Discrete Mathematics and its Applications

Ngày 17 tháng 9 năm 2011

SETS

Definition

A set **B** *is a collection of objects such that for every object X in the universe the statement:*

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4. $A_4 = \{A, B, C, \ldots, Z\}$

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8. $\emptyset$: *the set with no members* ($x \notin \emptyset \ \forall x$)

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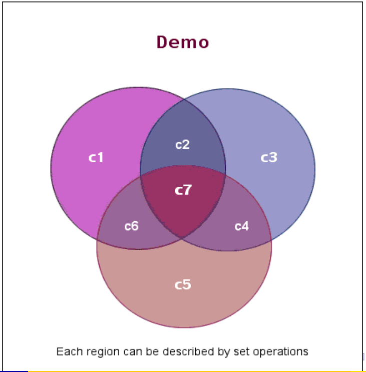
4. **SYMMETRIC DIFFERENCE**: $A \oplus B = A \setminus B \vee B \setminus A$.
5. **COMPLEMENT**: $\overline{A} = \{x \mid x \notin A\}$ ($\overline{A} = U \setminus A$) (U the universe).

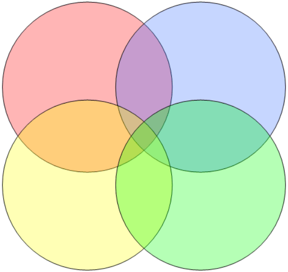
Venn Diagrams

Venn diagrams, were used a long time ago to visualize different relations, particularly among sets. John Venn formalized them in 1880. He did not call them Venn Diagrams but rather Euler Diagrams. Euler used them previously.



Euler's four cycles

This is Euler's drawing of a diagram for four sets: **B**, **P**, **Y**, **G**.



Hình: What is missing from this diagram?

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Theorem (De Morgan's Law)

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NOTE: $a \notin P(\{a,b\})$, $\{a\} \in P(\{a,b\})$. $P(\emptyset) = \{\emptyset\}$. $\qquad |P(\emptyset)| = 1$.

Tôi nghĩ rằng tôi giải quyết nó

the Cartesian Product

Definition

Let A, B be two sets. $A \times B$, the **cartesian product** *of A and B is:*
$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$.

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Let A, B be two sets. $A \times B$, the **cartesian product** *of A and B is:*
$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$.

Comment

We introduce here a new concept: the ordered pair (a, b). We note that if $a \neq b$ then $(a, b) \neq (b, a)$ while $\{a, b\} = \{b, a\}$, also $(a, a) \neq (a)$ while $\{a, a\} = \{a\}$.

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For the mathematical purist we note that the introduction of the ordered pair is not necessary. We can use the sets notions to achieve the same goal as follows:
$A \times B = \{\{a, \{a, b\}\} \mid a \in A,\ b \in B$.

Exercise

Prove that if $\{a, \{a, b\}\} = \{c, \{c, d\}\}$ then $a = c$ and $b = d$.

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Definition

A relation $\mathbb{R}$ *from the set* A *to the set* B *is a subset of* $A \times B$ $(\mathbb{R} \subset A \times B)$