Sequences

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- A sequence may be finite or infinite.

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    3. $a_n = (n-1)(a_{n-1} + a_{n-2}), \quad a_1 = 0, a_2 = 1.$

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7. We shall start by examining a number of examples.

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