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CALCULATED PARTICLE COLLECTION EFFICIENCIES OF SINGLE DROPLETS INCLUDING INERTIAL IMPACTION, BROWNIAN DIFFUSION, DIFFUSIOPHORESIS AND THERMOPHORESIS

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Abstract—Particle collection efficiencies of a single 100 μm dia. water droplet were calculated with a Runge–Kutta numerical solution for the particle equation of motion. The bulk gas properties were held constant at 65°C and 100 per cent r.h. whereas the water droplet temperature ranged from 10°C (water vapor condensing on the droplet) to 82°C (water evaporating from droplet). Potential flow was assumed for the gas flow profile around the droplet moving at its settling velocity of 30.0 cm s^{-1}. The results show that over the 0.01–10 μm particle dia. range, the calculated particle collection efficiency is significantly increased with colder water temperatures and decreased with warmer water temperatures. The calculated results for the case with both the water and gas temperature at 65°C (only inertial impaction and Brownian diffusion mechanisms) compares reasonably with previous reports.

NOMENCLATURE

a	particle acceleration
a_c	accommodation coefficient of the gas molecules on the particle surface
$\bar{c}$	gas molecule mean velocity
C	Cunningham correction factor
C_p	gas heat capacity
C_∞	particle concentration outside the stagnant layer
d_P	particle dia.
d_{P_1}	vapor pressure difference between the droplet surface and surrounding gas
D	droplet dia.
D_p	Brownian diffusivity of particles
D_w	diffusion coefficient of water vapor in air
F_B	force on particle due to Brownian diffusion
F_D	diffusiophoretic force on particle
F_{Drag}	drag force on particle
F_{Ext}	sum of external forces on particle
$F_{\mathrm{Ext}\text{-}x}$	sum of external forces in X-direction
$F_{\mathrm{Ext}\text{-}y}$	sum of external forces in Y-direction
F_T	thermophoretic force on particle
k	Boltzmann Constant
K_g	thermal conductivity of gas
K_P	thermal conductivity of particle
K	Stokes number
m_P	mass of particle
m_1	molecular mass of water vapor
m_2	molecular mass of gas
M	gas molecular weight
P_2	partial pressure of gas
P_0	vapor pressure at drop surface
P_∞	vapor pressure of gas
P	total pressure of gas
Pr	Prandtl number
r_P	particle radius
R	ideal gas constant
R_d	droplet radius
Re	Reynolds number of the droplet
Sc_w	Schmidt number for the diffusing water vapor molecules
Sc_P	Schmidt number for the diffusing aerosol particles
t	time
T	nondimensional time
T_g	temperature of gas
∇T	temperature gradient between the droplet and the surrounding gas
u	fluid velocity
u_0	undisturbed fluid velocity
U_x	nondimensional fluid velocity in X-direction
U_y	nondimensional fluid velocity in Y-direction
v	particle velocity
V_B	particle velocity due to Brownian diffusion
V_D	particle velocity due to diffusiophoresis
V_T	particle velocity due to thermophoresis
V_x	nondimensional particle velocity in X-direction
V_y	nondimensional particle velocity in Y-direction
X	nondimensional distance in X-direction
ΔX_B	film thickness surrounding the droplet for particle Brownian diffusion
ΔX_m	film thickness surrounding the droplet for water vapor mass transfer
ΔX_H	film thickness surrounding the droplet for heat transfer
Y_0	initial Y position measured from the drop centerline of the particle trajectory that just grazes the droplet
Y	nondimensional distance in Y-direction

Greek letters

η	single droplet collection efficiency
ρ_g	density of gas
ρ_P	density of particle
γ_1	mole fraction of water vapor
γ_2	mole fraction of gas
λ	mean free path of gas molecules
θ	angle that the line joining the center of the particle to the center of the droplet makes with the droplet centerline
μ_g	viscosity of gas.

1. INTRODUCTION

The objective of this paper is to present the method and results for the calculations of the particle collection efficiency of single water droplets including the collection mechanisms of inertial impaction, Brownian diffusion, diffusiophoresis and thermophoresis. The particle collection efficiency of single droplets has relevance to both the control of particulate air pollutant emissions using droplet spray scrubbers and the scavenging of atmospheric aerosol particles by cloud and rain droplets.

There have been a considerable number of reports of calculated particle collection efficiencies of single droplets considering only the mechanism of inertial impaction including papers by Langmuir (1948), Johnstone and Roberts (1949), Ranz and Wong (1952), Pemberton (1960), Hocking and Jonas (1970) and Beard and Grover (1974). Some of these later reports used gas flow fields intermediate between viscous and potential conditions. Sparks and Pilat (1970) reported calculated particle collection efficiencies for single droplets considering both inertial impaction and diffusiophoresis and showed that positive diffusiophoresis (condensation) could substantially improve the collection efficiency. Slinn and Hales (1970) reported that thermophoresis was an important collection mechanism for submicron particles in below-cloud rain scavenging.

2. METHOD OF CALCULATING THE PARTICLE COLLECTION EFFICIENCY OF SINGLE DROPLETS

2.1 *Definition of single droplet collection efficiency*

The particle collection efficiency of a single droplet η can be given by

$$\eta = \frac{(Y_0 + r_p)^2}{R_d^2}, \qquad (1)$$

where Y_0 is the initial Y position (measured from the drop centerline) of the particle center trajectory that just grazes the droplet, r_p the particle radius, and R_d the droplet radius. These parameters are illustrated in Fig. 1. Note that the droplet collection efficiency defined in equation (1) considers the finite size of the particle (to obtain the area swept free of particles). The Y_0 is calculated using the particle equation of motion for a gas flowing around a spherical droplet.

2.2 *Equation of particle motion*

The X-and Y-components of the equations of particle motion (developed in the Appendix) are:

$$\frac{d^2X}{dT^2} = \frac{1}{2K}(U_x - V_x) - \frac{F_{\text{Ext-}x} R_d}{m_p u_0^2} \qquad (2)$$

$$\frac{d^2Y}{dT^2} = \frac{1}{2K}(U_y - V_y) - \frac{F_{\text{Ext-}y} R_d}{m_p u_0^2}. \qquad (3)$$

Where X and Y are nondimensional distances, T nondimensional time, K the Stokes number ($K = C\rho_p u_0 r_p^2/9\mu_g R_d$), U_x and U_y the nondimensional fluid velocity components, V_x and V_y the nondimensional particle velocity components, $F_{\text{Ext-}x}$ and $F_{\text{Ext-}y}$ and X and Y components of the sum of the external forces on the particle, R_d the droplet radius, m_p the particle mass, u_0 the undisturbed fluid velocity, C the particle Cunningham correction factor, and μ_g the fluid viscosity. After substituting in the initial conditions ($X = -4.0$, $T = 0$ and $Y = Y_0$), the appropriate fluid velocity flow field (U_x and U_y) and the external particle forces ($F_{\text{Ext-}x}$ and $F_{\text{Ext-}y}$) equations (2) and (3) can be solved using a Runge–Kutta numerical technique.

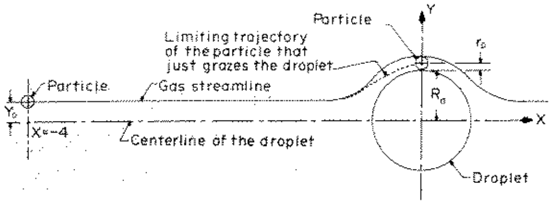

Fig. 1. Schematic of the model used for calculating single droplet collection efficiency.

2.3 *Fluid flow field*

For the collection of 0.01–10 μm dia. particles by a 100-μm dia. water droplet the particles are in general small compared to the droplet. Pemberton (1960) reported that with droplets much larger than the particles, the fluid flow field around the droplet can be assumed potential (gas considered to be inviscid, irrotational, and incompressible). For potential flow U_x and U_y are given by

$$U_x = 1 - \frac{2X^2 - Y^2}{2(X^2 + Y^2)^{2.5}} \qquad (4)$$

$$U_y = \frac{-3XY}{2(X^2 + Y^2)^{2.5}}. \qquad (5)$$

2.4 *External forces on particle*

2.4.1 *Thickness of droplet boundary layer.* The external forces on the particle of thermophoresis, diffusiophoresis and Brownian diffusion are the result of gradients of gas temperature, water vapor concentration and aerosol particle concentration. These gradients were assumed to be linear and to exist only within the boundary layer surrounding the water droplet. The linear gradients were assumed to be symmetrical around the droplet and thus the external forces act radially towards or away from the droplet center.

As reported by Johnstone and Roberts (1949) and Ead (1947) the boundary layer thickness due to the mass transfer of water vapor molecules to and from the droplet ΔX_m, due to heat transfer ΔX_H, and due to particle Brownian diffusion ΔX_B can be given by

$$\Delta X_m = D/(2 + 0.557\,Re^{0.5}Sc_w^{0.375}) \qquad (6)$$

$$\Delta X_H = D/(2 + 0.557\,Re^{0.5}Pr^{0.375}) \qquad (7)$$

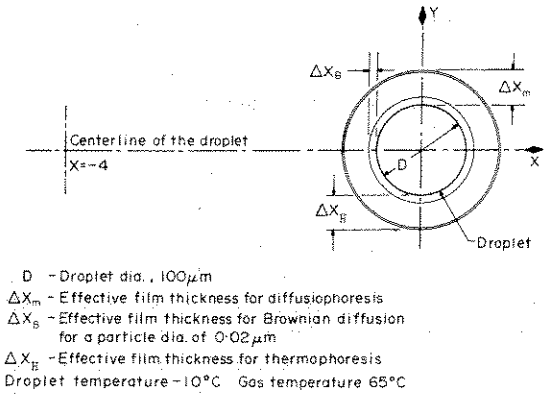

Fig. 2. Illustrations of films for heat transfer, water vapor diffusion and particle diffusion between the droplet and the bulk gas streams.

$$\Delta X_B = D/(2 + 0.557\,Re^{0.5}Sc_p^{0.375}), \tag{8}$$

where D is the water droplet dia. and Re the droplet Reynolds number. The Schmidt number for the water vapor Sc_w, the Prandtl number Pr and the Schmidt number for the aerosol particles Sc_p are given by:

$$Sc_w = \frac{\mu_g}{\rho_g D_w} \tag{9}$$

$$Pr = \frac{C_p \mu_g}{K_g} \tag{10}$$

$$Sc_p = \frac{\mu_g}{\rho_g D_p}, \tag{11}$$

where μ_g is the gas viscosity, ρ_g the gas density, D_w the water vapor diffusivity, D_p the particle diffusivity, C_p the gas heat capacity and K_g the gas thermal conductivity. The various film thicknesses surrounding the water droplet are illustrated in Fig. 2.

The boundary thickness surrounding the droplet due to diffusing aerosol particles is particle size dependent and is shown in Table 1. It is also dependent on the temperature of the boundary layer, but in the range of temperatures considered it is negligible. For example, the boundary thicknesses for a 0.03 μm particle at boundary layer temperatures of 38 and 65°C are 6.9 and 7.1 μm, respectively. The boundary thicknesses surrounding the droplet due to temperature and vapor pressure gradients are not particle size dependent but are somewhat temperature dependent, as shown in Table 2. This is due to the change of the water vapor diffusivity, thermal conductivity and viscosity of the gas with temperature.

2.4.2 *Brownian diffusion.* The pseudo-force on an aerosol particle due to particle Brownian diffusion was obtained using the Stokes–Cunningham equation

$$F_{\text{Brownian}} = \frac{6\pi\mu_g r_p V_B}{C}, \tag{12}$$

where V_B is the particle velocity towards the droplet due to the particle concentration gradient existing around the droplet. Note that V_B is not the instantaneous velocity of the particle undergoing a zig-zag Brownian motion. V_B, the particle velocity due to Brownian diffusion, may be calculated in a manner analogous to the rate of absorption of a solute gas from a gas mixture. That is the particles are considered to reach the drop surface entirely by diffusion through a stagnant gas layer, the effective thickness of which depends upon the gas velocity, shape of the object (droplet) and the properties of the gas and diffusing species. Outside the stagnant gas layer, the concentration of aerosol particles remains uniform, while at the surface of the droplet the equilibrium particle concentration is zero, that is the collision of the particles with the surface is assumed to be plastic (i.e. all particles that strike the droplet surface are collected by the droplet).

Einstein (1908) from the consideration of osmotic forces derived the following expression for the mass

Table 1. Parameters used in calculations of particle collection efficiency of a 100 μm dia. droplet at 65°C and gas at 65°C with 100 per cent r.h.

Particle dia. (μm)	Particle diffusivity (cm² s⁻¹)	Stokes No.	Droplet film thickness due to particle diffusion (μm)	Particle Cunningham correction factor
10	2.68×10^{-8}	0.9469	0.37	1.016
6	4.52×10^{-8}	0.3444	0.45	1.026
2	1.42×10^{-7}	0.0402	0.69	1.078
1	3.05×10^{-7}	0.0108	0.91	1.155
0.6	5.54×10^{-7}	0.0042	1.14	1.259
0.2	2.4×10^{-6}	0.0007	1.94	1.818
0.08	1.43×10^{-5}	0.0002	3.65	3.245
0.03	1.02×10^{-4}	0.00006	7.06	7.439
0.01	9.14×10^{-4}	0.00002	13.63	21.053

Table 2. Film thicknesses of 100 μm dia. droplet at a gas temperature of 65°C and 100 per cent r.h.

Droplet temp. (°C)	Average film temp. (°C)	Film viscosity at average film temp. (g cm s⁻¹)	Film gas density (g cm⁻³)	Film gas thermal conductivity (cal cm⁻¹ s⁻²)	Prandtl No.	Water vapor diffusivity (cm² s⁻¹)	Schmidt No. of water vapor	Drop film thickness due to water vapor diffusivity (cm)	Drop film thickness due to heat transfer (cm)
10	38	0·0001894	0·00113638	0·0000642	0·7080	0·267207	0·62375	0·003768	0·003723
38	50	0·0001949	0·00108766	0·0000667	0·7013	0·285320	0·62804	0·0037655	0·003727
48	57	0·0001992	0·00106934	0·0000680	0·7031	0·292674	0·636487	0·003761	0·003726
60	63	0·0002020	0·00105176	0·0000690	0·7026	0·300000	0·64020	0·003759	0·003726
65	65	0·0002041	0·0010430	0·0000693	0·7068	0·303818	0·64409	0·003757	0·003724
82	74	0·0002094	0·00102093	0·0000701	0·7169	0·315106	0·65091	0·003753	0·003719

transfer of a solute in a solution due to diffusion in a concentration gradient.

$$V_B C_\infty = -D_B \frac{\mathrm{d}c}{\mathrm{d}x}, \tag{13}$$

where

V_B is the velocity of the solute molecule.
C_∞ the concentration of solute molecule.
D_B the diffusivity of the solute molecule.
and
$\mathrm{d}c/\mathrm{d}x$ is the concentration gradient of the solute molecules.

In the above derivation it was assumed that the molecules of the solute are approximately spherical and are large compared to the molecules of solvent.

Equation (13) can also be used to calculate the movement of particles due to diffusion in a gas in which a particle concentration gradient exists. Referring to our model (Fig. 2) there is a stagnant gas layer surrounding the droplet of thickness ΔX_B across which the diffusion of the particles is taking place. In our model it was assumed that the concentration of particles outside the stagnant layer is uniform and is equal to C_∞. It was also assumed that the particle concentration gradient across the stagnant layer was linear and that the particle concentration at the drop surface was zero. Therefore, the concentration gradient of the particles across the stagnant layer reduces to

$$\frac{\mathrm{d}c}{\mathrm{d}x} = \frac{C_{\text{bulk gas}} - C_{\text{droplet surface}}}{\text{Boundary Layer Surface}} = \frac{C_\infty - 0}{\Delta X_B}. \tag{14}$$

Substituting equation (14) for the concentration gradient in equation (13) we get the velocity of the particle V_B due to diffusion in the stagnant layer surrounding the droplet.

$$V_B C_\infty = D_B \frac{(C_\infty - 0)}{\Delta X_B} \tag{15}$$

or

$$V_B = D_B / \Delta X_B. \tag{16}$$

The film thickness due to particle diffusion ΔX_B was obtained using equation (8). For particles of diameter greater than the gas mean free path of 0.062 μm the particle diffusivity was calculated using the equation reported by Einstein (1908)

$$D_p = \frac{CkT_g}{6\pi\mu_g r_p}, \tag{17}$$

where T_g is the gas temperature and k the Boltzmann constant. For particles of diameter less than the gas mean free path the particle diffusivity was calculated using Langmuir's (1942) equation based on Stephan–Maxwell diffusion theory

$$D_p = \frac{4kT_g}{3\pi d_p^2 P}\left(\frac{8RT_g}{\pi M}\right)^{1/2}, \tag{18}$$

where d_p is the particle dia., P the total gas pressure, R the ideal gas constant and M the gas molecular weight.

2.4.3 *Thermophoresis.* The thermophoretic force on the aerosol particle was obtained using the Stokes–Cunningham equation

$$F_{\text{Thermophoresis}} = \frac{6\pi\mu_g r_p V_T}{C}. \tag{19}$$

Where V_T is the particle velocity due to thermophoresis. For particle diameters much smaller than the gas mean force path Bakanov and Derjaguin (1959) and independently Waldmann (1959) theoretically derived equations for V_T considering the rate of total momentum transported to the particle by gas molecules striking the particle surface.

$$V_T = -\frac{15\pi\bar{c}\,\lambda}{16(8 + \pi\bar{a}_c)\,T_g}\nabla T, \tag{20}$$

where $\bar{c}$ is the gas molecule mean velocity, λ the gas mean free path, $\bar{a}_c$ the accomodation coefficient of the gas molecules on the particle surface (magnitude of a_c was 0.8 in our calculations), and ∇T is the temperature gradient.

For particle diameters larger than the gas mean free path, Derjaguin and Yalamov (1965) reported a theoretical equation for V_T as

$$V_T = -\frac{3\mu_g}{\rho_g T_g}\frac{[K_g + 2.16K_p(\lambda/r_p)]}{[2K_g + K_p + 4.32K_p(\lambda/r_p)]} \times \frac{\nabla T}{[1 + 2.26(\lambda/r_p)]}, \tag{21}$$

where K_g and K_p are the thermal conductivities of gas and particle, respectively.

2.4.4 *Diffusiophoresis.* The diffusiophoretic force on the aerosol partical was calculated using the particle velocity due to diffusiophoresis V_D and the Stokes–Cunningham Equation

$$F_{\text{Diffusiophoresis}} = \frac{6\pi\mu_g \mathbf{r}_\mathbf{p} \mathbf{V}_\mathbf{D}}{C}, \tag{22}$$

where V_D is the particle velocity due to diffusiophoresis. The velocity due to diffusiophoresis in the free molecule regime was calculated by Bakanov and Derjaguin (1959) and independently by Waldmann (1959) in a manner similar to that of thermophoretic velocity

$$V_D = -\frac{m_1^{0.5}}{\gamma_1 m_1^{0.5} + \gamma_2 m_2^{0.5}}\frac{D_w d_{P_1}}{P_2\,\Delta X_m}, \tag{23}$$

where D_w is the diffusion coefficient of vapor in gas, m_1 and m_2 are the molecular masses of vapor and gas, respectively, γ_1 and γ_2 are the mole fraction of vapor and gas respectively, P_2 is the partial pressure of gas, d_{p_1} is the vapor pressure difference between the droplet and the surrounding gas and ΔX_m is the film thickness across which the vapor pressure difference exists. On substitution of proper values for a water vapor–air system at 1 atm pressure equation (23) becomes

$$V_D = -[7.917 \times 10^{-4} D_w (P_0 - P_\infty)/\Delta X_m], \tag{24}$$

where P_0 and P_∞ are the vapor pressures at the drop surface and gas, respectively. Schmidt and Waldmann (1960) reported that equation (24) can be used for particles larger than the gas mean free path within an accuracy of 9 per cent. Goldsmith and May (1966) reported that their experimentally measured particle velocities were in excellent agreement with equation (24). This equation was used to obtain the particle diffusiophoretic velocity for our calculations.

2.5 *Computer program*

The computer program calculated the limiting trajectory of the particles that just grazes the droplet, which in turn was used to calculate the single droplet collection efficiency. The main program TRIAL calculates the settling velocities of the droplet and particle, film thicknesses surrounding the droplet (in which diffusiophoresis, thermophoresis and Brownian diffusion are active) and diffusiophoretic, thermophoretic and Brownian forces acting on the particle. All these calculated values are printed out. For droplets greater than 76 μm dia., the terminal settling velocity of the droplet is read in with other input data. Then the subroutine RUNGE is called.

Subroutine RUNGE calculated the limiting trajectory of the particle that just grazes the droplet, which in turn was used to calculate the single droplet collection efficiency. The particle trajectory was calculated by stepwise iteration of the equations of motion of the particles using a fourth order Runge–Kutta process. This was done by calculating the forces on the particle (which is done in function FORCE CALCULATIONS) and allowing it to move for a small increment of time after which the forces on the particle were recalculated. Since it was assumed that the forces on the particle were constant during the small increment of time, the selection of the increment of time was very important. The increment of time was calculated by the function STPSIZE. In the calculations, the diffusiophoretic, thermophoretic and Brownian forces exist only within the droplet film. These forces acted on the particle from the time it entered the film on the front side of the droplet until it departed from the film on the back side of the droplet (or it collided with the drop surface).

The function STPSIZE calculated the size of the time increment or stepsize by considering the radius of curvature of the particle trajectory. The greater the curvature, the smaller the stepsize. The radius of curvature of the particle trajectory is an indication of the direction of the forces and the change in the magnitude of the forces acting on the particle. The stepsize was calculated by assuming a constant subtended angle between the successive location points on the particle trajectory where the forces on the particle are calculated using function FORCE CALCULATIONS. Therefore, when the trajectory was straight, the time increment was large but when the trajectory curved the stepsize was decreased depending on the curvature of the particle trajectory.

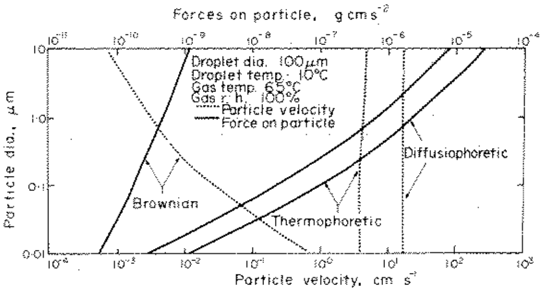

Fig. 3. Calculated particle velocities and forces due to Brownian diffusion, thermophoresis and diffusiophoresis.

In the function FORCE CALCULATIONS the acceleration of the particle in the X- and Y-directions was calculated. The calculations were performed by equating the drag force on the particle to the sum of the external forces acting on the particle.

Once the limiting trajectory for a given particle diameter was calculated, the calculations were repeated for another particle diameter. This continued until calculations for all the particle sizes of interest were completed.

3. DISCUSSION AND RESULTS

3.1 *Calculated particle velocities and forces*

The calculated particle velocities and forces due to the particle Brownian diffusion, thermophoresis and diffusiophoresis are presented in Fig. 3 for the case of a 10°C 100 μm water droplet in 65°C air at 100 per cent r.h. Note that the thermophoretic and the diffusiophoretic velocities of the particle are relatively constant with variation in the particle size. For these conditions the diffusiophoretic force is about three times the magnitude of the thermophoretic force. The diffusiophoretic and thermophoretic forces are larger than the Brownian diffusion force by about one order of magnitude at 0.01 μm particle dia. and by about four orders of magnitude at 10 μm particle dia.

3.2 *Calculated single droplet collection efficiencies*

The calculated particle collection efficiencies of a 100 μm dia. droplet at six water temperatures from 10 to 82°C and constant gas temperature of 65°C and 100 per cent r.h. are presented in Fig. 4. For the neutral case of both the gas and water droplet at 65°C the minimum particle collection efficiency of *ca.* 0.33 per cent occurs at a particle dia. of *ca.* 0.085 μm dia. With a water droplet only 5°C cooler at 60°C the particle collection efficiency at 0.085 μm dia. increases to *ca.* 68 per cent.

These calculated results indicate the particle collection efficiency of water droplets can be significantly enhanced when diffusiophoresis and thermophoresis are positive and greatly reduced when they are negative. For particles greater than 8 μm dia. the inertial impaction collection mechanism predominates, resulting in a lesser change in the particle collection efficiency (from *ca.* 26 per cent at 82°C water to 160 per cent with 10°C water).

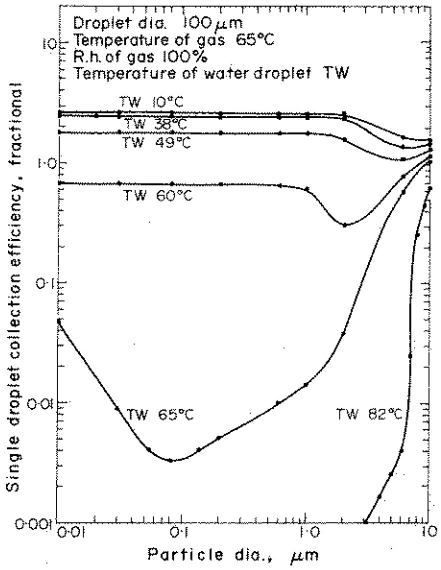

Fig. 4. Calculated particle collection efficiencies of a single 100 μm dia. droplet at 30 cm s^{-1} velocity, 65°C air temperature and various water droplet temperatures.

3.3 *Comparison with reported results*

During our search of journal literature, no experimental or calculated single droplet collection efficiencies considering simultaneously both inertial impaction and Brownian diffusion mechanisms over the 0.01–10 μm dia. particle range was found. In his doctoral dissertation Sparks (1971) presents a graph of the calculated particle collection efficiency of a single 200 μm dia. water droplet vs particle size including both Brownian diffusion and inertial impaction. Sparks' calculations used a droplet velocity of 100 cm s^{-1} and a potential gas flow field and therefore his curve shown in Fig. 5 is slightly shifted from ours with a 100 μm dia. droplet at a 30 cm s^{-1} velocity. Also presented in Fig. 5 are the experimental collection efficiencies reported by Ranz and Wong (1952) with a 900 μm dia. droplet, the empirical efficiencies from the equation reported by Johnstone and Roberts (1949) for Brownian diffusion, Fonda and Herne's (1957) calculated collection efficiencies for inertial impaction with a potential gas flow field, and Langmuir's (1948) calculated efficiencies with a 100 μm droplet and potential gas flow field.

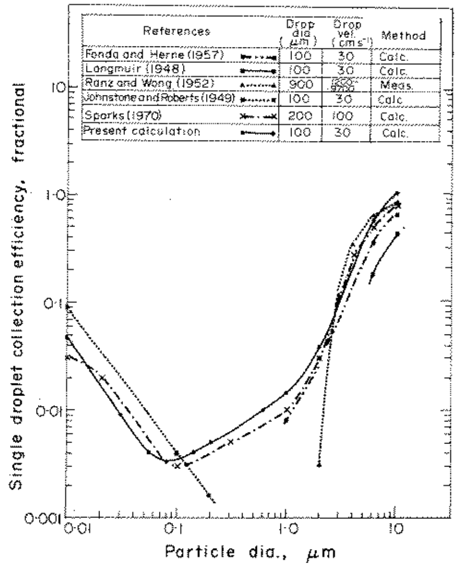

Fig. 5. Comparison of calculated particle collection efficiencies of a single 100 μm dia. droplet at 65°C air temperature and 65°C water droplet temperature with previous work.

In general our calculated efficiencies for the neutral case (no diffusiophoresis and/or thermophoresis) are in fairly good agreement with previously reported results. No experimental or calculated particle collection efficiencies of single water droplets at various particle sizes considering simultaneously inertial impaction, Brownian diffusion, and diffusiophoresis or thermophoresis was found in the literature and thus no comparison can be presented.

4. CONCLUSIONS

Calculated particle collection efficiencies of single water droplet including the mechanisms of inertial impaction, Brownian diffusion, diffusiophoresis, and thermophoresis, indicate that the particle collection efficiency can be significantly affected by the water temperature with respect to the gas temperature and r.h. With a 100 μm water droplet at a 30 cm s^{-1} velocity in 65°C air at 100 per cent r.h. the minimum particle collection efficiency of 0.33 per cent at 0.085 μm with 65°C water temperature increases to 68 per cent with 60°C water temperature.

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APPENDIX

DEVELOPMENT OF EQUATIONS OF MOTION FOR A SPHERICAL PARTICLE PAST A SPHERICAL COLLECTOR

A schematic of the model used for calculating single droplet collection efficiency is given in Fig. 1.

From Newton's Second Law

$$m_p \cdot a = F(t), \tag{1}$$

where m_p is the mass of the particle, a is its acceleration and $F(t)$ is the sum of the forces acting on the particle.

$$a = \mathrm{d}v/\mathrm{d}t, \tag{2}$$

where v is the particle velocity and t is the time.

$$F(t) = F_{\text{drag}} - F_{\text{Ext}}, \tag{3}$$

where F_{drag} is the force of drag on the particle and F_{Ext} is the sum of the external forces acting on the particle.

$$F_{\text{drag}} = \frac{6\pi\mu_g r_p}{C}(u - v), \tag{4}$$

where u is the fluid velocity, C is the Cunningham Correction, μ_g is the viscosity of the gas and r_p is the particle radius.

$$F_{\text{Ext}} = (F_B + F_T + F_D), \tag{5}$$

where F_B, F_T and F_D are the forces on the particle due to Brownian motion, thermophoresis and diffusiophoresis.

Substituting equations (2–4) in (1), we have

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{6\pi\mu_g r_p}{C \cdot m_p}(u - v) - \frac{F_{\text{Ext}}}{m_p}. \tag{6}$$

The x- and y-compnents of equation (6) are:

$$\frac{\mathrm{d}v_x}{\mathrm{d}t} = \frac{6\pi\mu_g r_p}{C \cdot m_p}(u_x - v_x) - \frac{F_{\text{Ext}}\text{-}x}{m_p} \tag{7}$$

$$\frac{\mathrm{d}v_y}{\mathrm{d}t} = \frac{6\pi\mu_g r_p}{C \cdot m_p}(u_y - v_y) - \frac{F_{\text{Ext-}y}}{m_p}. \tag{8}$$

Converting equations (7) and (8) to nondimensional form by the following substitutions.

$X = x/R_d$, $Y = y/R_d$, $V_x = v_x/u_0 = \mathrm{d}X/\mathrm{d}T$, $V_y = v_y/u_0 = \mathrm{d}Y/\mathrm{d}T$, $T = t \cdot u_0/R_d$, $U_x = u_x/u_0$ and $U_y = u_y/u_0$ we have

$$\frac{\mathrm{d}V_x}{\mathrm{d}T} = \frac{6\pi\mu_g r_p \cdot R_d}{C \cdot m_p}(U_x - V_x) - \frac{F_{\text{Ext-}x} \cdot R_d}{m_p \cdot u_0^2} \tag{9}$$

$$\frac{\mathrm{d}V_y}{\mathrm{d}T} = \frac{6\pi\mu_g r_p \cdot R_d}{C \cdot m_p}(U_y - V_y) - \frac{F_{\text{Ext-}y} \cdot R_d}{m_p \cdot u_0^2}. \tag{10}$$

We know that Stokes number, K is given as:

$$K = \frac{C \cdot m_p \cdot u_0}{12\pi\mu_g r_p \cdot R_d} = \frac{C \cdot \rho_p \cdot u_0 r_p^2}{9 \cdot \mu_g R_d}. \tag{11}$$

Therefore equations (9 and 11) can be written as:

$$\frac{\mathrm{d}V_x}{\mathrm{d}T} = \frac{1}{2K}(U_x - V_x) - \frac{F_{\text{Ext-}x} R_d}{m_p u_0^2} \tag{12}$$

$$\frac{\mathrm{d}V_y}{\mathrm{d}T} = \frac{1}{2K}(U_y - V_y) - \frac{F_{\text{Ext-}y} R_d}{m_p \cdot u_0^2}. \tag{13}$$

Where R_d is the radius of the droplet, u_0 is the fall speed of the droplet or the undisturbed fluid velocity.

In the model it is assumed that all the external forces act only within the film thickness surrounding the water droplet. It is also assumed that these forces act radially—that is they act either towards the center of the droplet or away from it.

$$\begin{aligned} F_{\text{Ext-}x} &= F_{\text{Ext}} \cos\theta \\ &= F_{\text{Ext}} \cdot X/(X^2 + Y^2)^{0.5} \end{aligned} \tag{14}$$

$$\begin{aligned} \text{and } F_{\text{Ext-}y} &= F_{\text{Ext}} \cdot \sin\theta \\ &= \mathrm{n}F_{\text{Ext}} \cdot\cdot Y/(X^2 + Y^2)^{0.5}, \end{aligned} \tag{15}$$

where θ is the angle that the line joining the center of the particle to the center of the droplet makes with the drop centerline.