

Discrete Mathematics and Applications

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1 Assignment No. 1

Due: Friday, Sep. 23

Please submit your answer in a neat, readable properly organized format. A * in an exercise indicates a more challenging problem. A ** is very challenging

2 Logic

1. Construct the truth table of $(p \rightarrow q) \vee (\neg p \rightarrow r)$
2. Construct the truth table of $(p \rightarrow q) \wedge (\neg p \rightarrow r)$
3. In a remote village in the mountains all men are either politicians or teachers. Politicians always lie while teachers always tell the truth. You meet two men, Hoang and Tu. Hoang says "we are both teachers". Tu says "Hoang is a politician". Determine who is a teacher and who is a politician.

3 Proofs

1. Prove that if a, b are real positive numbers then $a^2 + b^2 \geq 2ab$.
2. Prove that if a, b, c are real positive numbers then $a^3 + b^3 + c^3 \geq 3abc$.
3. * Construct 9 points in the plane R^2 , no three on a line, such all distances among them are integers.
4. Let S be the set of integers of the form $n^2 + 2m^2$. For instance, 9, 22 are in S : $9 = 1^2 + 2 \times 2^2$, $22 = 2^2 + 2 \times 3^2$. Find five consecutive integers in S and prove that S does not contain six consecutive integers (7, 8, 9, 10 are consecutive integers).

4 SAGE exercises

1. Let n be the sum of the digits of your cell phone number. Let m be the number obtained from n by appending the digit 7 at the end. For example, if $n = 46$ then $m = 467$. Find an integer q such that $mq = 11\dots 1$.
2. Create two sets:
 $A = \{n \mid n \in \mathbb{N}, n = a^2 + b^2, a, b \in \mathbb{N}, n \leq 10000\}$
 $B = \{n \mid n \in \mathbb{N}, n \bmod 4 = 1, n \leq 10000\}$
Let $C = A \cap B$.
Compare the sets B and C . Can you draw a conclusion?