Discrete Mathematics and its Applications

Ngày 14 tháng 9 năm 2011

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Comment

To show that a proposition is satisfiable we need to find only one assignment of truth values that renders it true while to show that it is not satisfiable we need to check **all** *possible truth assignments. Thus for a proposition of "only" 50 variables we need to perform* 2^{50}

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    - *You can try to factor it.*
    - *You can calculate* $2^{n-1} \bmod n \neq 1$ *which calculates much faster.*

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Theorem

SAT *is reducible to* **3-SAT**

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6. Verifying the factorization of $2^{256} + 1$ given above can be efficiently calculated. So this problem is in the class **NP**. But is it in **P**?

the Millenium Problems

The Millenium problems is a list of what is considered the most challenging problems in mathematics.
The list contains seven problems. A solution to any one of them carries a prize of $ 1,000,000 (USD). The list was published in 2000. For more details go to: http://www.claymath.org/millennium/.

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P = NP is one of the remaining six unsolved Millenium problems.

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The program logicsim.exe is a nice simulation for constructing logical gates and analyzing compound propositions. It has a nice tutorial. We shall see it at work in class.