isolated spherical conductor at its surface is related to its radius:

$$V = \frac{kQ}{R} \Rightarrow Q = \frac{RV}{k}$$

where R is the radius of the spherical conductor.

Substitute numerical values and evaluate Q :

$$\begin{aligned} Q &= \frac{(10.0 \text{ cm})(2.00 \text{ kV})}{8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}} \\ &= 22.25 \text{ nC} = \boxed{22.3 \text{ nC}} \end{aligned}$$

(b) Express the electrostatic potential energy of the isolated spherical conductor as a function of its charge Q and potential V :

$$U = \frac{1}{2}QV$$

Substitute numerical values and evaluate U :

$$U = \frac{1}{2}(22.25 \text{ nC})(2.00 \text{ kV}) = \boxed{22.3 \mu\text{J}}$$

70 •• Consider two point particles that each have charge $+e$, are at rest, and are separated by 1.50×10^{-15} m. (a) How much work was required to bring them together from a very large separation distance? (b) If they are released, how much kinetic energy will each have when they are separated by twice their separation at release? (c) The mass of each particle is 1.00 u (1.00 AMU). What speed will each have when they are very far from each other?

Picture the Problem (a) In the absence of other charged bodies, no work is required to bring the first proton from infinity to its initial position. We can use the work-energy theorem to find the work required to bring the second proton to a position 1.50×10^{-15} m away from the first proton. (b) and (c) We can apply conservation of mechanical energy to the two-proton system to find the kinetic energy of each proton when they are separated by twice their separation at release and when they are separated by a large distance.

(a) Apply the work-energy theorem to the second proton to obtain:

$$W_{\text{ext}} = \Delta K + \Delta U = 0 + \frac{ke^2}{r} = \frac{ke^2}{r}$$

Substitute numerical values and evaluate W_{ext} :

$$\begin{aligned} W_{\text{ext}} &= \frac{\left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) (1.602 \times 10^{-19} \text{ C})^2}{1.50 \times 10^{-15} \text{ m}} = 1.538 \times 10^{-13} \text{ J} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \\ &= \boxed{960 \text{ keV}} \end{aligned}$$

(b) Apply conservation of mechanical energy to the separating protons to obtain:

$$\begin{aligned} \Delta K + \Delta U &= 0 \Rightarrow K_f - K_i + U_f - U_i = 0 \\ \text{or, because } K_i &= 0, \\ K_f + U_f - U_i &= 0 \end{aligned}$$

Substituting for U_i and U_f and simplifying yields:

$$\begin{aligned} K_f = U_i - U_f &= \frac{ke^2}{r_i} - \frac{ke^2}{r_f} = \frac{ke^2}{r} - \frac{ke^2}{2r} \\ &= \frac{ke^2}{2r} \end{aligned}$$

Remembering that K_f is the kinetic energy of both protons, substitute numerical values and evaluate K_f , each proton:

$$\begin{aligned} K_{f, \text{ each proton}} &= \frac{1}{2} \frac{\left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) (1.602 \times 10^{-19} \text{ C})^2}{2(1.50 \times 10^{-15} \text{ m})} \\ &= 3.844 \times 10^{-14} \text{ J} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \\ &= \boxed{240 \text{ keV}} \end{aligned}$$

(c) Apply conservation of mechanical energy to the separating protons to obtain:

$$\begin{aligned} \Delta K + \Delta U &= 0 \Rightarrow K_f - K_i + U_f - U_i = 0 \\ \text{or, because } K_i &= U_f = 0, \\ K_f - U_i &= 0 \Rightarrow \frac{1}{2} m_p v_\infty^2 - U_i = 0 \end{aligned}$$

Solving for v_∞ yields:

$$v_\infty = \sqrt{\frac{2U_i}{m_p}} \text{ where } U_i \text{ is half the initial potential energy of the two-proton system.}$$

Substitute numerical values and evaluate v_∞ :

$$\begin{aligned}v_\infty &= \sqrt{\frac{2 \left(480 \text{ keV} \times \frac{1.602 \times 10^{-19} \text{ J}}{\text{eV}} \right)}{1.673 \times 10^{-27} \text{ kg}}} \\ &= \boxed{9.59 \times 10^6 \text{ m/s}}\end{aligned}$$