Discrete Mathematics and Applications

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1 Introduaction

What is mathematics?

One classical definition is:
"Mathematics is the study of numbers, shapes and patterns."

I also like the following quote by Sun-Yung Alice Chang Professor of Mathematics, Princeton University:

"Mathematics is a language like music. To learn it systematically, it is necessary to master small pieces and gradually add another piece and then another. Mathematics is like the classical Chinese language - very polished and very elegant. Sitting in a good mathematics lecture is like sitting in a good opera. Everything comes together."

So where does Discrete Mathematics fit in?

According to Wikipedia: "discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous." More formally, discrete mathematics has been characterized as the branch of mathematics dealing with countable sets. However, there is no exact, universally agreed, definition of the term **"discrete mathematics."**

Cryptology based on number theory, linear programming, computing theory, scheduling are some exampples of very important modern applications of mathematics. Most of the mathematics used in these applications is Discrete Mathematics.

So rather than spending time to try and define or describe what Discrete Mathmatics is, let us talk about what we will try to learn.

We will look at logic, elementary set theory, combinatorics, number thoery, and graph theory. We shall see how these topics are used in related applications. The heart and soul of mathematics is the ***Proof***. In almost all instances, proofs of theorems will be provided.

1.1 Class work

We shall try to emphasize *explorations and discovery.* To implement this we shall use the mathematical package SAGE developed at the University of Washington.

Class presentations will be a mixture of blackboard and Power Point slides. Slides used in class will be posted on the class web site:

Your grade will be determiend as follows:

1. Assignments: 15%
2. Mid Term: 35%
3. Final: 50%

Office hours
My office is in the media center office number 505.
Office hours:
Wednesday: 1:00 - 2:30
Friday: 1:00 - 2:30

Please feel free to stop by any time.

There are many textbooks for Discrete Mathematics. Below please find a small list of books. A large part of the material covered in this class can be found in Rosen's book.

1. Discrete Mathematics and Its Applications, 4th edition, K.H. Rosen, McGraw Hill, 1999. Web site: http://www.mhhe.com/math/advmath/rosen/
2. Discrete and Combinatorial Mathematics (an Applied Introduction) 4th Edition, R. P. Grimaldi, Addison-Wesley, 1999
3. Concrete Mathematics, R.L.Graham, D.E.Knuth and O. Patashnik, Addison-Wesley
4. Invitation to Discrete Mathematics, J. Matousek and J. Nesetril, OXFORD

1.2 Topics

1. Logic.
2. Sets.
3. Counting.
4. A sample of scheduling problems.
5. Modular arithmetic
6. Sselected topics in graph theory.

Al topics will include examples of related applications.
To be added later: book information, gate simulator http://ozark.hendrix.edu/ burch/logisim

2 Permutations

This short section will be devoted to a short review of permutations. It is a topic you are expected to be familiar with. We shall start with the basic definitions, fundamental properties, examples and in a separate pdf file a sample of questions, hints and answers.

Definition 1. *A bijection on a set A is called a* **permutation**

If $|A| = k$ then a permutation on A is a k-permutation. In most cases, the word permutation is used for permutations on finite sets.

The usual notation for a permutation is:

$$S = \begin{pmatrix} 1 & 2 & ,\dots, & k \\ a_1 & a_2 & ,\dots, & a_k \end{pmatrix} \tag{1}$$

An alternative simpler notation we shall use is $a_1 a_2 a_3 \dots a_k$.

Definition 2. *The product of two permutations $\pi_1 \times \pi_2$ is the composition of the two bijections definig them.*

Corolary 1. *The set of n-permutations on a set of cardinality n forms a group, the symmetric group S_n.*

Example 1. •

- *The permutation: $\pi = 1\ 3\ 5\ 4\ 2\ 6\ 8\ 10\ 9\ 7$ is a 10-permutation.*
- $\pi(1) = 1,\ \pi(5) = 2,\ \pi(7) = 8$

Definition 3.

1. *Let π be a k-permutation. A cycle is a subsequence $a_1\ a_2\ \dots\ a_m$ of π such that $\pi(a_i) = a_{i+1},\ i < m,$ and $\pi(a_m) = a_1$..*
2. *We denote the cyclic permutaions by $(a_1, a_2, \dots, a_m)$.*
3. *When it is clear from the context that we are dealing with n-permutations when we write $(a_1, a_2, \dots, a_m)$. we mean an n-permutation μ such that $\mu(j) = j\ if\ j \notin \{a_1, a_2, \dots, a_m\}$*

Example 2. *The 10-permutation π in example 1. Contains the cycels* $(1),\ (2, 3, 5),\ (4)$ *etc.*

Exercise 1. •

- *Find all cycles in the permutation π in example 1.*
- *Check that they are pairwise disjoint.*
- *Check that π is the product of these cycles.*

Definition 4. *A cycle* (a, b) *is called a* **transposition***.*

Note that if π is a transposition then π^2 is the identity permutation.

Remark 1. •

- *Every permutation is a unique product of disjoint cycles.*
- *Every cycle is a product of transpositions.*
- *Every permutation is a product of transposition.*
- *While the representation of permutations as a product of transpositions is not unique the parity is invariant. This means that all representations of a permutation as a product of transpositions are either all a product of an even number of transpositions, or and odd number of transpositions.*

Definition 5. •

- *An n-permutation that is a product of an even number of transpositions is called an even permutation.*
- *the set of all even n-permutations is a group denotde by* A_n*.*
- $|S_n| = n!, \ |A_n| = \frac{n!}{2}$.

Exercise 2. •

- *Is the permutation in example 1 odd or even?*
- *Construct a 12-permutation which is the product of 3 disjoint 4-cycles.*
- *Determine whether your permutation is odd or even.*
- *Find its inverse.*
- *Is the cycle* $(1, 2, \ldots, n)$ *odd or even?*

We conclude this brief review of permutations by introducing an order on S_n.

Definition 6. *Given two n-permutations* $\pi = a_1\ a_2\ \ldots a_n$ *and* $\mu = b_1\ b_2\ \ldots b_n$ *we say that* $\pi \prec \mu$ *if* $a_i = b_i, \quad for\ i < j$ *and* $a_j < b_j$*.*

Example 3. 7 3 4 8 1 5 6 2 $\prec$ 7 3 4 8 2 1 6 5

This is a total order on the set of n-permutations. It is called the *l*exicographic order.

Exercise 3. *Can you find a permutation* α *between* π *and* μ*?*