Discrete Optimization
Lecture-15

Ngày 29 tháng 11 năm 2011

Vertex and edge coloring and their computational complexity

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König: If G is bipartite then $\chi_1(G) = \Delta(G)$.

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Remark

Brooks' theorem gives us an effcient algorithm that will guarantee a coloring by no more than $\Delta(G)$ colors. This can be quite far from the chromatic number. It is not difficult to construct a sequencing for a bipartite graph for which a particular sequence-coloring will yield a coloring by close to $\Delta(G)$ colors.

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Surprisingly, for edge coloring, which is NP-Complete, there is an effcicient algorithm that will color the edges of a simple graph G by no more than $\Delta(G) + 1$ colors. That is by using no more than one extra color than the absolute necessary number of colors.

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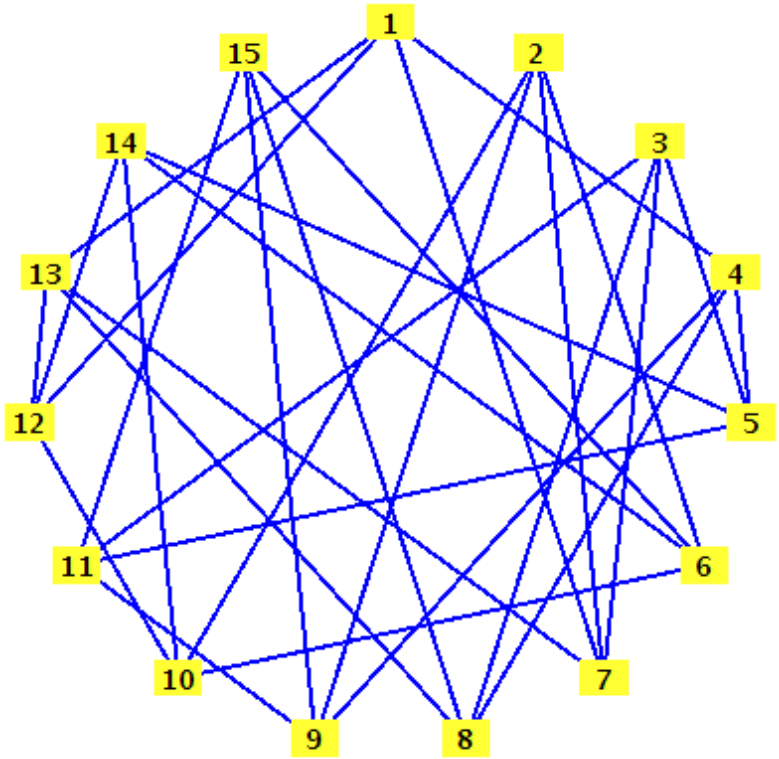
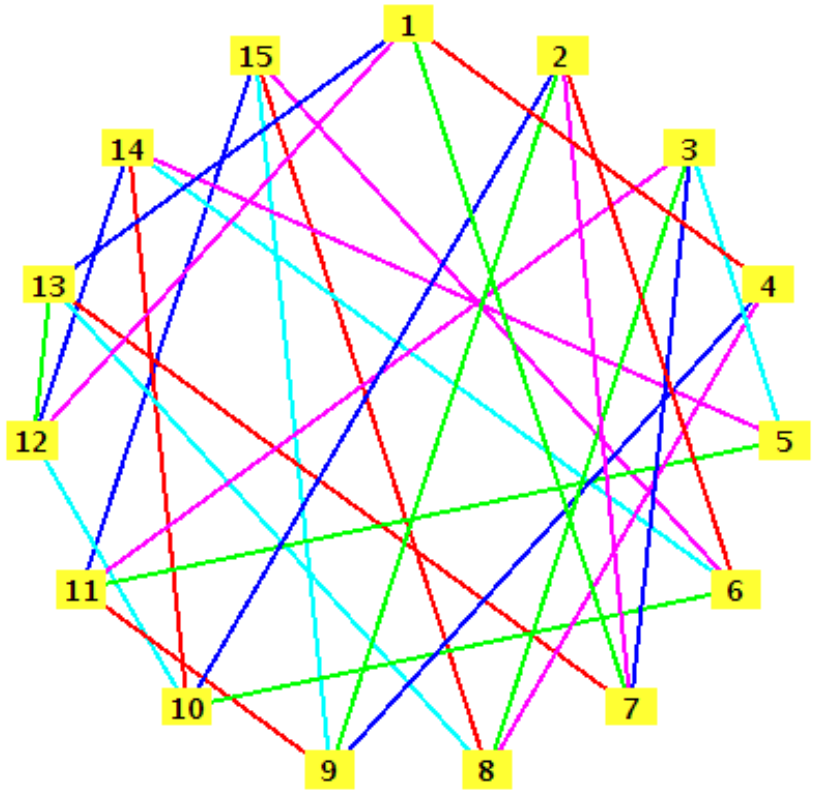
Discussion

To understand the proof, we shall start with an example. Consider the following graph of order 15 regular of degree 4. We shall try to color the edges using 5 colors.

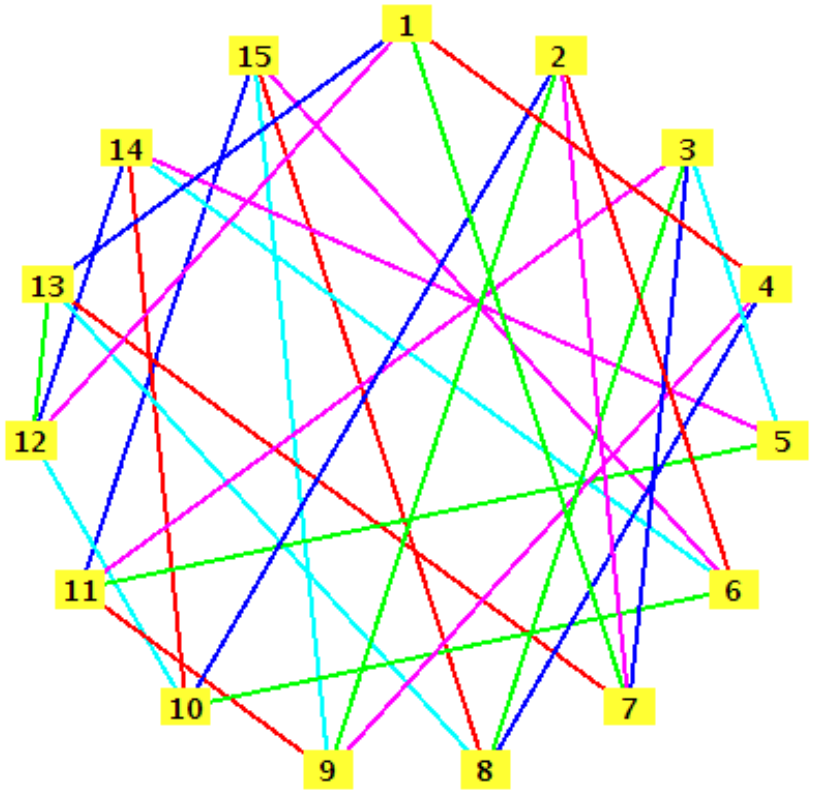
My program had difficulties coloring the edges of G in five colors.



But it had no difficulties coloring the edges of $G - (4, 5)$ in five colors.



Changing the colors prior to building the path.



Computational complexity

The core issues of Discrete Optimization is efficient computations. Most of the problems we encountered are conceptually simple. There is a finite number of feasible solutions. All we have to do is generate all of them and choose the optimal. As we studied a variety of problems, we realized that this approach is almost never feasible. So we needed to find some more clever ways to do it.

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Convetional wisdom is that if the number of steps is bounded by $p(n)$ a plynomial of a fixed degree k then the algorithm is feasible.

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- *Does G have a vertex cover of size k?*