

Sequences

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- A sequence may be finite or infinite.

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- 2 Answer: a_n is “The n^{th} non perfect square.”
- 3 a_n is the number of different ways to write n as a sum of no more than $\lfloor \sqrt{n} \rfloor$ positive integers.

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- 6 Do you recognize this sequence?

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- 6 There are many other “named sequences’.” We shall study some of them.
- 7 We shall start by examining a number of examples.

Examples

For the following sequences try to find a “simple” explicit rule:

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- 7 1, 5, 19, 65, 211, 665, 2059, 6305, 19171, 58025 ... $a_n = ?$

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Remark

Consider the last example. It was not too difficult to see that $a_n = 3^n - 2^n$

You are probably still struggling with the sequence preceding it.

Do you see any relation between it and the last sequence?

Can you see it now once your attention was called to it?

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Answer

All three are correct.

We can find a polynomial $p(x)$ of degree 2 such that $p(1) = 1$, $p(2) = 2$, $p(3) = 4$.

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- 4 2, 3, 5, 7, 11, ... the prime numbers.
- 5 Given a sequence a_n define a new sequence: $s_n = \sum_{k=1}^n a_k$.

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You will see many more sequences through out this class and in many other classes.

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Let $(a_n) = 2, 5, 10, 17, \dots, n^2 + 1$. The sequence $2, 5, 17, 37$ is a subsequence of (a_n) of length 4.

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A sequence (a_i) is monotonically increasing if $a_{i+1} > a_i$.
(Monotonically, decreasing ($<$), non-decreasing (\geq), non-increasing (\leq) are defined similarly).

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Let $(a_n) = 2, 5, 10, 17, \dots, n^2 + 1$. The sequence $2, 5, 17, 37$ is a subsequence of (a_n) of length 4.

$$n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 6.$$

If we wanted to extend this subsequence (can you propose a "rule"?) what will be n_5 ?

Definition

A sequence (a_i) is monotonically increasing if $a_{i+1} > a_i$.
(Monotonically, decreasing ($<$), non-decreasing (\geq), non-increasing (\leq) are defined similarly).

Question

A little puzzle: 10 policemen stand in a line. Can you prove that there are at least four policemen whose heights are monotonical?

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Example

- 1 *How many binary sequences of length $2n$ have exactly n 0's?*
- 2 *How many binary sequences of length n do not contain the pattern 010?*
- 3 *Can you construct a circular binary sequence of length 32 so that each binary sequence of length 5 is a segment of it? (01001 is a segment of 1001001101).*