Discrete Mathematics and Applications

Moshe Rosenfeld

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moishe@u.washington.edu

1 Preparation for the final exam

Date: Wednesday, 22 Dec. 2:00 P.M.
Duration: 2 hours.

1.1 List of topics:

1. Logic
   - Propositions
   - Boolean operations
   - Truth Tables
   - Logical Equivalence
   - Conjunctions and Disjunctions
   - Logic Gates
2. Sets
   - Set Builder
   - Set Operations
   - Characteristic Vectors
   - Venn Diagrams
   - Power Set
   - Cardinality
   - Countable sets
   - The Diagonal Process
   - Set Systems (Linear Algebra and Sets)
   - Finite Projective Geometries.
3. Counting

- Sum Rule
- Product Rule
- Pigeon Hole Principle
- Inclusion Exclusion

4. Permutations, Combinations, Binomials

   - Derangements
   - Enumerating permutations (lexicographic, Cantor Digits)
   - Lexicographic Order of Combinations
   - Binomial Theorem, $\binom{n}{k}$
   - Laplace Triangle
   - Estimates of $n!$, $\binom{n}{k}$

5. Recurrence Relations

   - What is a *solution* to a recurrence relation.
   - Telescoping
   - Build a recurrence relation to solve a problem.
   - Linear recurrence relations.
   - Homogeneous and Non-Homogeneous Recurrence Relations.
   - Order of a Linear Recurrenec Relation.
   - Solving Recurrence Relations.
   - Generating Functions
   - Generalized Binomial Coefficients
   - Catalan Numbers

6. Discrete Optimization: Scheduling problems.

7. Number Theory Applications

   - Modular Arithmetic
   - Prime Numbers.
   - GF(q)
   - Fermat's Theorem
   - Chinese Remainder Theorem
   - RSA
   - Factoring
   - SQRT mod p and mod pq

1.2 Description of Test

Questions in the test will be of three categories:

1. Basic knowledge of concepts.
   - Examples:
   - Build the truth table of $(p \to q) \wedge (p \vee s) \vee (q \to s)$
   - Draw a Venn Diagram
   - How many integers < 1000 are not divisible by 7 or 11?
   - Which combination precedes $\{5, 6, 9, 11, 17\}$
   - Check whether a given relation is transitive.
2. Applying what we learned.
   - Examples
   - Build a 3-SAT equivalent to a given boolean function (given its Truth Table).
   - From a class of 13 students arrange a list of 13 teams, each wtih 4 students such that each two teams have exactly one student in common.
   - In how many ways can you give 13 children 50 identical coins?
   - Calculate $7^{35467828} \bmod 11$
   - Find a_{1000} given: $a_n = a_{n-1} - 6a_{n-2} + 2n,\ a_0 = 0, a_1 = 1$
   - Given a list of jobs $J_k(p, d)$ with processing time p and deadline d. Schedule them to minimize the number of late jobs.
   - Given 4 matrices, find the smallest number of multiplications of real numbers needed to calculate their product.
   - Which permutaion of order 10 is permutation number 234,000 in the Cantor Digits enumeration.
   - Given 34 red balls, 45 blue balls and 50 green balls. In how many ways can you select 80 balls?
3. Proofs
   - Examples
   - Prove Fermat's Theorem.
   - Prove that if $n \bmod 10 = 7$ then there is an integer k such that $n \cdot k = 11\ldots1$
   - Prove that if A is a finite set with $2n$ members then the maximum number of subsets such that no set contains another set is $\binom{2n}{n}$.

All questions will be based on the class notes, examples discussed in class, and the assignments.