Atmospheric Environment Pergamon Press 1970, Vol. 4, pp. 163–173. Printed in Great Britain.

PLUME OPACITY AND PARTICULATE MASS CONCENTRATION

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(*First received* 12 *June* 1969, *and in final form* 8 *August* 1969)

Abstract—A general theoretical relationship between plume opacity and the properties of particulate air pollutants has been developed. These results are in agreement with previously reported theoretical equations for specific emissions and with the known measurements of plume opacity and particle properties. A parameter K, defined as the specific particulate volume/light extinction coefficient ratio ($cm^3\ m^{-3}m$), was used to relate the plume opacity to the particle properties. Graphs of K *vs.* the particle geometric mass mean radius at geometric standard deviations of 1, 1.5, 2, 3, 4 and 5 are presented for particles of refractive index 1.33 (water) and $1.95-0.66i$ (carbon). An example is included illustrating the use of the theoretical results to calculate the maximum allowable particle concentration which will meet a given Ringelmann number.

NOMENCLATURE

a_t, b_t	Mie amplitude coefficients
B_E	Extinction coefficient
cm	Centimeter
$f(r)$	Particle number fraction frequency distribution
I	Intensity of transmitted light
I_0	Intensity of incident light
I/I_0	Light transmittance
i	Imaginary unit $(-1)^{1/2}$
g	Gram
K	Specific particulate volume/extinction coefficient ratio
L	Illumination path length
m	Refractive index of particle relative to air
m	Meter
N	Total particle number concentration
$n(r)$	Particle number frequency distribution, which, multiplied by the radius increment, dr, is the number of particles between r and $r+\mathrm{d}r$
Q_E	Extinction efficiency factor
r	Particle radius
Δr	Particle radius increment
r_{gn}	Geometric number mean radius
r_{gw}	Geometric mass mean radius
S_E	Extinction cross-section per particle
t	Order of function
W	Total particle mass concentration
$Z_t^{(1)}(y)$	Spherical Bessel function of the first kind
$Z_t^{(3)}(y)$	Spherical Bessel function of the third kind

Greek symbols

α	Particle size parameter, $2\pi r/\lambda$
$\eta_t^{(1)}(y)$	Derivative of spherical Bessel function of the first kind
$\eta_t^{(3)}(y)$	Derivative of spherical Bessel function of the third kind
λ	Wavelength of light
π	3.14159
ρ	Particle density
σ_g	Particle size geometric standard deviation.

1. INTRODUCTION

AIR POLLUTION control laws may limit the emission of particulate matter into the atmosphere by four types of regulations:

1. visible emission (Ringelmann number)
2. particulate concentration in the stack (gr ft^{-3})
3. process weight code (lb particulate matter/lb process material).
4. particulate concentration at the property line boundary ($\mu g\ m^{-3}$).

Because of the ease of making a visual observation of a plume with a Ringelmann number, it is the particulate emission evaluation technique most frequently used. However, the design of particulate air pollutant control equipment is based on a removal efficiency for a certain inlet particulate weight concentration, size distribution, and density. Thus for equipment design purposes it would be useful to know the relationship between the plume visual properties (Ringelmann number) and the particulate properties (weight concentration and size distribution).

Relationships between plume opacity and particulate mass concentration have been reported for specific emissions. HAWKSLEY *et al.* (1961) estimated the relationship of the particulate mass concentration to the plume opacity for two sizes of light absorbing particles of 2 g cm^{-3} density. The relationship for particles much smaller than the light wavelength is

$$\ln(I/I_0) = -4.0\,WL, \tag{1}$$

where I/I_0 is the fraction of incident light which is transmitted (light transmittance), L is the illumination path length in meters (usually stack diameter), and W is the total particle mass concentration in g m^{-3}. The equation for particles much larger than the wavelength of light (average specific surface area diameter of 12 μm) is

$$\ln(I/I_0) = -0.125\,WL, \tag{2}$$

HAWKSLEY *et al.* (1961) reported good agreement between the relationship for small particles (1) and the data of HURLEY and BAILEY (1958). Measurements of plume opacity and particulate mass concentration have been reported by CONNER and HODKINSON (1967). Some attempts to correlate the measured light extinction and particle concentration in stacks have been reported by STOECKER (1950) and HURLEY and BAILEY (1958). These studies indicated that in a single source the light extinction of the particles is directly related to the particle mass concentration. Scatter in the experimental data was attributed to changes in the particle size distribution. The inability to control or measure the particle size distribution was cited by ENGDAHL (1951) and by MITCHELL and ENGDAHL (1963) as a problem in developing a relationship.

The Bay Area Pollution Control Board Regulation 2 (1962) has a requirement limiting the concentration of particulate matter emissions to that given by

$$W[\text{g m}^{-3}] = \frac{1.01}{L(\text{m})} \tag{3}$$

ROBINSON (1962) reported that this equation was developed for a plume opacity of Ringelmann number 2 (40 per cent extinction) and a hypothetical oil aerosol (mass mean radius of 0.23 μm and a geometric standard deviation of 3.4).

This paper presents a general theoretical analysis of the relationship between particulate air pollutant properties and plume opacity. These results should assist in: (1) developing regulations for the control of particulate air pollutants, (2) designing particulate control equipment which can meet opacity standards, and (3) explaining the results of attempts to correlate particulate mass concentrations and light extinction measurements.

2. THEORY

(a). *Development of equation relating plume opacity to particle properties*

A general relationship can be developed between plume opacity (I/I_0), the plume path length, and the particle properties (particle size distribution, density, mass concentration, and refractive index). The transmission of light through a volume containing an aerosol is described by the Lambert–Beer law

$$I/I_0 = \exp(-B_E L), \tag{4}$$

where L is the illumination path length and B_E is the extinction coefficient. Assuming that the light extinction in the volume is entirely due to aerosol particles of constant concentration throughout the illumination path length, B_E can be defined in terms of the extinction cross-section per particle S_E and $n(r)$, the particle number frequency distribution (number of particles/volume of air between r and $r+dr$).

$$B_E = \int_0^\infty S_E(r, \lambda, m)\, n(r)\, \mathrm{d}r. \tag{5}$$

The extinction cross-section per particle S_E is related to the extinction efficiency factor Q_E (total light flux scattered and absorbed by a particle divided by the light flux incident on the particle) and the projected cross-sectional area of a spherical particle by

$$S_E = \pi r^2 Q_E. \tag{6}$$

Therefore the extinction coefficient of a polydisperse aerosol is given by

$$B_E = \int_0^\infty Q_E(r, \lambda, m)\, \pi r^2\, n(r)\, \mathrm{d}r. \tag{7}$$

The extinction efficiency factor Q_E can be computed using the Mie equations (VAN DER HULST, 1957).

$$Q_E = \frac{2}{\alpha^2} \sum_{t=1}^{\infty} (2t+1)\, Re(a_t + b_t) \tag{8}$$

The term Re means a real part of the complex number in parenthesis and α is the size parameter $2\pi r/\lambda$.

The complex Mie amplitude coefficients a_t and b_t are defined as:

$$a_t = \frac{\eta_t^{(1)}(m\alpha)\, Z_t^{(1)}(\alpha) - m\, Z_t^{(1)}(m\alpha)\, \eta_t^{(1)}(\alpha)}{\eta_t^{(1)}(m\alpha)\, Z_t^{(3)}(\alpha) - m\, Z_t^{(1)}(m\alpha)\, \eta_t^{(3)}(\alpha)}, \tag{9}$$

$$b_t = \frac{m\, \eta_t^{(1)}(m\alpha)\, Z_t^{(1)}(\alpha) - Z_t^{(1)}(m\alpha)\, \eta_t^{(1)}(\alpha)}{m\, \eta_t^{(1)})m\alpha(\, Z_t^{(3)}(\alpha) - Z_t^{(1)}(m\alpha)\, \eta_t^{(3)}(\alpha)}. \tag{10}$$

Where: $Z_t^{(1)}(y)$ = Spherical Bessel function of the first kind.
$Z_t^{(3)}(y)$ = Spherical Bessel function of the third kind.
$\eta_t^{(1)}(y)$ = Derivative of Spherical Bessel function of the first kind.
$\eta_t^{(3)}(y)$ = Derivative of Spherical Bessel function of the third kind.

The particle size frequency distribution $n(r)$ is related to the total particle number concentration N and the particle number fraction frequency distribution $f(r)$ (which, multiplied by the particle radius increment, dr, gives the fraction of particles between r and $r+\mathrm{d}r$)

$$n(r) = Nf(r). \tag{11}$$

The total particle number concentration N is defined as

$$N = \int_0^\infty n(r)\,\mathrm{d}r, \tag{12}$$

and the particle number fraction frequency distribution $f(r)$ as

$$\int_0^\infty f(r)\,\mathrm{d}r = 1.0. \tag{13}$$

Substituting (7) for B_E into the Lambert–Beer law (4) gives

$$\ln(I/I_0) = -L\int_0^\infty \pi r^2 Q(r,\lambda,m)\,n(r)\,\mathrm{d}r. \tag{14}$$

Substituting (11) for the particle size number frequency distribution $n(r)$ in (14) produces.

$$\ln(I/I_0) = -NL\int_0^\infty \pi r^2 Q(r,\lambda,m) f(r)\,\mathrm{d}r. \tag{15}$$

The particle mass concentration W is related to $n(r)$ and the particle density ρ by

$$W = \int_0^\infty \tfrac{4}{3}\pi r^3 \rho\, n(r)\,\mathrm{d}r. \tag{16}$$

and can also be given in terms of the total particle number concentration N by substituting (11) for $n(r)$ into (16)

$$W = \tfrac{4}{3}\pi N\rho \int_0^\infty r^3 f(r)\,\mathrm{d}r. \tag{17}$$

An equation relating the plume opacity I/I_0 to the aerosol mass concentration W is obtained by dividing (17) by (15)

$$\frac{W}{\ln(I/I_0)} = -\frac{\tfrac{4}{3}\rho\int_0^\infty r^3 f(r)\,\mathrm{d}r}{L\int_0^\infty r^2 Q(r,\lambda,m) f(r)\,\mathrm{d}r}. \tag{18}$$

A parameter K defined as the specific particulate volume (cm^3 particles/m^3 air) divided by the extinction coefficient (m^{-1}) is given by

$$K = \frac{\frac{4}{3}\int_0^\infty r^3 f(r)\,dr}{\int_0^\infty r^2 Q(r, \lambda, m) f(r)\,dr}. \tag{19}$$

The relationship of K to the plume and particle parameters can be shown by an equation of the form of the Lambert–Beer law (4) and the other reported specific emission relationships (2) and (3)

$$\ln(I/I_0) = -\frac{WL}{K\rho}, \tag{20}$$

or of a form relating K to the extinction coefficient

$$B_E = \frac{W}{K\rho}. \tag{21}$$

For the calculation of the maximum allowable particle concentration for a given plume opacity, particle density, extinction path length, and parameter K, (20) can be rearranged into the following working equation

$$W = -\frac{\rho K \ln(I/I_0)}{L}. \tag{22}$$

(b). *Calculation of typical magnitudes of* K

The parameter K is a function of the particle size distribution, the particle refractive index, and the wavelength of light. The particle size distribution is very important in the determination of K. The size distribution of particulate air pollutants is usually reported in a log normal form. As the size distributions are commonly reported as cumulative particle mass vs. the particle radius or diameter, a transformation of these parameters to cumulative number distribution parameters is necessary. The size distribution parameters of the geometric mass mean radius r_{gw} and the geometric standard deviation of σ_g are determined by plotting on log probability paper a cumulative curve of particulate mass vs. the log of the particle radius. The geometric mass mean radius occurs at the 50 per cent cumulative point and the ratio of the 84.1 per cent radius to the 50 per cent radius gives the geometric standard deviation. A detailed explanation of particle size analyses and statistics has been published by HERDAN (1960) and by CADLE (1965). A log normal frequency is unique in that simple mathematical transformations exist between the various types of distributions (particle number, surface area, or mass). The geometric standard deviation remains the same for all types of distributions

The transformation between number and mass geometric mean radii, r_{gn} and r_{gw}, respectively, is

$$\ln r_{gn} = \ln r_{gw} - 3\ln^2 \sigma_g. \tag{23}$$

The equation for a log normal distribution of the particle number fraction frequency distribution $f(r)$ is

$$f(r) = \frac{1}{r\sqrt{2\pi}\ln\sigma_g} \exp - \left[\frac{\ln^2 r/r_{gn}}{2\ln^2\sigma_g}\right], \tag{24}$$

substituting (24) for $f(r)$ gives an equation for the parameter K

$$K = \frac{\frac{4}{3}\int_0^\infty r^2 \exp - \left[\frac{\ln^2 r/r_{gn}}{2\ln^2\sigma_g}\right] dr}{\int_0^\infty r\, Q_E(r, \lambda, m) \exp - \left[\frac{\ln^2 r/r_{gn}}{2\ln^2\sigma_g}\right] dr}. \tag{25}$$

The parameter K was calculated using a digital computer (IBM 7094). Equations (8), (9), (10) and (23), were used to convert the input data (geometric mass mean radius r_{gw}, geometric standard deviation σ_g, particle refractive index m, and light wavelength λ) into the variables needed in (25). Equation 25 was evaluated numerically with the trapezoidal rule which is reported by DAVIS and RABINOWITZ (1967) to be a suitable numerical technique for periodic functions. The inputs of the light extinction efficiency factor $Q_E(r, \lambda, m)$ and the particle number fraction frequency distribution $f(r)$ were computed for discrete values of r over the radius range from 0.001 to 1000 μm. Because r extended over 7 orders of magnitude the value of Δr was maintained at about 1 per cent of r (Δr varied from 0.00001 to 10 μm) in order not to exceed the computer memory capacity. The spherical Bessel functions were computed for higher orders of t by forward recursion formulas and the initial values at $t = 0$ and $t = 1$. The extinction efficiency factor Q_E was set equal to 2.0 for magnitudes of the particle size parameter α greater than 85 to avoid convergence problems. A value of 2.0 for Q_E was based on the assumption that the illuminated path is long compared with the diameter of the light source and thus the light scattered by the aerosol particles should not reach the detector.

3. DISCUSSION OF RESULTS

(a). *Effect of particle refractive index*

The parameter K was calculated as a function of the geometric mass mean radius r_{gw} and the geometric standard deviation σ_g for homogeneous spherical particles of refractive indices of 1.33 (pure light scatterer) and of 1.95–0.66i (light absorber) at a light wavelength of 0.5 μm. These refractive indices were selected to represent the extremes of the particulate pollutant emissions. The effect of non-homogeneous refractive index is significant only for particles smaller than the wavelength of light as reported for concentric spheres by FENN and OSER (1965) and PILAT (1967).

The computed results for K at a particle refractive index of 1.33 (liquid water) are presented in FIG. 1. Curves for geometric standard deviations of 1.0 (monodisperse) 1.5, 2.0, 3.0, 4.0, and 5.0 are included. The oscillating nature of the monodisperse curve reflects the inverse of the familiar plot of the extinction efficiency factor vs. the size parameter. In FIG. 2 K is plotted vs. the geometric mass mean radius for a particle refractive index of 1.95–0.66i (carbon or soot). It is evident from these two figures that above a geometric mean radius of 0.5 μm the particle refractive index does not significantly influence the plume opacity. The major variation in K is caused by changes in the particle size distribution.

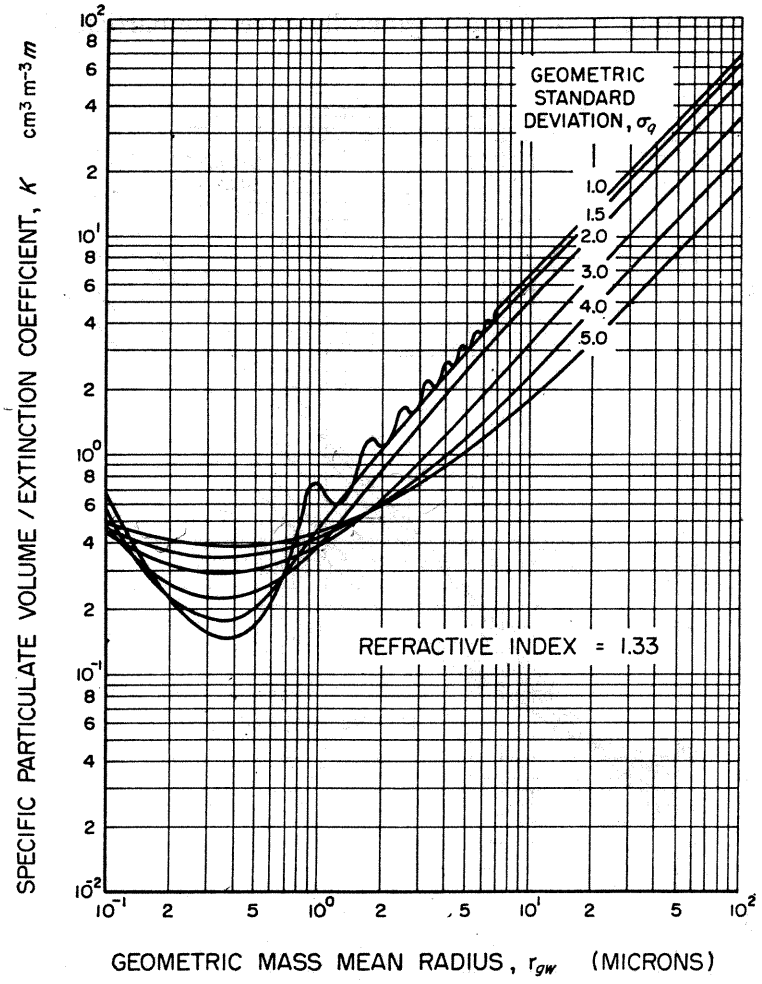

FIG. 1. Relationship of K to particle size distribution parameters for a white aerosol.

TABLE 1. PARTICULATE AIR POLLUTANT SIZE DISTRIBUTION DATA

Source	Geometric mass mean radius (μ)	Geometric standard deviation	Reference
Electric steel furnace	1.1	8.2	*Air Pollution Engineering Manual* (1967)
Cement dust	8.5	3	KREICHELT *et al.* (1967)
Wood smoke	0.035	1.7	FOSTER (1960)
Pulverized coal	9.5	4	SMITH and GRUBER (1966)

(b). *Particulate air pollutant size distributions of emission sources*

As the particulate size distribution has a significant effect on K an examination of typical source size distributions is appropriate. Particulate air pollutant size distribution data for emission sources is presented in TABLE 1. The geometric standard deviation ranges from 1.0 for monodisperse particles to about 8 for very polydisperse emission with the average σ_g around 3. The geometric mass mean radius ranges from about

0.04 to 10 μm. Therefore FIGS. 1 and 2 generally cover the particle size distribution range of interest.

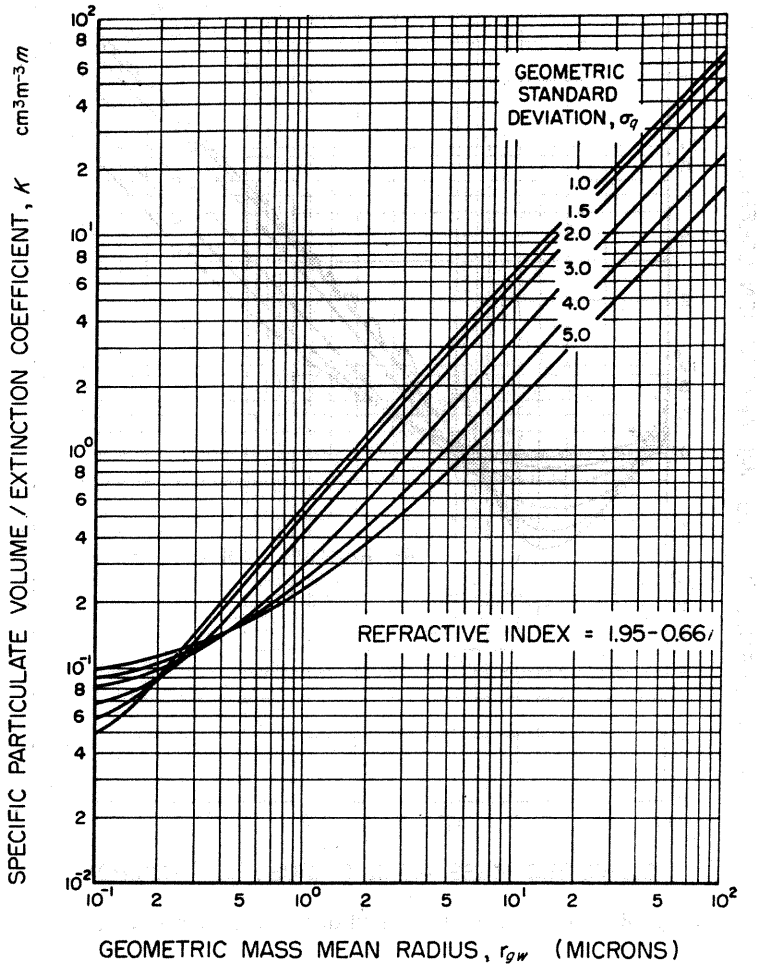

FIG. 2. Relationship of K to particle size distribution parameters for a black aerosol.

C. *Comparison of calculated K with previously reported relationships*

The theoretical results for K presented in FIGS. 1 and 2 are in good agreement with the relationships reported by HAWKSLEY *et al.* (1961). Equation 1 is for light absorbing particles smaller than the wavelength of light and of density 2 g cm^{-3}. Assuming a mass mean radius of 0.2 μm and a standard deviation of 1.0 (monodisperse) a K of 0.09 is obtained from FIG. 2. Substituting these magnitudes for K and ρ into (20) gives

$$\ln(I/I_0) = -5.55\,WL, \tag{26}$$

which is approximately the same as (1).

Equation 2 is for light absorbing particles larger than the wavelength of light having an average specific surface area diameter (i.e. diameter of a sphere with a surface area/volume ratio equal to the average of all the particles) of 12 μm and a particle density of 2 g cm^{-3}. Assuming a standard deviation of 1.0 and a mass mean radius of 6 μm, K from FIG. 2 is 4.0. Substituting these variables into (20) gives

$$\ln(I/I_0) = -0.125\,WL, \tag{27}$$

which is exactly equal to (2).

Equation 3 is for an oil aerosol (probably a pure scatterer), with a mass mean radius of 0.23 μm, and a geometric standard deviation of 3.4 which gives a K of about 0.33 from FIG. 1. Assuming a particle density of 0.9 g cm^{-3} and substituting into (20) gives

$$\ln(I/I_0) = -3.36\,WL, \tag{28}$$

changing into the form of (3) which is for a Ringelmann number 2

$$W = \frac{1.63}{L}. \tag{29}$$

This equation disagrees somewhat with (3). However, (3) was developed using only 5 size distribution increments. Also the refractive index of the oil was probably around 1.5 whereas FIG. 1 is for a refractive index of 1.33.

(d). *Comparison of calculated and experimental magnitudes of* K

To completely verify the theoretical results, independent simultaneous measurements of the particle mass frequency distribution (r_{gw} and σ_g), the light transmittance (I/I_0), and the stack diameter (L) at a particulate air pollutant source are necessary. Unfortunately no reports of these measurements have been found. However, there are reports of light extinction studies to which particle size distribution have been approximated. A summary of the applicable data is presented in TABLE 2. A comparison of

TABLE 2. PLUME OPACITY AND PARTICULATE CONCENTRATION DATA

Source	L (m)	ρ (g m^{-3})	W (g m^{-3})	I/I_0	K (cm^3 m^{-2})	d (μ)	Reference
White smoke	0.2	0.87	0.21	0.90	0.46	0.3	CONNER and
Generator			0.47	0.70	0.30		HODKINSON
			1.00	0.31	0.20		(1967)
Black smoke	0.2	1.95	0.13	0.80	0.060	0.23	CONNER and
(oil fired power			0.49	0.40	0.055		HODKINSON
plant)			0.95	0.20	0.061		(1967)
Black smoke	0.15	1.95	0.46	0.75	0.12	<1	STOECKER
(coal stoker)			0.92	0.52	0.11		(1950)
			1.40	0.34	0.10		
Black smoke	0.30	1.95	0.46	0.43	0.084	—	HURLEY and
(coal stoker)			0.92	0.18	0.083		BAILEY
			1.4	0.08	0.085		(1958)

the measured (TABLE 2) and calculated (FIGS. 1 and 2) magnitudes of K indicates qualitative agreement. The parameter K from the data of STOECKER (1950) and of HURLEY and BAILEY (1958) are in close agreement (K ranges from 0.083 to 0.12 cm^3 m^{-3} m). Both of these sources were coal fired stokers and thus the particulate emissions probably had very similar size distributions. The magnitudes of K calculated from the data of CONNER and HODKINSON (1967) and of STOECKER (1950) decrease with increasing particle mass concentration W. For white smoke it can be seen by examining FIG. 1 that this could only occur if the mass mean radius was less than the minimum in the curve of K vs. mass mean radius (point of maximum optical activity) and the particle size was increasing with increased mass concentration. The value of K calculated from the data of CONNER and HODKINSON (1967) for an oil fired power

plant (black smoke) is 0.06 which is in close agreement with the K of about 0.05 obtained from FIG. 2 at a mass mean diameter of 0.23 μm and a standard deviation of 1.0.

E. *Calculation of maximum particle concentration to meet plume opacity regulations*

An example will be presented to illustrate the practical application of the theoretical results presented in FIGS. 1 and 2. Assume the problem is to determine the maximum allowable particle mass concentration for a plume which will meet a Ringelmann number 1 requirement (80 per cent light transmittance). The information required is the particle properties (mass mean radius r_{gw}, particle size standard deviation σ_g, particle density ρ, particle refractive index m) and the plume dimensions (stack diameter L). Assume that the particle properties are: r_{gw} of 2 μm, σ_g of 3, ρ of 2 g cm^{-3}, m of 1.95–0.66i, and L of 3.28 m (10 ft). From FIG. 2 a K of 0.59 cm^3 m^{-3} m is obtained. The maximum particle mass concentration can be calculated by substituting the proper parameters into (22)

$$W = -\frac{(2\,\mathrm{g\,cm^{-3}})(0.59\,\mathrm{cm^3\,m^{-2}})\ln 0.8}{3.28\,\mathrm{m}} \tag{30}$$

$$W = 0.08\,\mathrm{g\,m^{-3}}. \tag{31}$$

Thus the maximum allowable particle concentration is 0.08 g m^{-3} or 0.035 grains ft^{-3} to meet a Ringelmann number 1.

4. CONCLUSIONS

A general theoretical relationship between plume opacity and particle properties has been developed which is in agreement with similar equations for specific particle size distributions and with the known measurements of plume opacity and particle properties. Above a geometric mass mean radius of 0.5 μm, the particle refractive index does not significantly influence the plume opacity relationships. The major variations in the parameter K are caused by changes in the particle size distribution.

Acknowledgements—This research was supported by the United States Public Health Service, National Air Pollution Control Administration Training Grant AP-29. The assistance of and discussions with ROBERT J. CHARLSON, KENNETH E. NOLL, RUDOLF F. PUESCHEL, AUGUST T. ROSSANO, and LESLIE E. SPARKS are gratefully acknowledged.

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THE DERIVATION of the opacity of an aerosol cloud from the size distribution and optical properties of aerosol particles given by Pilat and Ensor has the merit of being entirely rigorous under certain conditions, while at the same time minimising the amount of computation left to the experimentalist to convert particle concentrations to a light transmittance, or vice versa. Unfortunately, the accurate application of the data presented in Figs. 1 and 2 is restricted by two underlying assumptions which have not been sufficiently stressed. In the first place, it is assumed that polydisperse aerosols follow strictly a log normal distribution. Such a law may well apply to powders prepared by comminution, or liquid aerosols produced by atomization. However, when such particles are burnt, deviations from the log normal distribution can occur in the plume particles produced. Even so, probably useful approximate results can be obtained from a judicious linearization of the actual size distribution plotted on log-probit paper in the usual way.

A more serious limitation is that the Lambert–Beer (or Bouguer) extinction formula only refers to the light flux received directly from a source of light, such as a lamp or a small portion of the sky behind the plume. The light from a plume reaching an observer includes, in general, important contributions due to sunlight and skylight scattered by plume particles into the eye. On a sunny day, this in-scattering can more than compensate for transmission losses through the plume from power stations, which can then appear brighter than its background (JARMAN and DE TURVILLE, 1969). This phenomenon would not arise if the Lambert–Beer law was applicable under all viewing conditions. For this reason, it is suggested that the results obtained by Pilat and Ensor can only be related directly to Ringelmann numbers for a plume of light-absorbing particles. They are also suitable for the interpretation of in-stack transmissometer measurements of either light absorbing or dielectric aerosols, where all other light sources are absent.

The transmittance of a plume can, however, be measured directly. A telephotometer is sighted to measure the brightness difference between a pair of contrasting targets, directly and through the plume, in the way described by CONNER and HODKINSON (1967). With this method, interference from light scattered by the plume is fully compensated, so the analysis of Pilat and Ensor will then apply.

If Ringelmann charts only are used, some correction for sky and sunlight scattering will be necessary. This limitation unfortunately restricts somewhat the use of this theoretical analysis in the development of regulations for the control of particulate air pollutants.

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AUTHORS' REPLY

WE AGREE with Jarman and de Turville that an understanding of the simplifying assumptions used in any calculation is necessary for intelligent application of the calculation results. We believe that our calculations are of sufficient accuracy and completeness for use in the air pollution control field. The effects on the plume opacity–particulate mass concentration relationship of such non-ideal factors as deviations from log normal size distributions, particle shape, particle concentration gradients, etc. can only be determined by the collection of additional data. It should be noted that the mathematical derivation of K (equations 4 through 19) in our paper is not limited to log normal particle size distributions.

The changes in the plume appearance caused by changes in the background lighting as described by Jarman and de Turville are not due to failures of the Lambert–Beer law. The Lambert–Beer law is used to describe plume opacity. The plume contrast (plume to background luminance contrast) is expressed as a sum of the plume opacity and the ratio of the light scattered to the observer from the plume to the background luminance (CONNER and HODKINSON, 1967).

$$\text{Plume contrast} = \text{plume opacity} + \frac{\text{light scattered from plume to observer}}{\text{background luminance}}$$

Thus our calculation results may be used to compute the plume opacity (light extinction) contribution to the plume contrast for a given plume diameter, particle size distribution, and particle mass concentration.

* M. J. PILAT and D. S. ENSOR (1970) *Atmospheric Environment* 4 (2), 163–173.

The applicability of our results to plume opacities determined by trained smoke inspectors depends on the relationship between the visually measured plume opacities (Ringelmann numbers or percentage smoke density) to the light transmittance of the plume measured in the stack. In the United States smoke inspectors are carefully trained by air pollution control agencies to qualify as expert smoke readers (qualified to testify in court concerning plume opacity). This training usually consists of learning to read the opacity of both black and white smokes at a wide variety of plume opacities and lighting conditions to within $\pm$ 10 per cent opacity (0·5 Ringelmann number) with an instrumentally measured in-stack plume opacity as the primary standard. WEISBURD (1962) and ROM (1968) presented descriptions of smoke reading schools. To minimize the errors in reading plume opacities smoke inspectors are instructed to:

(1) Read gray and black smoke in densities and to record in Ringelmann numbers.
(2) Read emissions colored other than gray or black in opacities and record in percentages.
(3) The light source (sun) should be behind the observer during daylight hours.
(4) The light source should be behind the plume during hours of darkness.
(5) Readings should be taken at right angles to the wind direction and from any distance necessary to obtain a clear view of the stack.
(6) Readings should be made through the densest part of the plume and where the plume is no wider than the stack diameter.

Thus when reading plumes, smoke inspectors are trained to use their judgement to compensate for variations in the plume contrast due to light scattered to the observer from the plume and to report the plume opacity *not the plume contrast.*

We feel that our mathematical analysis is applicable to both black and non-black plume emissions within the legal limits of plume opacity readings by qualified smoke inspectors.

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