Discrete Mathematics and its Applications

Ngày 18 tháng 9 năm 2011

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7. *Let G be a group and H a subgroup of G.* $\mathbb{R}_8 = \{(r,s) \mid r \cdot s^{-1} \in H\}$ *is a relation on G.*

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We say that the relation $\mathbb{R}$ partitions A into equivalence classes.

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Alternatively, $f \subset A \times B$ such that $((a, b) \in f) \wedge ((a, c) \in f) \to b = c$.

In other words, a function $f : A \to B$ is a "restricted" binary relation between A and B.
Common notation: $f(a) = b$ b is the image of a under the function f.

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Which of the relations in our sample of 8 relations is a function?

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Ex. 4. f assigns to every citizen of Vietnam his I.D number.
Domain: the $90,000,000$ citizens of Vietnam. Range; I.D numbers.