

ERDÖS VIGNETTES

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Unexpectedly — somehow, I always assumed that he will be among us indefinitely — Paul Erdős died three months ago. Many of his close collaborators and many other prominent mathematicians have written or are writing fitting tributes and obituaries. This short note does not mean to compete with any of these, but simply to tell of some of my recollections of Erdős.

In the mid 1950's, while I was a student at the Hebrew University in Jerusalem, Erdős gave a series of talks on combinatorics and geometry. Among the topics were the "selection theorems", an extension of the Ramsey theory that Erdős and Richard Rado were working on at that time. The last talk was devoted to a particular case, involving the transfinite ordinal ω^3 . This was quite abstract, and the proof — which involved three nested inductive arguments — took the whole hour. It was quite a shock when Erdős said, at the end of the lecture, that he has not written down the proof because it seemed too complicated to be intelligibly conveyed in writing. I was very impressed by the beauty of the result and the refinement of its proof. With the exuberance of youth, and jumping in where angels fear to tread, I offered to write it up. Erdős agreed, and gave me the addresses of his stops for the next few weeks.

Very diligently I went to work, and although the argument extended over six or eight pages, by paying careful attention to notation and to the proper order of argument, the paper was shaping up very well. All, that is, except for the last inductive step, deep in the

third induction, which I had troubles to streamline into a conclusive form. After a week or so of unsuccessful attempts, I gave up and wrote to Erdős admitting my failure to nail down this last step.

Erdős' answer arrived soon, praising my carefulness, and saying that it is fortunate that I did not manage to complete a proof: Someone (I think it was Schinzel) has meanwhile sent Erdős a counterexample to the supposed theorem!

A moral I learned from this, which served me well ever since, is that one can easily be misled by oral arguments, and that the validity of a statement cannot be taken for granted unless all the steps have been clearly understood and digested. Later I learned, from sometimes painful experiences, that even written presentations very often appear to prove what turns out to be false statements.

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Close to twenty years later, during one of the many times we met at conferences or symposia, I told Erdős of a nice little result I had proved which solved a problem that has been mentioned in the literature several time over many years. This result gave the precise maximum $\mu(n)$ for the number of touching points in families of n simple curves with disjoint interiors; surprisingly, the maximum can be attained even if all curves are circles. He liked this very much, and at once asked what happens if one allows the interiors to overlap. We started investigating this question, and continued by correspondence over the next several months. Our most interesting result was that, for circles, $\mu(n)/n$ grows to infinity with n . We also found estimates for $\mu(n)$ for the case of general curves. The aspect of this collaboration which impressed me most was the inequality of the mathematical aptitude of the coauthors. At several steps, Erdős wrote that now that have established this or that part, the next step should follow by such-and-such simple arguments. Since I could not see the "simple

arguments", several more letters were exchanged before I was able to grasp the "simple arguments". His patience during this correspondence was really exemplary. In the end, we were satisfied that there is not much chance of tightening the bounds any more, and we published our results in 1973.

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The last contact I wish to mention occurred on September 16, 1996. I received the following e-mail from Erdős:

Dear Branko,

Sorry that I disturb you with this message. Please phone Moshe Rosenfeld that if he can finish our paper fast he should send it to Schinzel for Acta Arithmetica. I would like to dedicate it to the 75th birthday of Cassels.

Regards to all shalom lehitraot dod zaken.

Paul

The remarkable thing about this message, sent just four days before his death, is that he sent it to me and not directly to Rosenfeld: he knew Rosenfeld's email address as well as he knew mine. Both Rosenfeld and I have tried to puzzle out why he chose to send this message in an indirect way, but could not find a convincing explanation. I am inclined to think that Erdős suffered a mild stroke a few days before the fatal ones, and some memories were erased. The knowledge that I can contact Rosenfeld lay probably deeper, since Rosenfeld used to be my student some forty years ago, when Erdős first met him.

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Paul Erdős will be missed by many, both for his mathematics, and for his humanity.