## Discrete Optimization

## Moshe Rosenfeld

Hanoi 2011 moishe@u.washington.edu

Name:

## 1 Assignment-7

Due: Thursday Nov. 3

Please submit your answer in a neat, readable properly organized format.

This assignment includes questions from previous topics in preparation for the coming mid-term.

- 1. Suppose  $A_i \subset \{1, 2, ..., n\}$  is a collection of n subsets such that  $|A_i \cap A_j| = 1$ . Does this collection have an SDR?
- 2. Prove that if M is an  $n \times n$  doubly stochastic matrix then it is a convex combination of permutation matrices.
- 3. Is Herschel's graph (see the folder Supplements in Week-8) Hamiltonian?
- 4. Generate a weighted graph with 10 vertices (use density 3). In this graph find a MCST. Now double every edge of the graph and construct an Eulerian cycle in this new graph.
- 5. Prove that if G is a 2k-regular graph then it has a spanning 2-regular subgraph.
- 6. Recall that if G is bipartite then  $\nu(G) = \tau(G)$  (König's theorem). We say that G has the König property if  $\nu(G) = \tau(G)$ . Prove that every graph G is an induced subgraph of a graph with König's property.