Sequences

October 1, 2011

Basics

Sequences are used in many areas in mathematics, computer science, economics and almost all sciences.

Basics

Sequences are used in many areas in mathematics, computer science, economics and almost all sciences.

Definition

A seqeunce is a function $f : N \to A$.

Basics

Sequences are used in many areas in mathematics, computer science, economics and almost all sciences.

Definition

A seqeunce is a function $f : N \to A$.

- A common notaion for a sequence is $a_1, a_2, \ldots a_n, \ldots$.

Basics

Sequences are used in many areas in mathematics, computer science, economics and almost all sciences.

Definition

A seqeunce is a function $f : N \to A$.

- A common notaion for a sequence is $a_1, a_2, \ldots a_n, \ldots$.
- a_n is usually called *the* general term

Basics

Sequences are used in many areas in mathematics, computer science, economics and almost all sciences.

Definition

A seqeunce is a function $f : N \to A$.

- A common notaion for a sequence is $a_1, a_2, \ldots a_n, \ldots$.
- a_n is usually called *t*he general term
- Sequences do not necessarily start with a_1. They may start with any other number.

Basics

Sequences are used in many areas in mathematics, computer science, economics and almost all sciences.

Definition

A seqeunce is a function $f : N \to A$.

- A common notaion for a sequence is $a_1, a_2, \ldots a_n, \ldots$.
- a_n is usually called *the* general term
- Sequences do not necessarily start with a_1. They may start with any other number.
- A sequence may be finite or infinite.

Describing sequences

There are three common ways to describe sequences:

Describing sequences

There are three common ways to describe sequences:

Explicitely:

Describing sequences

There are three common ways to describe sequences:

Explicitely:

$1, 3, 5, 7, \ldots, (2n-1), \ldots$

Describing sequences

There are three common ways to describe sequences:

Explicitely:

$1, 3, 5, 7, \ldots, (2n-1), \ldots$

$a_n = 2n - 1$

Describing sequences

There are three common ways to describe sequences:

Explicitely:

$1, 3, 5, 7, \ldots, (2n-1), \ldots$

$a_n = 2n - 1$

$0, 3, 8, 15, \ldots$

Describing sequences

There are three common ways to describe sequences:

Explicitely:

$1, 3, 5, 7, \ldots, (2n-1), \ldots$
$a_n = 2n - 1$

$0, 3, 8, 15, \ldots$
Can you suggest an explicit expression for the general term a_n?

Describing sequences

There are three common ways to describe sequences:

Explicitely:

$1, 3, 5, 7, \ldots, (2n-1), \ldots$
$a_n = 2n - 1$

$0, 3, 8, 15, \ldots$
Can you suggest an explicit expression for the general term a_n?

$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$

Describing sequences

There are three common ways to describe sequences:

Explicitely:

$1, 3, 5, 7, \ldots, (2n-1), \ldots$
$a_n = 2n - 1$

$0, 3, 8, 15, \ldots$
Can you suggest an explicit expression for the general term a_n?

$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$

$$a_n = \frac{1}{n^2}.$$

Describing sequences

There are three common ways to describe sequences:

Explicitely:

$1, 3, 5, 7, \ldots, (2n-1), \ldots$
$a_n = 2n - 1$

$0, 3, 8, 15, \ldots$
Can you suggest an explicit expression for the general term a_n?

$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$

$$a_n = \frac{1}{n^2}.$$

By a "rule":

Describing sequences

There are three common ways to describe sequences:

Explicitely:

$1, 3, 5, 7, \ldots, (2n-1), \ldots$

$a_n = 2n - 1$

$0, 3, 8, 15, \ldots$

Can you suggest an explicit expression for the general term a_n?

$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$

$$a_n = \frac{1}{n^2}.$$

By a "rule":

1. $2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, \ldots \; a_n = ?$

Describing sequences

There are three common ways to describe sequences:

Explicitely:

$1, 3, 5, 7, \ldots, (2n-1), \ldots$
$a_n = 2n - 1$

$0, 3, 8, 15, \ldots$
Can you suggest an explicit expression for the general term a_n?

$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$
$a_n = \frac{1}{n^2}$.

By a "rule":

1. $2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, \ldots$ $a_n = ?$
2. Answer: a_n is "The n^{th} non perfect square."

Describing sequences

There are three common ways to describe sequences:

Explicitely:

$1, 3, 5, 7, \ldots, (2n-1), \ldots$
$a_n = 2n - 1$

$0, 3, 8, 15, \ldots$
Can you suggest an explicit expression for the general term a_n?

$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$
$$a_n = \frac{1}{n^2}.$$

By a "rule":

1. $2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, \ldots\ a_n = ?$
2. Answer: a_n is "The n^{th} non perfect square."
3. *a_n is the number of different ways to write n as a sum of no more than $\lfloor sqrt(n) \rfloor$ positive integers.*

Recursively:

1. $1, 2, 6, 24, 120, \ldots \quad a_1 = 0, a_n = na_{n-1}.$

Recursively:

1. $1, 2, 6, 24, 120, \ldots \quad a_1 = 0, a_n = na_{n-1}.$
2. $1, 2, 3, 5, 8, \ldots, \quad a_{n+2} = a_{n+1} + a_n, \ a_1 = 1, a_2 = 2$

Recursively:

1. $1, 2, 6, 24, 120, \ldots \quad a_1 = 0, a_n = na_{n-1}.$
2. $1, 2, 3, 5, 8, \ldots, \quad a_{n+2} = a_{n+1} + a_n, \; a_1 = 1, a_2 = 2$
3. $a_{n,k} = a_{n-1,k-1} + a_{n-k,k}, \quad a_{n,0} = 0, a_{n,n} = a_{n,1} = 1 \; n \geq k$

Recursively:

1. $1, 2, 6, 24, 120, \ldots \quad a_1 = 0, a_n = na_{n-1}.$
2. $1, 2, 3, 5, 8, \ldots, \quad a_{n+2} = a_{n+1} + a_n, \; a_1 = 1, a_2 = 2$
3. $a_{n,k} = a_{n-1,k-1} + a_{n-k,k}, \quad a_{n,0} = 0, a_{n,n} = a_{n,1} = 1 \; n \geq k$
4. $a_n = (n-1)(a_{n-1} + a_{n-2}), \quad a_1 = 0, a_2 = 1.$

Recursively:

1. $1, 2, 6, 24, 120, \ldots \quad a_1 = 0, a_n = na_{n-1}.$
2. $1, 2, 3, 5, 8, \ldots, \quad a_{n+2} = a_{n+1} + a_n,\ a_1 = 1, a_2 = 2$
3. $a_{n,k} = a_{n-1,k-1} + a_{n-k,k}, \quad a_{n,0} = 0, a_{n,n} = a_{n,1} = 1\ n \geq k$
4. $a_n = (n-1)(a_{n-1} + a_{n-2}), \quad a_1 = 0, a_2 = 1.$
5. $a_{n,k} = a_{n-1,k} + a_{n-1,k-1},\ a_{1,1} = 1,\ a_{n,k} = 0\ \mathit{if}\ n < k.$

Recursively:

1. $1, 2, 6, 24, 120, \dots \quad a_1 = 0, a_n = na_{n-1}$.
2. $1, 2, 3, 5, 8, \dots, \quad a_{n+2} = a_{n+1} + a_n,\ a_1 = 1, a_2 = 2$
3. $a_{n,k} = a_{n-1,k-1} + a_{n-k,k}, \quad a_{n,0} = 0, a_{n,n} = a_{n,1} = 1\ n \geq k$
4. $a_n = (n-1)(a_{n-1} + a_{n-2}), \quad a_1 = 0, a_2 = 1.$
5. $a_{n,k} = a_{n-1,k} + a_{n-1,k-1},\ a_{1,1} = 1,\ a_{n,k} = 0\ if\ n < k.$
6. Do you recognize this sequence?

Recursively:

1. $1, 2, 6, 24, 120, \ldots \quad a_1 = 0, a_n = na_{n-1}.$
2. $1, 2, 3, 5, 8, \ldots, \quad a_{n+2} = a_{n+1} + a_n,\ a_1 = 1, a_2 = 2$
3. $a_{n,k} = a_{n-1,k-1} + a_{n-k,k}, \quad a_{n,0} = 0, a_{n,n} = a_{n,1} = 1\ n \geq k$
4. $a_n = (n-1)(a_{n-1} + a_{n-2}), \quad a_1 = 0, a_2 = 1.$
5. $a_{n,k} = a_{n-1,k} + a_{n-1,k-1},\ a_{1,1} = 1,\ a_{n,k} = 0\ if\ n < k.$
6. Do you recognize this sequence?

Remark

The process of constructing a sequence from a given collection $\mathbb{C}$, *that is building a bijection between* Z^+ *and* $\mathbb{C}$ *is called* **enumeration** *or sequencing.*

Recursively:

1. $1, 2, 6, 24, 120, \ldots \quad a_1 = 0, a_n = na_{n-1}.$
2. $1, 2, 3, 5, 8, \ldots, \quad a_{n+2} = a_{n+1} + a_n, \; a_1 = 1, a_2 = 2$
3. $a_{n,k} = a_{n-1,k-1} + a_{n-k,k}, \quad a_{n,0} = 0, a_{n,n} = a_{n,1} = 1 \; n \geq k$
4. $a_n = (n-1)(a_{n-1} + a_{n-2}), \quad a_1 = 0, a_2 = 1.$
5. $a_{n,k} = a_{n-1,k} + a_{n-1,k-1}, \; a_{1,1} = 1, \; a_{n,k} = 0 \; if \; n < k.$
6. Do you recognize this sequence?

Remark

The process of constructing a sequence from a given collection $\mathbb{C}$, that is building a bijection between Z^+ and $\mathbb{C}$ is called **enumeration** *or sequencing.*

Example

$(0, 0), (0, 1), (1, 0), (0, 2), (1, 1), (2, 0), (0, 3), (1, 2), (2, 1), (3, 0), \ldots$ *is an enumeration of $N \times N$.*

Goals

1. Throughout this class and in many other classes you will be using sequences to model and solve problems.

Goals

1. Throughout this class and in many other classes you will be using sequences to model and solve problems.
2. One major goal is to find "simple" rules for given sequences.

Goals

1. Throughout this class and in many other classes you will be using sequences to model and solve problems.
2. One major goal is to find “simple” rules for given sequences.
3. Another goal is to build sequences that will help solve problems.

Goals

1. Throughout this class and in many other classes you will be using sequences to model and solve problems.
2. One major goal is to find “simple” rules for given sequences.
3. Another goal is to build sequences that will help solve problems.
4. For instance calculating integrals is based on building sequences and finding their limit.

Goals

1. Throughout this class and in many other classes you will be using sequences to model and solve problems.
2. One major goal is to find "simple" rules for given sequences.
3. Another goal is to build sequences that will help solve problems.
4. For instance calculating integrals is based on building sequences and finding their limit.
5. Calculating the number of rabbits starting with one pair is done using the Fibonacci sequence.

Goals

1. Throughout this class and in many other classes you will be using sequences to model and solve problems.
2. One major goal is to find "simple" rules for given sequences.
3. Another goal is to build sequences that will help solve problems.
4. For instance calculating integrals is based on building sequences and finding their limit.
5. Calculating the number of rabbits starting with one pair is done using the Fibonacci sequence.
6. There are many other "named sequences'." We shall study some of them.

Goals

1. Throughout this class and in many other classes you will be using sequences to model and solve problems.
2. One major goal is to find "simple" rules for given sequences.
3. Another goal is to build sequences that will help solve problems.
4. For instance calculating integrals is based on building sequences and finding their limit.
5. Calculating the number of rabbits starting with one pair is done using the Fibonacci sequence.
6. There are many other "named sequences'." We shall study some of them.
7. We shall start by examining a number of examples.

Examples

For the following sequences try to find a "simple" explicit rule:

1. $1.0.1.0.1.0....$ $\quad a_n = ?$

Examples

For the following sequences try to find a "simple" explicit rule:

1. $1.0.1.0.1.0. \ldots \quad a_n = ?$
2. $1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, \ldots \quad a_n = ?$

Examples

For the following sequences try to find a "simple" explicit rule:

1. $1.0.1.0.1.0.\dots \quad a_n = ?$
2. $1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, \dots \quad a_n = ?$
3. $3, 6, 11, 18, 27, 38, 51, \dots \quad a_n = ?$

Examples

For the following sequences try to find a "simple" explicit rule:

1. $1.0.1.0.1.0....\quad a_n =?$
2. $1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, \ldots\quad a_n =?$
3. $3, 6, 11, 18, 27, 38, 51, \ldots\quad a_n =?$
4. $2, 4, 16, 256, 65536, 4294967296, \ldots\quad a_n =?$

Examples

For the following sequences try to find a “simple” explicit rule:

1. $1.0.1.0.1.0.\ldots \quad a_n = ?$
2. $1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, \ldots \quad a_n = ?$
3. $3, 6, 11, 18, 27, 38, 51, \ldots \quad a_n = ?$
4. $2, 4, 16, 256, 65536, 4294967296, \ldots \quad a_n = ?$
5. $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, \ldots a_n = ?$

Examples

For the following sequences try to find a "simple" explicit rule:

1. $1.0.1.0.1.0.\ldots \quad a_n = ?$
2. $1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, \ldots \quad a_n = ?$
3. $3, 6, 11, 18, 27, 38, 51, \ldots \quad a_n = ?$
4. $2, 4, 16, 256, 65536, 4294967296, \ldots \quad a_n = ?$
5. $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, \ldots a_n = ?$
6. $1, 2, 4, 8, 14, 25, 45, 79, 138, 240, \ldots \quad a_n = ?$

Examples

For the following sequences try to find a "simple" explicit rule:

1. $1.0.1.0.1.0.\dots \quad a_n = ?$
2. $1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, \dots \quad a_n = ?$
3. $3, 6, 11, 18, 27, 38, 51, \dots \quad a_n = ?$
4. $2, 4, 16, 256, 65536, 4294967296, \dots \quad a_n = ?$
5. $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, \dots a_n = ?$
6. $1, 2, 4, 8, 14, 25, 45, 79, 138, 240, \dots \quad a_n = ?$
7. $1, 5, 19, 65, 211, 665, 2059, 6305, 19171, 58025 \dots \quad a_n = ?$

Examples

For the following sequences try to find a “simple” explicit rule:

1. $1.0.1.0.1.0.\dots \quad a_n =?$
2. $1,0,1,1,0,0,1,1,1,0,0,0,\dots \quad a_n =?$
3. $3,6,11,18,27,38,51,\dots \quad a_n =?$
4. $2,4,16,256,65536,4294967296,\dots \quad a_n =?$
5. $1,2,2,3,3,3,4,4,4,4,5,5,5,5,5,\dots a_n =?$
6. $1,2,4,8,14,25,45,79,138,240,\dots \quad a_n =?$
7. $1,5,19,65,211,665,2059,6305,19171,58025\dots \quad a_n =?$

Remark

Consider the last example. It was not too difficult to see that
$a_n = 3^n - 2^n$
You are probably still struggling with the sequence preceding it.
Do you see any relation between it and the last sequence?
Can you see it now once your attention was called to it?

In general, if a finite number of terms of a given sequence are given, we may have multiple rules that will conform to the given data.

Example

What are the rules that will satisfy the sequence $1, 2, 4, \ldots$*?*
Here are a few rules:

In general, if a finite number of terms of a given sequence are given, we may have multiple rules that will conform to the given data.

Example

What are the rules that will satisfy the sequence $1, 2, 4, \ldots$*?*
Here are a few rules:

1. $a_n = 2^{n-1}$.

In general, if a finite number of terms of a given sequence are given, we may have multiple rules that will conform to the given data.

Example

What are the rules that will satisfy the sequence $1, 2, 4, \ldots$*?*
Here are a few rules:

1. $a_n = 2^{n-1}$.
2. $a_n = \binom{n}{2} + 1$

In general, if a finite number of terms of a given sequence are given, we may have multiple rules that will conform to the given data.

Example

What are the rules that will satisfy the sequence $1, 2, 4, \ldots$*?*
Here are a few rules:

1. $a_n = 2^{n-1}$.
2. $a_n = \binom{n}{2} + 1$
3. $a_n =$ *the (smallest prime* $> n$*) - 1.*

In general, if a finite number of terms of a given sequence are given, we may have multiple rules that will conform to the given data.

Example

What are the rules that will satisfy the sequence $1, 2, 4, \ldots$*?*
Here are a few rules:

1. $a_n = 2^{n-1}$.
2. $a_n = \binom{n}{2} + 1$
3. $a_n =$ *the (smallest prime* $> n$*) - 1.*

Question

So which one is the "correct" answer?

In general, if a finite number of terms of a given sequence are given, we may have multiple rules that will conform to the given data.

Example

What are the rules that will satisfy the sequence $1, 2, 4, \ldots$*?*
Here are a few rules:

1. $a_n = 2^{n-1}$.
2. $a_n = \binom{n}{2} + 1$
3. $a_n =$ *the (smallest prime* $> n$*) - 1.*

Question

So which one is the "correct" answer?

Answer

All three are correct.
We can find a polynomial $p(x)$ *of degree* 2 *such that* $p(1) = 1,\ p(2) = 2,\ p(3) = 4.$

Common Sequences

1. Arithmetic progression: $a_n = \alpha + (n-1)d, \quad (a_n - a_{n-1} = d)$

Common Sequences

1. Arithmetic progression: $a_n = \alpha + (n-1)d, \quad (a_n - a_{n-1} = d)$
2. Geometric progression: $a_n = a_0 \cdot q^{n-1} \quad (\frac{a_n}{a_{n-1}} = q)$

Common Sequences

1. Arithmetic progression: $a_n = \alpha + (n-1)d, \quad (a_n - a_{n-1} = d)$
2. Geometric progression: $a_n = a_0 \cdot q^{n-1} \quad (\frac{a_n}{a_{n-1}} = q)$
3. Binomial coefficients: $a_{n,k} = \binom{n}{k}$

Common Sequences

1. Arithmetic progression: $a_n = \alpha + (n-1)d, \quad (a_n - a_{n-1} = d)$
2. Geometric progression: $a_n = a_0 \cdot q^{n-1} \quad (\frac{a_n}{a_{n-1}} = q)$
3. Binomial coefficients: $a_{n,k} = \binom{n}{k}$
4. $2, 3, 5, 7, 11, \ldots$ the prime numbers.

Common Sequences

1. Arithmetic progression: $a_n = \alpha + (n-1)d, \quad (a_n - a_{n-1} = d)$
2. Geometric progression: $a_n = a_0 \cdot q^{n-1} \quad (\frac{a_n}{a_{n-1}} = q)$
3. Binomial coefficients: $a_{n,k} = \binom{n}{k}$
4. $2, 3, 5, 7, 11, \ldots$ the prime numbers.
5. Given a sequence a_n define a new sequence: $s_n = \sum_{k=1}^{n} a_k$.

Sums

Question

Let $a_n = \alpha + (n-1)d, \;\; s_n = \sum_{i=1}^{n} a_n$.
What is s_n*?*

Sums

Question

Let $a_n = \alpha + (n-1)d, \; s_n = \sum_{i=1}^{n} a_n$.
What is s_n*?*

Sums

Question

Let $a_n = \alpha + (n-1)d, \; s_n = \sum_{i=1}^{n} a_n$.
What is s_n*?*
Let $b_n = \alpha \cdot q^{n-1}, \; S_n = \sum_{i=1}^{n} b_i$.
What is S_n*?*

1. Let $\{a_n\} = \{\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \ldots\}$.

Sums

Question

Let $a_n = \alpha + (n-1)d, \; s_n = \sum_{i=1}^{n} a_n$.
What is s_n*?*
Let $b_n = \alpha \cdot q^{n-1}, \; S_n = \sum_{i=1}^{n} b_i$.
What is S_n*?*

1. Let $\{a_n\} = \{\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \ldots\}$.
2. What is a_n?

Sums

Question

Let $a_n = \alpha + (n-1)d, \; s_n = \sum_{i=1}^{n} a_n.$
What is s_n*?*
Let $b_n = \alpha \cdot q^{n-1}, \; S_n = \sum_{i=1}^{n} b_i.$
What is S_n*?*

1. Let $\{a_n\} = \{\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \ldots\}$.
2. What is a_n?
3. What is $s_n = \sum_{i=1}^{n} a_n$?

Sums

Question

Let $a_n = \alpha + (n-1)d,\ s_n = \sum_{i=1}^{n} a_n$.
What is s_n*?*
Let $b_n = \alpha \cdot q^{n-1},\ S_n = \sum_{i=1}^{n} b_i$.
What is S_n*?*

1. Let $\{a_n\} = \{\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \dots\}$.
2. What is a_n?
3. What is $s_n = \sum_{i=1}^{n} a_n$?
4. What is $\sum_{i=1}^{n} \frac{1}{n^2}$?

Sums

Question

Let $a_n = \alpha + (n-1)d,\ s_n = \sum_{i=1}^{n} a_n$.
What is s_n*?*
Let $b_n = \alpha \cdot q^{n-1},\ S_n = \sum_{i=1}^{n} b_i$.
What is S_n*?*

1. Let $\{a_n\} = \{\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \dots\}$.
2. What is a_n?
3. What is $s_n = \sum_{i=1}^{n} a_n$?
4. What is $\sum_{i=1}^{n} \frac{1}{n^2}$?
5. An interesting sequence: (it has a limit!) $\gamma_n = \log n - \sum_{i=1}^{n} \frac{1}{i}$

Sums

Question

Let $a_n = \alpha + (n-1)d,\ s_n = \sum_{i=1}^{n} a_n.$
What is s_n*?*
Let $b_n = \alpha \cdot q^{n-1},\ S_n = \sum_{i=1}^{n} b_i.$
What is S_n*?*

1. Let $\{a_n\} = \{\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \dots\}$.
2. What is a_n?
3. What is $s_n = \sum_{i=1}^{n} a_n$?
4. What is $\sum_{i=1}^{n} \frac{1}{n^2}$?
5. An interesting sequence: (it has a limit!) $\gamma_n = \log n - \sum_{i=1}^{n} \frac{1}{i}$

Sums

Question

Let $a_n = \alpha + (n-1)d,\ s_n = \sum_{i=1}^{n} a_n$.
What is s_n*?*
Let $b_n = \alpha \cdot q^{n-1},\ S_n = \sum_{i=1}^{n} b_i$.
What is S_n*?*

1. Let $\{a_n\} = \{\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \ldots\}$.
2. What is a_n?
3. What is $s_n = \sum_{i=1}^{n} a_n$?
4. What is $\sum_{i=1}^{n} \frac{1}{n^2}$?
5. An interesting sequence: (it has a limit!) $\gamma_n = \log n - \sum_{i=1}^{n} \frac{1}{i}$

You will see many more sequences thorugh out this class and in many other classes.

Definition

Let $n_1 < n_2 < \ldots n_k \subset N$. $a_{n_1}, a_{n_2}, \ldots, a_{n_k}$ *is a subsequence of the sequence* (a_i).

Definition

Let $n_1 < n_2 < \ldots n_k \subset N$. $a_{n_1}, a_{n_2}, \ldots, a_{n_k}$ *is a subsequence of the sequence* (a_i).

Example

Let $(a_n) = 2, 5, 10, 17, \ldots n^2 + 1$. *The sequence* $2, 5, 17, 37$ *is a subsequence of* (a_n) *of length 4.*

$$n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 6.$$

Definition

Let $n_1 < n_2 < \dots n_k \subset N$. $a_{n_1}, a_{n_2}, \dots, a_{n_k}$ *is a subsequence of the sequence* (a_i).

Example

Let $(a_n) = 2, 5, 10, 17, \dots n^2 + 1$. *The sequence* $2, 5, 17, 37$ *is a subsequence of* (a_n) *of length 4.*

$$n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 6.$$

Definition

Let $n_1 < n_2 < \dots n_k \subset N$. $a_{n_1}, a_{n_2}, \dots, a_{n_k}$ *is a subsequence of the sequence* (a_i).

Example

Let $(a_n) = 2, 5, 10, 17, \dots n^2 + 1$. *The sequence* $2, 5, 17, 37$ *is a subsequence of* (a_n) *of length 4.*

$$n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 6.$$

If we wanted to extend this subsequence (can you propose a "rule"?) what will be n_5*?*

Definition

Let $n_1 < n_2 < \dots n_k \subset N$. $a_{n_1}, a_{n_2}, \dots, a_{n_k}$ *is a subsequence of the sequence* (a_i).

Example

Let $(a_n) = 2, 5, 10, 17, \dots n^2 + 1$. *The sequence* $2, 5, 17, 37$ *is a subsequence of* (a_n) *of length 4.*

$$n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 6.$$

If we wanted to extend this subsequence (can you propose a "rule"?) what will be n_5*?*

Definition

A sequence (a_i) *is monotonically increasing if* $a_{i+1} > a_i$. *(Monotonically, decreasing* $(<)$*, non-decreasing* $(\geq)$*, non-increasing* $(\leq)$ *are defined similarly).*

Definition

Let $n_1 < n_2 < \dots n_k \subset N$. $a_{n_1}, a_{n_2}, \dots, a_{n_k}$ *is a subsequence of the sequence* (a_i).

Example

Let $(a_n) = 2, 5, 10, 17, \dots n^2 + 1$. *The sequence* $2, 5, 17, 37$ *is a subsequence of* (a_n) *of length 4.*

$$n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 6.$$

If we wanted to extend this subsequence (can you propose a "rule"?) what will be n_5*?*

Definition

A sequence (a_i) *is monotonically increasing if* $a_{i+1} > a_i$. *(Monotonically, decreasing* $(<)$*, non-decreasing* $(\geq)$*, non-increasing* $(\leq)$ *are defined similarly).*

Question

A little puzzle: 10 policemen stand in a line. Can you prove that there are at least four policemen whose heights are monotonic?

Binary Sequences

1. Binary sequences play important roles in many areas such as electronics, medicine, economics, engineering, computer science and of course mathematics.

Binary Sequences

1. Binary sequences play important roles in many areas such as electronics, medicine, economics, engineering, computer science and of course mathematics.
2. Integers have a binary representation.

Binary Sequences

1. Binary sequences play important roles in many areas such as electronics, medicine, economics, engineering, computer science and of course mathematics.
2. Integers have a binary representation.
3. There are 2^n distinct binary sequences of length n.

Binary Sequences

1. Binary sequences play important roles in many areas such as electronics, medicine, economics, engineering, computer science and of course mathematics.
2. Integers have a binary representation.
3. There are 2^n distinct binary sequences of length n.
4. In many applications we look for binary sequences with particular properties.

Binary Sequences

1. Binary sequences play important roles in many areas such as electronics, medicine, economics, engineering, computer science and of course mathematics.
2. Integers have a binary representation.
3. There are 2^n distinct binary sequences of length n.
4. In many applications we look for binary sequences with particular properties.

Example

Binary Sequences

1. Binary sequences play important roles in many areas such as electronics, medicine, economics, engineering, computer science and of course mathematics.
2. Integers have a binary representation.
3. There are 2^n distinct binary sequences of length n.
4. In many applications we look for binary sequences with particular properties.

Example

1. *How many binary sequences of length $2n$ have exactly n 0's?*

Binary Sequences

1. Binary sequences play important roles in many areas such as electronics, medicine, economics, engineering, computer science and of course mathematics.
2. Integers have a binary representation.
3. There are 2^n distinct binary sequences of length n.
4. In many applications we look for binary sequences with particular properties.

Example

1. *How many binary sequences of length $2n$ have exactly n 0's?*
2. *How many binary sequences of length n do not contain the pattern 010?*

Binary Sequences

1. Binary sequences play important roles in many areas such as electronics, medicine, economics, engineering, computer science and of course mathematics.
2. Integers have a binary representation.
3. There are 2^n distinct binary sequences of length n.
4. In many applications we look for binary sequences with particular properties.

Example

1. *How many binary sequences of length* $2n$ *have exactly n 0's?*
2. *How many binary sequences of length n do not contain the pattern* 010*?*
3. *Can you construct a circular binary sequence of length* 32 *so that each binary sequence of length* 5 *is a segment of it? (01001 is a segment of 1001001101).*