Discrete Optimization

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1 Assignment - 5

Due: Tuesday 20 Oct.

1. A bipartite graph G has 20 vertices labeled by $a, b, \ldots, j,\ 1, 2, \ldots 10$. Its adjacency list is:

a: $1, 3, 4$

b: $1, 2, 8$

c: $1, 10$

d: $3, 7$

e: $2, 5, 6$

f: $7, 9, 4$

g: $10, 1$

h: $8, 6, 9$

i: $10, 5$

j: 3

Find a maximumsize matching and a minimum size cover in this graph.

2. The adjacency matrix of a graph G is:

0: 0 58 0 0 0 53 25 0 0 0 27
1: 58 0 0 0 95 0 0 0 50 59 0
2: 0 0 0 0 0 56 68 22 99 29 0
3: 0 0 0 0 0 16 0 0 0 0 68
4: 0 95 0 0 0 17 48 76 49 0 0
5: 53 0 56 16 17 0 0 0 85 0 26
6: 25 0 68 0 48 0 0 0 0 0 93
7: 0 0 22 0 76 0 0 0 21 75 0
8: 0 50 99 0 49 85 0 21 0 0 65
9: 0 59 29 0 0 0 0 75 0 0 0
10: 27 0 0 68 0 26 93 0 65 0 0

- Is the graph Eulerian?
- Find the shortest walk that uses every edge of the graph.

3. The politburo of the land of OZ has 13 members. The secretary of the party has issued an order to form 13 commitees with the following structure:

1. Every committee has 5 members.
2. Every two commitees must have at least one person in common.
3. Every member of the politburo has to serve on five different committees.
4. Every committee must select a chairman of the committee.
5. No person shall be chairing more than one committee.

Can you help design these committes? Or at least explain how it can be done.

4. Let $A = \{1, 2, 3, \ldots, 100\}$. We partiton A into 10 subsets $A_1, A_2, \ldots, A_{10}$ each of size 10. A second partition into 10 sets of size 10 each is given by $B_1, B_2, \ldots, B_{10}$. Prove that we can rearrange the indices of the second partition so that $A_i \cap B_i \neq \emptyset$.