OPTIC FLOW

Problem:

A camera at point C moves at velocity \( \mathbf{v} \) and angular velocity \( \omega \).
What is the "optic flow" caused by the terrain at point P, angle \( \gamma \)?

Solution:

First consider a simpler problem:

C is fixed, P is moving at \( \mathbf{v} \).

Define \( \mathbf{\Omega} = -\omega \mathbf{k} \) as the optic flow "vector": want \( -\omega \mathbf{k} \) for point P at \( \gamma \).

Note that \( \mathbf{v} \times \mathbf{r} = \mathbf{l} \mathbf{r} = \frac{\mathbf{r}}{1} \Rightarrow -\omega = \frac{\mathbf{v} \times \mathbf{r}}{r} \).

Now, suppose that the camera is moving at \( -\mathbf{v} \) and P is fixed. This is indistinguishable from "simpler" case.

\[ -\omega = -\frac{\mathbf{v} \times \mathbf{r}}{r} \]

Next, suppose the camera has \( \mathbf{v} = 0 \) but \( \omega \neq 0 \Rightarrow -\omega = -\omega \)

(note: effect of \( \omega \) does not depend on \( r \))

Sum these two effects to get \( -\omega = -\omega - \frac{\mathbf{v} \times \mathbf{r}}{r} \).

Component form: given \( \mathbf{v}' = v_x \mathbf{i} + v_z \mathbf{k} = [v_x, v_z] \), and \( \mathbf{\gamma}' = [-\cos \gamma, \sin \gamma] \)

Then \( v_y = \mathbf{v} \cdot \mathbf{\gamma}' = -v_x \cos \gamma + v_z \sin \gamma \Rightarrow -\omega = -\omega + \frac{v_x}{r} \cos \gamma - \frac{v_z}{r} \sin \gamma \)

Remark: in full 3D case, is 2D vector field on sphere \( \mathbf{\Omega} = -\omega \mathbf{r} + \frac{1}{r} (I - \mathbf{r} \mathbf{r}^T) \mathbf{v} \).