This file contains a collection of graphs for practicing some of the algorithms we studied.

1. The adjacency matrix of a weighted directed graph is:

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 24 & 29 & 0 & 0 & 12 & 13 & 0 & 14 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 27 & 0 & 0 & 0 & 24 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 29 & 0 & 12 \\
0 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25 & 0 & 14 & 0 & 0 & 0 & 0 & 16 & 16 \\
0 & 0 & 19 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 25 & 0 & 0 \\
0 & 30 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25 \\
0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
24 & 0 & 0 & 0 & 0 & 0 & 25 & 0 & 0 & 17 & 0 & 0 & 0 & 29 & 14 & 18 & 0 & 0 \\
0 & 14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 26 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 24 & 0 & 0 \\
0 & 0 & 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 15 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 12 & 0 & 15 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 15 & 0 & 0 & 0 & 0 & 16 \\
26 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 18 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 12 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

   a. Show the adjacency list of the underlying digraph.
   b. Find the shortest distance from vertex 1 to vertex 2.
   c. Show the steps used by Dijkstra's algorithm.

2. The adjacency matrix of a graph is:

$$
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

a. Is this graph connected?

b. Find as large a matching as you can.

c. Is this graph bipartite? If yes, color its vertices by two colors, if not show why.

d. Is this graph Eulerian? Add the smallest number of new edges to make it Eulerian.

3. The following matrix is a weighted version of the previous graph.

$$
\begin{bmatrix}
0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 41 & 0 & 18 & 0 & 0 \\
0 & 0 & 29 & 0 & 0 & 0 & 31 & 0 & 0 & 0 & 45 & 27 & 0 & 0 & 0 & 0 & 33 \\
19 & 29 & 0 & 66 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 66 & 0 & 35 & 75 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 69 & 0 & 0 \\
0 & 0 & 0 & 35 & 0 & 0 & 53 & 50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 75 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25 & 0 \\
0 & 31 & 0 & 0 & 53 & 0 & 0 & 0 & 0 & 16 & 59 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 50 & 0 & 0 & 0 & 0 & 0 & 71 & 0 & 0 & 35 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 42 & 0 & 44 & 0 & 0 & 19 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 42 & 0 & 0 & 0 & 0 & 46 & 0 & 0 & 0 \\
0 & 45 & 0 & 0 & 0 & 0 & 59 & 71 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 26 & 45 \\
0 & 27 & 25 & 0 & 0 & 0 & 0 & 0 & 44 & 0 & 0 & 0 & 58 & 26 & 0 & 0 & 0 \\
41 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 58 & 0 & 0 & 0 & 72 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 35 & 0 & 46 & 0 & 26 & 0 & 0 & 0 & 0 & 0 \\
18 & 0 & 0 & 69 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 25 & 0 & 0 & 0 & 0 & 26 & 0 & 72 & 0 & 0 & 0 & 0 \\
0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 45 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

a. Find a MCST using Prim’s algorithm.

b. * Add edges to make it Eulerian. Your goal is to add edges whose combined weight is as small as possible.

c. Identify all vertices of odd degrees. Randomly break them into pairs and find the shortest distance among the pairs. For instance, if you identified only 8 vertices find the shortest distance between 1 – 4, 2 – 7, 3 – 6, 5 – 8

4. The following connected graph has 11 vertices. It is regular of degree 4. So it has an Eulerian cycle. Construct it.