Discrete Mathematics Midterm

Friday, 21 Oct. 2010

Answer all questions. Your answers will be evaluated on correctness and clarity. Start by reading **all questions** first. Select the question you feel most comfortable with and start by answering it.

Name:

1. Recall: "$p \to q$" (p implies q) is **False** only when p = **True** and q = **False**.
   A conjunction is a Boolean expression of the form $(a \wedge b \wedge (\neg c) \wedge \ldots)$. Also written as $(a \cdot b \cdot \bar{c} \ldots)$
   A disjunction is a similar boolean expression: $(a + b + \bar{c} \ldots)$
   (a) construct the truth table for: $(p \to q) \wedge (\neg p \to \neg q)$
   (b) Construct an equivalent conjunction of disjunctions.

2. Draw the Venn diagram for: $(A \setminus B) \cup (C \setminus A) \cup (B \setminus C)$

3. Let $A = \{1.2, 3, 4\} \times \{1, 2, 3, 4, 6\}$. A relation on A is defined by: $(x, y) \propto (s, t)$ if $x \cdot t = y \cdot s$.
   (a) Show that $\propto$ is an equivalence relation.
   (b) For each equivalence class list one memebr belonging to it.

Name

4. Let X be a set. Prove that there is an injunction $f : P(X) \to X$ but there is no injunction $g : X \to P(X)$.

5. Prove that $5^{n+1} + 2 \cdot 3^n + 1$ is divisible by 8 $\forall n \geq 1$.

6. Prove that if $p > 3$ is a prime then $p^2 \bmod 12 = 1$.

7. (a) Prove that for every finite set A it is possible to construct n subsets $A_1, A_2, \ldots, A_n$, $(n = |A|)$ such that $|A_i \cap A_j| = 1 \ if \ i \neq j$.

   (b) Prove that it is not possible to construct more than n subset such that each two different subsets have exactly one element of A in common.

8. (a) What is the line in the Projective Plane PG(5) through the point $(2, 0, 1)$ *and* $(0, 1, 2)$

   (b) Give a brief explanation how we can partition a set of 31 players into 31 teams such that each team has six players and every two teams have exactly one player in common.