mid-Term Discrete Optimization

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Name:

1 Instructions

1. **There are six questions in this test. Try to answer as much as you can.**
2. **Read ALL questions before you start. Select first the question you feel most comfortable with.**
3. **THINK before you start answering any question. Have a strategy (steps) to construct your answer.**
4. **If you want to disprove a statement produce a counter example.**
5. **Where appropriate, especially when defining things consider giving an example.**
6. **Use blank pages for your answers. Mark clearly which question is being answered.**

Question 1. Execute the Hungarian method and find an optimal assignment for the following assignment problem:
Precede each step by a brief sentence describing what you do.

	C1	C2	C3	C4	C5
J1	10	15	10	10	15
J2	20	30	20	10	20
J3	15	10	10	10	30
J4	35	10	10	30	45
J5	20	10	15	15	35

Question 2. a. **True or false: if in an assignment problem M the cheapest cost is $M_{1,1}$ then an optimal assignment will assign Job1 to company 1.**

b. **Recall: Switching an assignment means that if Job_i is assigned to Com_j and Job_k is assigned to Com_m we reassign Job_i to Com_m and Job_k to Com_j.**
Prove or disprove: If for agiven assignment no switch results in a cheaper cost then the current assignment is optimal.

Question-3. a. **What is a tree?**

b. **What is a matching in a graph?**

c. **What is an M augmenting path?**

d. **What is a bipartite graph?**

Question-4. Use the attached weighted graph to answer the following:

a. **In the simple graph underlying the weighted graph, determine whether it is bopartite. If so, draw it as a bipartite graph. If no, show why it is not.**

b. **Find the shortest path between vertex 3 and vertex 7.**

c. **Find a MCST in this graph.**

Question-5. **In the attached pages titled Floyd's algorithm you are given the initial cost of direct travel between any pair of cities. The next table produces the the chepeast way to fly between cities. Use it to determine the cost of the cheapest path between cities** 0 **and** 20**.**

Question-6. **a. What is an SDR? (system of distinct representatives)?**

b. Let $A_1, A_2, \ldots, A_{20}$ **be a family of disjoint subsets of** $\{1, 2, \ldots, 400\}$ **each containing** 20 **numbers, and let** $B_1, B_2, \ldots, B_{20}$ **be a similar family. Prove that we can match the two families so that** $A_i \cap B_{j_i} \neq \emptyset$**.**