Discrete Optimization

Ngày 13 tháng 7 năm 2011

1. Introduction

Our life is a long sequence of optimization decisions. When we travel from point A to point B we wish to do it in the fastest way; when we want to fly to another city we want to find the cheapest fare; when we look for a job we wish to find the best paying suitable job in a certain region; when an investor invests money he may wish to maximize his profit and minimize the risk. In short, most of our time (while awake) is spent trying to optimize something.

1. Introduction

Our life is a long sequence of optimization decisions. When we travel from point A to point B we wish to do it in the fastest way; when we want to fly to another city we want to find the cheapest fare; when we look for a job we wish to find the best paying suitable job in a certain region; when an investor invests money he may wish to maximize his profit and minimize the risk. In short, most of our time (while awake) is spent trying to optimize something.

Many optimization problems can be modeled as mathematical optimization problems.

1. Introduction

Our life is a long sequence of optimization decisions. When we travel from point A to point B we wish to do it in the fastest way; when we want to fly to another city we want to find the cheapest fare; when we look for a job we wish to find the best paying suitable job in a certain region; when an investor invests money he may wish to maximize his profit and minimize the risk. In short, most of our time (while awake) is spent trying to optimize something.

Many optimization problems can be modeled as mathematical optimization problems.

Example

A manufacturer of mango (xoai) jam wishes to package it in metal cans whose volume is 1000 *cubic cm. He is considering rectangular or cylindrical cans.*

Example

A manufacturer of mango (xoai) jam wishes to package it in metal cans whose volume is 1000 *cubic cm. He is considering rectangular or cylindrical cans.*

Example (Boxes)

If you decide to make a rectangular can. What should be the length of the three sides of the can so that the total area of the sides will be as small as possible?

Example

Example

A manufacturer of mango (xoai) jam wishes to package it in metal cans whose volume is 1000 *cubic cm. He is considering rectangular or cylindrical cans.*

Example (Boxes)

If you decide to make a rectangular can. What should be the length of the three sides of the can so that the total area of the sides will be as small as possible?

Example (Cylinders)

How about making a cylindrical can, will it use less material then the optimal rectangular can?

Discussion

To solve these problems we can construct a function that describes the area and find its minimum.

Discussion

To solve these problems we can construct a function that describes the area and find its minimum.

Discussion

To solve these problems we can construct a function that describes the area and find its minimum.

For instance, for boxes, if the sides of the rectangular can are a, b, c *then the area will be* $f(a, b, c) = 2(ab + ac + bc)$ *and the volume restriction is* $abc = 1000$ $a, b, c > 0$.

Discussion

To solve these problems we can construct a function that describes the area and find its minimum.

For instance, for boxes, if the sides of the rectangular can are a, b, c *then the area will be* $f(a, b, c) = 2(ab + ac + bc)$ *and the volume restriction is* $abc = 1000\ a, b, c > 0$.

So we need to minimize $2(ab + ac + bc)$ *subject to the constraints* $abc = 1000,\ \ a, b, c > 0$.

Discussion

To solve these problems we can construct a function that describes the area and find its minimum.

For instance, for boxes, if the sides of the rectangular can are a, b, c *then the area will be* $f(a, b, c) = 2(ab + ac + bc)$ *and the volume restriction is* $abc = 1000\ a, b, c > 0$.

So we need to minimize $2(ab + ac + bc)$ *subject to the constraints* $abc = 1000,\ a, b, c > 0$.

For the cylindrical can if the radius of the base is r *and the height is* h *we need to minimize:*

$g(r, h) = 2\pi(r^2 + rh)$ *subject to* $\pi r^2 h = 1000\ r, h > 0$.

Basic vocabulary

For rectangular boxes:

1. The conditions $abc = 1000, \; a, b, c \geq 0$ are the constraints

Basic vocabulary

For rectangular boxes:

1. The conditions $abc = 1000,\ a, b, c \geq 0$ are the constraints
2. $f(a, b, c) = 2(ab + ac + bc)$ is the objective function

Basic vocabulary

For rectangular boxes:

1. The conditions $abc = 1000,\ a, b, c \geq 0$ are the *constraints*
2. $f(a, b, c) = 2(ab + ac + bc)$ is the *objective function*
3. a triple such as $a = 20,\ b = 5,\ c = 10$ is a *feasible solution*.

Basic vocabulary

For rectangular boxes:

1. The conditions $abc = 1000,\ a, b, c \geq 0$ are the constraints
2. $f(a, b, c) = 2(ab + ac + bc)$ is the objective function
3. a triple such as $a = 20,\ b = 5,\ c = 10$ is a feasible solution.

Basic vocabulary

For rectangular boxes:

1. The conditions $abc = 1000,\ a, b, c \geq 0$ are the constraints
2. $f(a, b, c) = 2(ab + ac + bc)$ is the objective function
3. a triple such as $a = 20,\ b = 5,\ c = 10$ is a feasible solution.

For cylinders:

1. The constraints are $\pi r^2 h = 1000,\ h, r \geq 0$

Basic vocabulary

For rectangular boxes:

1. The conditions $abc = 1000,\ a, b, c \geq 0$ are the constraints
2. $f(a, b, c) = 2(ab + ac + bc)$ is the objective function
3. a triple such as $a = 20,\ b = 5,\ c = 10$ is a feasible solution.

For cylinders:

1. The constraints are $\pi r^2 h = 1000,\ h, r \geq 0$
2. The objective function is: $g(r, h) = 2\pi(r^2 + rh)$

Basic vocabulary

For rectangular boxes:

1. The conditions $abc = 1000,\ a, b, c \geq 0$ are the constraints
2. $f(a, b, c) = 2(ab + ac + bc)$ is the objective function
3. a triple such as $a = 20,\ b = 5,\ c = 10$ is a feasible solution.

For cylinders:

1. The constraints are $\pi r^2 h = 1000,\ h, r \geq 0$
2. The objective function is: $g(r, h) = 2\pi(r^2 + rh)$
3. A pair such as $r = 10, h = \frac{10}{2\pi}$ is a feasible solution.

(Summary)

A typical optimization problem will usually have three components:

(Summary)

A typical optimization problem will usually have three components:

- **Constraints**

(Summary)

A typical optimization problem will usually have three components:

- **Constraints**
- **Objective Function**

(Summary)

A typical optimization problem will usually have three components:

- **Constraints**
- **Objective Function**
- **Feasible Solutions**

(Summary)

A typical optimization problem will usually have three components:

- **Constraints**
- **Objective Function**
- **Feasible Solutions**
- *In our example, we identified the objective functions, the constraints and feasible solutions.*

Summary

(Summary)

A typical optimization problem will usually have three components:

- **Constraints**
- **Objective Function**
- **Feasible Solutions**
- *In our example, we identified the objective functions, the constraints and feasible solutions.*
- *The feasible solutions are regions in* $\mathbb{R}^3$ *and* $\mathbb{R}^2$ *respectively.*

Summary

(Summary)

A typical optimization problem will usually have three components:

- **Constraints**
- **Objective Function**
- **Feasible Solutions**
- *In our example, we identified the objective functions, the constraints and feasible solutions.*
- *The feasible solutions are regions in* $\mathbb{R}^3$ *and* $\mathbb{R}^2$ *respectively.*
- *In other words, there are infinitely many feasible solutions and our goal is to find the best.*

Summary

(Summary)

A typical optimization problem will usually have three components:

- **Constraints**
- **Objective Function**
- **Feasible Solutions**
- *In our example, we identified the objective functions, the constraints and feasible solutions.*
- *The feasible solutions are regions in* $\mathbb{R}^3$ *and* $\mathbb{R}^2$ *respectively.*
- *In other words, there are infinitely many feasible solutions and our goal is to find the best.*
- *In contrast, typical Discrete Optimization problems usually have only finitely many feasible solutions.*