Generating Membership Functions for a Noise Annoyance Model from Experimental Data

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Summary:
The success of fuzzy expert systems could be mainly attributed to the inclusion of linguistic terms into their reasoning scheme. This allows reasoning about complex issues within a certain (tolerated) degree of imprecision. Hence, an important issue in the development of such systems is the choice of the membership functions that model the linguistic terms involved in the application. In this chapter we will describe several methods for the construction of these membership functions (which represent information) from measurements obtained in psycholinguistic experiments. Special attention will be paid to the inclusive and the non-inclusive interpretation of linguistic terms. Secondly, these techniques are applied to data gathered in an International Annoyance Scaling Study, where the relationship between more than 20 different linguistic terms and their corresponding noise annoyance level was under survey.

Keywords: fuzzy expert system, linguistic term, membership function, inclusive and non-inclusive interpretation, noise annoyance

1 Introduction

People tend to express real-life information by means of natural language. It allows them to reason about everyday issues within a certain (tolerated) degree of imprecision. It is, therefore, not surprising that the introduction of the fuzzy set theory as a framework for the mathematical representation of linguistic concepts has given a rise to an important evolution in the field of computer science. The fuzzy expert systems that emerged in this context have proved to be a useful tool in many real-life applications. A fundamental issue in the development of such systems, is the design of the membership functions that model the linguistic terms involved in the application. In this chapter we will describe several methods for the construction of those functions (which represent information) from measurement results obtained in psycholinguistic experiments.

The chapter is structured as follows: after presenting the problem of noise annoyance modelling (Section 2) and describing the form of the experimental data at hand (Section 3), we will briefly recall how linguistic terms are represented in fuzzy expert systems, thereby stressing the difference between an inclusive and a non-inclusive interpretation (Section 4). The main part of this chapter is an overview of well-known, improved and new methods for the construction of membership functions; we explain in detail how they can be applied for noise annoyance terms (Sections 5, 6, 7, 8).

2 Noise Annoyance Modelling

As an environmental factor, noise has several adverse effects on man. Annoyance or disturbance is commonly used as an impact indicator for these effects. Noise surveys can be used as a measurement tool. One of the most important questions in such a survey sounds out about the level in which someone is annoyed by the noise, namely "Thinking about the last 12 months, when you were here at home, how much did noise bother, disturb, or annoy you?" If the survey is conducted by telephone, the subjects are given a set of linguistic terms to choose from: e.g. *not at all annoyed*, *slightly annoyed*, *very annoyed*,... Throughout this chapter we will refer to the linguistic terms that are generated by applying an adverb to the base term *annoyed* as **annoyance terms**. In postal or face-to-face surveys, a numerical or graphical scale can be used. Even in this case the question must be asked whether the mark on a numerical or graphical scale is not a forced expression of a feeling that is more easily expressed using natural language.

The goal of an annoyance model is to predict the outcome of annoyance surveys. Such models can be used by noise policy makers to make strategic decisions. For instance, they can help to choose the "best" route for a new railway by comparing the predicted level of annoyance that the population living along two different possible routes will experience. Fuzzy techniques are very well suited for this modelling purpose: it is far more easy for humans to express annoyance by means of a linguistic term - which is intrinsically vague - than by some crisp number.

The output of a linguistic noise annoyance model will be a noise annoyance term. The input can be either crisp or vague facts. Examples of crisp facts that may influence noise annoyance are age, number of children under 18, average daytime noise exposure level, average nighttime noise exposure level, etc. An example of a vague fact is a person's sensitivity to noise or the state of the environment in general. Rules used in the model are always fuzzy. For more details on a fuzzy system to model noise annoyance we refer to [2].

3 Experimental Data: an International Scaling Study.

To get a better understanding of what people really mean when they use some annoyance term, an International Annoyance Scaling Study was conducted (see [5]). In 9 different languages for 21 annoyance terms people were asked to indicate with a mark on a continuous line what each term meant to them, with the most left-hand side being no annoyance at all and the most right-hand side being the most possible degree of annoyance one can imagine. While processing the data, each mark on the line was converted into the distance (expressed in centimetres) from the left-hand side on the 10 centimetres long line. This resulted in a continuous numerical domain ranging from 0 to 10 and a dataset containing 21 values for each subject. In the same study, people were also asked to select five annoyance terms dividing the annoyance level scale in equal intervals. With these data and classical statistical analysis, five terms for each language were selected to globally represent five equidistant portions of the annoyance scale. For the English language they are: *not at all, slightly, moderately, very* and *extremely annoyed.* These five terms are the ones which will be used throughout this chapter. We will refer to them as A_1, A_2, A_3, A_4 and A_5 respectively. Finally, it should be stressed that these data were not gathered with the purpose of membership function construction in mind.

4 Representing Annoyance Terms

Representing terms. In fuzzy set theoretical contexts, a linguistic term is usually represented by a fuzzy set on the suitable universe X. This fuzzy set is characterized by a $X \rightarrow [0,1]$ mapping A which is called the membership function. For each x in X, A(x) is called the membership degree of x in the fuzzy set A. The class of all fuzzy sets on X will be denoted **F**(X). Throughout this chapter we will use the same symbol (e.g. A) to denote the term being modelled, the fuzzy set and the membership function. Recall that for A and B fuzzy sets on X, the inclusion is defined as follows: $A \subseteq B$ iff $(\forall x \in X)(A(x) \leq B(x))$.

The height of A is defined by $\text{hgt } A = \sup\{A(x) \mid x \in X\}$. If hgt A = 1 then A is called normalized. For the representation of annoyance terms, we will use the universe X = [0,10] which corresponds to the real interval denoted by the 10 centimetres long line used in the scaling study as described in the previous section.

If A and B are two (annoyance) terms then the terms “A *and* B” and “*not* A” are usually represented by the fuzzy sets A ∩ B and co(A) respectively, defined by (A ∩ B)(x) = min(A(x),B(x)) and co(A)(x) = 1-A(x), for all x in X.

Representing modified terms. In the literature, adverbs such as *slightly, very*,... are often interpreted as linguistic modifiers that alter the meaning of a linguistic term. Therefore, they are represented by a fuzzy modifier on X, i.e. a **F**(X)-**F**(X) mapping (for an overview we refer to [11]). For instance, in *very young*, the adverb *very* is represented by an operator that transforms the fuzzy set *young* into another fuzzy set which is then interpreted as *very young*. In the main part of this chapter however, we will consider annoyance terms as linguistic terms "as such": e.g. *very annoyed* will be considered as one term rather than the term *annoyed* being modified by *very*. In Section 8 this approach is compared to an approach based on modifiers.

General Shape Functions. For the representation of linguistic terms by fuzzy sets on a numerical universe, usually a general shape function is chosen. This allows for the fuzzy set to be fully described by only a small number of parameters. In this chapter we will use sigmoidal and bell-shaped functions, characterized by real parameters μ, σ, and δ, and defined by

$$SIGM(\mu,\sigma,x) = \frac{1}{1+\exp(\mu-\sigma x)}$$

$$BELL(\mu,\sigma,\delta,x) = \begin{cases} \exp\left(\dfrac{-(x-\mu)^2}{2\sigma^2}\right) & if\ x \le \mu \\ \exp\left(\dfrac{-(x-\mu)^2}{2\delta^2}\right) & if\ x > \mu \end{cases}$$

for all x in **R.** For positive σ, a sigmoidal function is increasing. A bell-shaped function is partly increasing and partly decreasing: μ corresponds to the “top” of the bell, while σ and δ determine the width of the left and the right flank of the bell respectively. The shape of these functions is considered to be acceptable from a psycholinguistic point of view [7] for the representation of linguistic terms.

Inclusive and Non-Inclusive Interpretation.

Linguistic models found in literature are usually based on one of two different interpretations of the linguistic terms involved, each characterized by a typical shape for the corresponding membership functions.

In the **inclusive interpretation**, we assume that the membership function for *not at all annoyed* is decreasing, while the membership functions for the four other terms are increasing. Furthermore

extremely annoyed ⊆ *very annoyed* ⊆ *moderately annoyed* ⊆ *slightly annoyed*

The underlying semantics is that e.g. everybody who is *very annoyed* is also *moderately annoyed.* In this interpretation the membership degree of x in A clearly corresponds to the degree to which x **satisfies** the term modelled by A: indeed the degree to which somebody is *moderately annoyed* is always larger than or equal to the degree to which he or she is *very annoyed*. To distinguish this interpretation more clearly, the membership functions could be labelled with a preceding *at least*.

In the **non-inclusive interpretation**, *not at all annoyed* is decreasing and *extremely annoyed* is increasing, but the membership functions for the other three terms are partly increasing, partly decreasing (e.g. bell-shaped). They denote neither subsets nor supersets of each other, but different, possibly overlapping categories. In this interpretation the membership degree of x in A corresponds to the degree to which x is **representative** for the term modelled by A.

For a more detailed discussion about the inclusive and the non-inclusive interpretation we refer to [7,11,16]. In fuzzy control applications the non-inclusive interpretation is the most popular one (see e.g. [9]), although the inclusive interpretation is sometimes also used (see [15]).

5 Overview of existing methods

In this section, we will briefly describe the two broad categories of methods for the construction of membership functions that are commonly found in the literature. The first category constructs membership functions based on inquiries, solely done for this purpose. A second, fundamentally different approach, obtains the membership functions as a by-product of fuzzy clustering techniques.

5.1 Inquiry-driven methods

The bottomline of the inquiry is always the same: for a certain linguistic term A, we want for each element x of the universe, the degree of compatibility A(x) between the element and the term ("compatibility" may both refer to "satisfiability" or "representativity" as mentioned in the previous section, depending how the question is posed). However, the way in which the questions are formulated has a serious impact on the size and the position of the resulting membership function (see [12] for a more detailed discussion).

1. **Direct rating.** One possibility is to directly ask one or more experts, or the group in which the experiment is conducted, for the membership degree of some elements, for example: "How A is x?".

2. **Reverse rating.** In this approach, the questions are formulated in reverse form: “Which element x has a given degree A(x) of membership in A?”.

3. **Polling.** In the two methods described above, people are very directly asked to assign a certain degree of membership to some elements. This is for most domains very difficult and at the same time somewhat arbitrary. Hersh and Caramazza [7] overcome this, by asking only yes/no questions of the form: “Does x belong to A?”. Afterwards, A(x) is calculated as the total number of “yes” responses for x divided by the total number of responses for x (yes and no together).

4. **Indirect rating.** These methods also try to replace the direct assignments of degrees with simpler tasks, for instance with pairwise comparisons which are generally easier to estimate. One such method is the analytic hierarchy process (AHP) (see [14]), where questions are asked as “To what degree does x_1 imply A in comparison with x_2 ?”. If the cardinality of the universe is an integer n, then all those answers, for instance on a scale from 1 till 10, result in a square matrix P of order n with $P_{ij} = 1/P_{ji}$ for all i=1,…,n and j=1,…,n. After column-wise normalisation, $A(x_i)$ is then calculated as the (if desirable, again normalized) row average of the i-th row of P.

Final construction phase. Whatever method above used, we always end up with couples (x, A(x)). Finally, the resulting membership function can then be constructed using some well-known curve fitting method, for instance Lagrange interpolation and least-square error method. Other techniques such as learning through neural networks can be also applied for this purpose (see [12]).

Annoyance terms. In the case of annoyance the universe is quite abstract: the scale from 0 to 10 does not correspond to any quantity people are used to experience. Therefore, it is not possible to use the techniques described above in their direct form.

5.2 Methods based on fuzzy clustering

Fuzzy clustering. The primary goal of fuzzy clustering algorithms, such as the fuzzy c-means (FCM) algorithm (see [8]), is to partition a given set of data or objects into fuzzy subsets called clusters such that objects strongly belonging to the same cluster (the membership degree of both objects to that cluster are close to 1) are as “similar” as possible and objects that belong to different clusters (the membership degree of one of those objects to that cluster is close to 0) are as “different” as possible. The notions “similar” and “different” are defined by a user given dissimilarity measure d (for instance, the Euclidean distance in a metric space). The aim of the clustering procedure is then to globally minimise this dissimilariy between elements belonging to the same cluster. In a two-dimensional universe X x Y, clustering is often used to extract the relationship between a variable **u** on X and a variable **v** on Y. Each cluster C (which is a fuzzy set on X x Y) gives rise to a fuzzy rule of the form: “IF **u** is A THEN **v** is B”, in which A and B are fuzzy sets on X and Y respectively, obtained by "projecting" C on X and Y respectively. A common practice in the field of fuzzy clustering is to assign “ad hoc” linguistic terms to the obtained fuzzy sets A and B. This way, the resulting rules are fully linguistical and easy to understand by domain experts (not necessarily having much knowledge of fuzzy logic).

Annoyance terms. Suppose, we have data from an annoyance survey that contains the noise level the subject is exposed to (expressed in decibels) as well as the experienced annoyance level (expressed on a continuous 0-10 scale). We would then be able to use fuzzy clustering (where the variable **u** is the exposure and **v** is the annoyance), generate fuzzy rules describing that relationship and derive the necessary membership functions used in the rules. If five membership functions for the consequent variable **v** would emerge from the clustering process, we could consider to label them with the five annoyance terms introduced in Section 3. However, it can be expected that the generated rules and membership functions are optimally suited for the training data, but will not necessarily behave well in the general case. The membership functions associated with the annoyance terms would not really represent the meaning that people give to that term. Therefore, fuzzy clustering is less suitable for a real-world annoyance model with linguistic rules.

6 A Probabilistic Approach

The construction of membership functions presented in this section is based on a pure probabilistic approach, which resembles to the polling method. The underpinnings of these methods can be found in the statement by Zadeh that the probability of a fuzzy event is equal to the expected grade of membership of the event itself (see [17]). In the polling method as presented in [7], for each linguistic term A, 13 elements x_j of the universe were evaluated. Hence 13 pairs (x_j, <answer>) were obtained (j=1,..,13 and <answer> either "yes" or "no"). In the annoyance scaling study for each term A_m, (m=1,..,5) the informant placed a mark at a point x_m on the 0-10 scale. This corresponds to the pair (x_m,yes) for A_m. For all other x in $[0,10]\backslash\{x_m\}$, however, the scaling study does not give any information about the second part of the pair (x,<answer>). We can, however, make some intuitively justifiable assumptions about this, thereby forcing either the inclusive or the non-inclusive interpretation.

6.1 Inclusive interpretation

Assume that for all $x < x_m$ (m=2,..,5) the informant would have meant "no," and for all $x \geq x_m$ he would have answered "yes." In other words, he placed a mark on the line to indicate the annoyance level corresponding to *at least* A_m. Computing the average number of "yes"-responses for each x-value taken over all informants then comes down to constructing the cumulative distribution function. The inclusive membership functions is then created by fitting a sigmoidal S-function $SIGM(\mu,\sigma,.)$ on the cumulative distribution function.

This approach works well for all the terms, except for A_1 = *not at all annoyed.* Indeed for this term, the informant would have answered "yes" for all $x \leq x_5$ and "no" for all $x > x_5$, thereby indicating with his mark the annoyance level corresponding to *at most* A. To obtain the curve of *not at all annoyed*, therefore the reversed cumulative distribution function (thus moving from 10 down to 0 in our domain) was used and fitted on the reflected sigmoidal function $co(SIGM(\mu,\sigma,.))$.

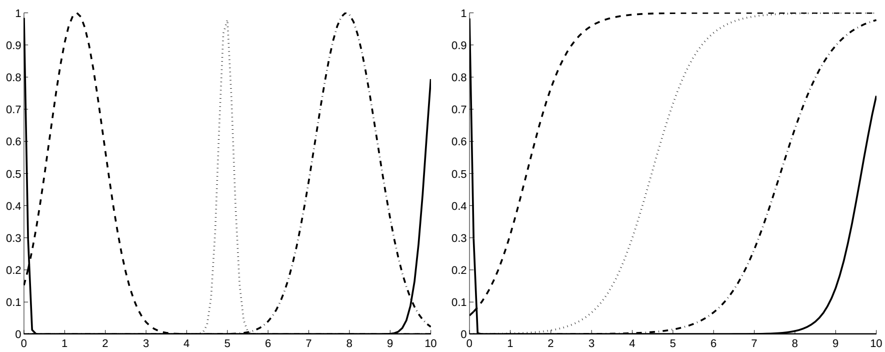

Figure 1: a) Non-inclusive (left) and b) inclusive (right) membership functions for *not at all*, *slightly*, *moderately*, *very* and *extremely annoyed* (probabilistic)

	Non-inclusive		Inclusive	
	μ	σ	μ	σ
not at all	-0.0128	0.0723	4.0456	49.0496
slightly	1.2928	0.6662	2.7888	1.9846
moderately	4.9634	0.1742	7.9807	1.7820
very	7.9206	0.7561	12.2168	1.5988
extremely	10.2227	0.3276	27.3687	2.8424

Table 1: Numerical parameters for the fitted curves (probabilistic)

The generated membership functions are depicted in Figure 1b; the values of the parameters are given in Table 1. Please note, that not every fitted function A is normalized. If necessary, this can be solved by dividing all membership degrees of A by hgt A.

6.2 Non-inclusive interpretation

Another possibility is that the informant wanted to indicate with his mark on x_m (m=1,..,5): "this (and the surrounding) annoyance level(s) I call A_m, but the other ones not." This means that the answer is "yes" for x_m (and perhaps for the levels in a small interval containing x_m) but "no" for the others. Computing the average number of "yes" answers for every x taken over all informants now corresponds to constructing the (normalized) probabilistic histogram. The non-inclusive membership function is then derived from the histogram by fitting an exponential function $BELL(\mu,\sigma,\sigma,.)$ on it with the least-square error method. See Figure 1a for the membership functions and Table 1 for the obtained parameters of those functions.

7 Aggregation of individual curve methods

The probabilistic method does not use any information about how far the marks for all linguistic terms placed on the 0-10 scale by an informant, lie from each other. Each term is modelled as such, without using the relationships between the terms. A method taking this relationship into account is explained in [3] for the non-inclusive interpretation. For every term first an individual curve for each informant is constructed. Then all these curves are combined into the final membership function for that term. The curves generated with the approach in [3] however tend to overlap each other a lot, which makes them less suitable for practical purposes. In this section we will present a variant to this technique, in which the overlap is intrinsically smaller and can even be controlled by a parameter. Furthermore we explain how the same approach can be used for the inclusive interpretation as well.

7.1 Non-inclusive interpretation

The general underlying idea is that the mark x_m (m=1,..5) placed by an informant for a term A_m can be considered as a value with some uncertainty. In the non-inclusive interpretation this uncertainty is proportional to the distance from x_m to the previous mark x_{m-1} (for term A_{m-1}) and the next mark x_{m+1} (for term A_{m+1}). Since there is no previous mark for x_1, we can only take the distance to the next mark x_2 into account to construct a curve for A_1. Likewise there is no next mark for x_5; hence we will only use the distance to the previous mark x_4.

Individual curves. First a fixed membership degree α is chosen. The individual curves will be constructed such that the height of the intersection of two succeeding curves A_m and A_{m+1} is α. For each informant and for each term A_m (m = 2,..,4) an individual bell-shaped function $A_m = BELL(x_m, \sigma_m, \delta_m,.)$ is constructed such that for $\Delta x_m = (x_{m+1} - x_m)/2$

$$A_m(x_m+\Delta x_m) = \alpha = A_{m+1}(x_{m+1}-\Delta x_m)$$

The top of the bell for A_m corresponds to the mark x_m placed by the informant, and the width of the flanks is determined by the distance of the mark x_m to the previous mark x_{m-1} and the next mark x_{m+1}, as well as by the parameter α. Solving this equation results in the value for δ_m and σ_{m+1}, namely:

$$\delta_m = \sigma_{m+1} = \left(\frac{1}{\sqrt{-2\ln(\alpha)}}\right)\Delta_m .$$

For the left-most and the right-most terms A_1 and A_5:we will use the functions $co(SIGM(x_1, \delta_1,.))$ and $SIGM(x_5, \sigma_5,.)$ respectively.

Aggregation. After all the individual curves for a linguistic term A_m (m = 1,..,5) are calculated, they are numerically added and normalized. Finally, the curve is fitted to the appropriate shape function to produce the final membership function.

Remark. It can be observed that this method comes down to the probabilistic histogram method for the limit value 0 of parameter α, where $\lim_{\alpha\to 0}\frac{1}{\sqrt{-2\ln(\alpha)}} = 0$, which means that the width of all flanks is 0. Stated otherwise: no flanks are added at all, hence only the given crisp points are summed, which is exactly the same as the histogram approach.

The results of this method, fitted on the same exponential and sigmoidal functions, for parameter α=0.1 are shown in Figure 2a. The exact parameters of the fitted functions are given in Table 2.

Compared to Figure 1a, the curves in Figure 2a cover the whole universe in a uniform manner. This covering is induced by taking the distance between the marks into account when constructing the membership functions, and make them far more suitable for practical purposes such as an annoyance model. For if we would use the curves in Figure 1a, what would happen for an annoyance level of e.g. 4? It does not belong to any fuzzy set to a degree greater than 0, so how can we call it, how should we treat it?

On the other hand the functions in Figure 1a are more desirable from a linguistic point of view. The curves might in fact indicate that the number of terms taken into account is too small to cover the whole universe. Perhaps two more terms should be taken into consideration, namely one "concentrated" around 4, and one around 6.

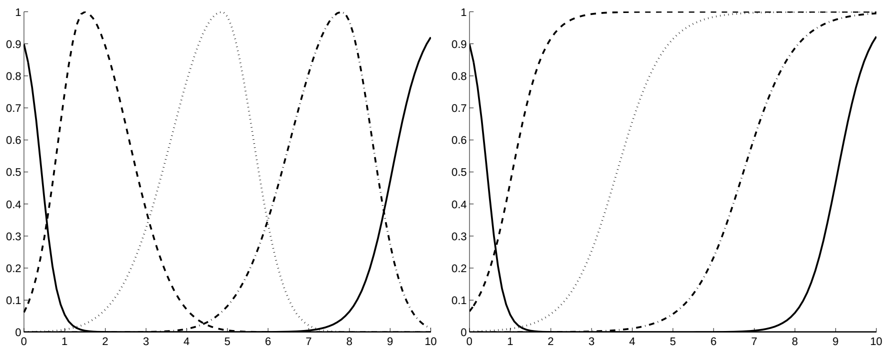

Figure 2: a) Non-inclusive (left) and b) inclusive (right) membership functions for *not at all, slightly, moderately, very* and *extremely annoyed* (aggregation)

	Non-inclusive			Inclusive	
	μ	σ	δ	μ	σ
Not at all	2,1832	5,0436	-	2,1832	5,0436
Slightly	1.4777	0.6261	1.0959	2,6666	2,5325
Moderately	4.8681	1.2452	0.7677	6,2700	1,7293
Very	7.8198	1.2588	0.7340	10,9270	1,6229
Extremely	23.1984	2.5647	-	23,6321	2,6111

Table 2: Numerical parameters for the fitted curves (aggregation)

7.2 Inclusive interpretation.

Like in the previous section for the inclusive interpretation we will assume that, by placing a mark x_m (m=2,..,5), the informant indicates that all levels greater than that mark are surely A_m (A_m to degree 1). Hence the only uncertainty is situated left from the mark x_m. As a membership function for A_m we will therefore use a sigmoidal curve $SIGM(x_m,\sigma_m,.)$, in which σ_m is determined by the distance between x_m and the previous mark x_{m-1}, as well as by a parameter α. For the term A_1 on the other hand we will use a reflected sigmoidal curve, with a flank determined by the distance between x_1 and x_2 and the parameter α.

Those results for the same parameter value α=0.1, again after fitting to a sigmoidal function, are shown in Figure 2b and given in Table 2.

8 Using Fuzzy Logical Operators and Modifiers

8.1 Logical operators

From inclusive to non-inclusive Please note that the membership functions for A_1 = not at all annoyed and A_5 = extremely annoyed in the inclusive and the non-inclusive interpretation are practically interchangeable. It is worth mentioning that also very acceptable membership functions for the terms A_2, A_3, and A_4 in the non-inclusive interpretation can be derived from those in the inclusive interpretation, using the fuzzy logical operations *not* and *but* (=*and*). E.g. the term *very annoyed* in the non-inclusive interpretation corresponds to *very annoyed but not extremely annoyed* in the inclusive interpretation. Formally - for m=2,..,4 -

$$(A_m)_{NI} = (A_m)_I \cap co((A_{m+1})_I)$$

in which NI stands for the non-inclusive and I for the inclusive interpretation.

Note that in the inclusive interpretation co(*slightly annoyed*) is also a possible membership function for *not at all annoyed*.

8.2 Fuzzy Modifiers

As briefly mentioned in Section 4, fuzzy modifiers (mappings from **F**(X) to **F**(X)) can be used to derive the membership function for "<adverb> A" from that of A. In the five terms of the scaling study, the base term *annoyed* is lacking. Nevertheless it is interesting to consider A_3 = *moderately annoyed* as base term instead, and to try to derive the membership functions for the terms A_2, A_4 and A_5 from A_3 (generated with a method described in Section 6 or 7), using fuzzy modifiers.

Powering modifiers

The oldest and most popular modifiers used in the inclusive interpretation are the powering modifiers P_α originally developed by Zadeh [18] and defined by (for α in $[0,+\infty[$): $P_\alpha(A)(x) = (A(x))^\alpha$, for all A in **F**(X), x in X. The most important shortcoming is that for all x in X, $P_\alpha(A)(x) = 1$ iff $A(x) = 1$ and that $P_\alpha(A)(x) = 0$ iff $A(x) = 0$. Looking at the Figures 1b and 2b one immediately sees that the intervals in which the inclusive membership functions are 0 (1 respectively) are usually different. Powering modifiers are therefore not really suitable.

Shifting modifiers

The shifting modifiers S_α, informally suggested by Lakoff [13] and more formally developed by Hellendoorn [6], Bouchon-Meunier [1] and Kerre [10], are defined by (for α in **R**): $S_\alpha(A)(x) = A(x-\alpha)$, for all A in **F**(**R**), x in **R**. They simply shift the original membership function of A to the left or the right (for a positive and negative α respectively) and can be used in both the inclusive and the non-inclusive interpretation. In Figures 3a and 3b the membership functions for A_2, A_3, A_4 and A_5 from Figures 1a and 1b are repeated. The dashed curves correspond to shifted versions of A_3 that are most suitable to represent the other terms. Although the shape of all curves in Figure 1b is not really the same, the functions obtained by shifting A_3 in Figure 3b are very good approximations. The same holds for the membership functions in Figures 2a and 2b. In Figure 3a however, we are not able to derive from the small curve for A_3 the wide functions for *slightly annoyed* and *very annoyed* simply by using a shifting operator.

Modifiers based on fuzzy relations

In [4] a new class of fuzzy modifiers is introduced. They are based on fuzzy relations R on X, i.e. fuzzy sets on X x X. For y in X, the R-foreset of y is denoted Ry and defined by Ry(x) = R(x,y), for all x in X. Furthermore the concepts of *degree of inclusion* and *degree of overlap* are used. For A and B two fuzzy sets on X are defined by

$$\mathrm{INCL}(A,B) = \inf\{\min(1-A(x)+B(x),1) \mid x \in X\}$$

$$\mathrm{OVERL}(A,B) = \sup\{\max(A(x)+B(x)-1,0) \mid x \in X\}$$

Inclusive interpretation. For the inclusive interpretation a resemblance relation E_1 is used, i.e. for all x and y in X, $E_1(x,y)$ is the degree to which x and y resemble to each other. Hence E_1y is the fuzzy set of objects resembling to y. The general idea is that an object y can be called *slightly* A if it resembles to an object that can be called A; in other words if the set of objects resembling to y overlaps with A (cfr. a man can be called *slightly old* if he resembles to somebody who is *old*). On the other hand y can be called *very* A if every object resembling to y can be called A; in other words if the set of objects resembling y is included in A (cfr. a man can be called *very old* if everybody whom he resembles to is *old*). Formally and fuzzy:

$$\textit{slightly}\ A(y) = \mathrm{OVERL}(E_1y,A)$$

$$\textit{very}\ A(y) = \mathrm{INCL}(E_1y,A)$$

Extremely A is modelled in a similar, but with a looser resemblance relation E_2 (i.e. $E_1 \subseteq E_2$):

$$\textit{extremely}\ A(y) = \mathrm{INCL}(E_2y,A)$$

Following this scheme and using the resemblance relations

$$E_1(x,y) = \min(1, BELL(x,1.5,1.5,y)*10)$$
$$E_2(x,y) = \min(1, BELL(x,2.5,2.5,y)*10)$$

the dotted membership functions in Figure 3b were obtained.

Non-inclusive interpretation. As stated above, shifting modifiers work well for Figure 1b, 2a, and 2b. It is explained in [4] that S_α is actually a fuzzy modifier based on the fuzzy relation G_α defined by

$$G_\alpha(x,y) \begin{matrix} = 1 & \textit{if } x = y - \alpha \\ = 0 & \textit{otherwise} \end{matrix}$$

namely $S_\alpha(A)(y) = OVERL(G_\alpha y, A)$. G_α is based on the crisp equality between x and y - α. If this equality is fuzzified by means of a resemblance relation E, a more general kind of relation F_α arises, defined by $F_\alpha(x,y) = E(x,y - \alpha)$. The fuzzy modifier built on this relation, namely $WS_\alpha(A)(y) = OVERL(F_\alpha y, A)$ does not only have a shifting but also a widening effect on the membership function of A. Hence it can be applied to A_3 to obtain approximations for the curves in Figure 1a. Using the resemblance relation $E(x,y) = BELL(x,0.7,0.7,y)$ the dotted curves in Figure 3a were obtained. The α-values were chosen as –3.7, 3, and 5.2 respectively. Note that the obtained membership function for A_2 even coincides in the picture with the original membership function generated with the probabilistic approach; therefore the dotted curve for A_2 is not visible.

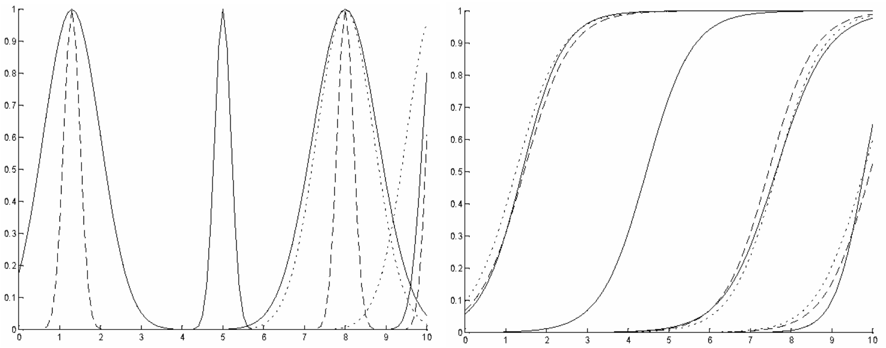

Figure 3: a) Non-inclusive (left) and b) inclusive (right) membership functions (fuzzy modifiers)

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