

## CEE 483 Winter 2008 Midterm #2 solutions

1. The dimensionless concentration in the equilibrium constant expression is based on the concentration *in the phase where the constituent occurs*. For solids, the normalized concentration is expressed as a mole fraction. Pure, solid  $\text{CaCO}_3(s)$  is made up entirely of  $\text{CaCO}_3$  molecules, so the mole fraction of  $\text{CaCO}_3$  in the solid is 100%, or 1.0, and that is the value used for the dimensionless concentration.

2. Particles are rejected by porous membrane when a portion of the solid collides with the edge of the pore. Water can enter the pore across the whole pore cross-section, but if a particle's center approaches the pore within a distance  $r_p$  of the pore edge, a part of the particle will collide with the edge of the pore, preventing the particle from entering.

3. RO membranes are not considered to have discrete pores. Rather, the membrane is viewed as a continuous phase that is different from the water phase on either side. When molecules reach the membrane, some of them dissolve into the membrane and migrate across it by diffusion. The rejection of salt molecules is therefore attributed to a lower solubility of those molecules in the RO phase, and/or a lower rate of diffusion through the phase, when compared to water, not to their size relative to water molecules.

4. Struvite will be supersaturated when  $Q_{so}$ , represented by the product  $(\text{Mg}^{2+})(\text{NH}_4^+)(\text{PO}_4^{3-})$  is just barely greater than  $K_{so}$ . Since the three species in this product are all dissolved, the calculation uses their molar concentrations. According to the graph, at pH 9.0,  $\text{NH}_4^+$  accounts for 61% of  $TOTN$ . We are told that  $\text{Mg}^{2+}$  always represents 100% of  $TOTMg$ , so the  $\text{PO}_4^{3-}$  concentration when the solid is saturated is:

$$(\text{PO}_4^{3-}) = \frac{K_{so}}{(\text{Mg}^{2+})(\text{NH}_4^+)} = \frac{10^{-12.6}}{(5 \times 10^{-3})(0.61 \times 6 \times 10^{-4})} = 1.37 \times 10^{-7}$$

The concentration of the other species containing  $\text{PO}_4$  can be computed from the  $K_a$  values:

$$(\text{HPO}_4^{2-}) = \frac{(\text{H}^+)(\text{PO}_4^{3-})}{K_{a3}} = \frac{(10^{-9.0})(1.37 \times 10^{-7})}{10^{-12.2}} = 2.18 \times 10^{-4}$$

$$(\text{H}_2\text{PO}_4^-) = \frac{(\text{H}^+)(\text{HPO}_4^{2-})}{K_{a2}} = \frac{(10^{-9.0})(2.18 \times 10^{-4})}{10^{-7.2}} = 3.45 \times 10^{-6}$$

$$(\text{H}_3\text{PO}_4) = \frac{(\text{H}^+)(\text{H}_2\text{PO}_4^-)}{K_{a1}} = \frac{(10^{-9.0})(3.45 \times 10^{-6})}{10^{-2.2}} = 5.46 \times 10^{-13}$$

Finally, we can compute  $TOTPO_4$  as the sum of the species containing  $PO_4$ :

$$\begin{aligned}TOTPO_4 &= (H_3PO_4) + (H_2PO_4^-) + (HPO_4^{2-}) + (PO_4^{3-}) \\ &= 5.46 \times 10^{-13} + 3.45 \times 10^{-6} + 2.18 \times 10^{-4} + 1.37 \times 10^{-7} = 2.21 \times 10^{-4}\end{aligned}$$

Thus, struvite will be supersaturated when  $TOTPO_4$  is greater than  $2.21 \times 10^{-4}$  mol/L.

5. The total permeate flow is the product of the flux ( $J$ ) and the membrane area ( $A$ ):

$$Q_p = JA = \left(65 \frac{L}{m^2 \cdot h}\right) (200 m^2) = 13,000 \frac{L}{h}$$

The recovery is the ratio of  $Q_p$  to the feed flow rate,  $Q_f$ , and is given as 85%, so  $Q_f$  is:

$$Q_f = \frac{Q_p}{\text{recovery}} = \frac{13,000 L/h}{0.85} = 15,290 \frac{L}{h}$$

The flow rate of the concentrate is therefore:

$$Q_c = Q_f - Q_p = (15,290 - 13,000) \frac{L}{h} = 2,290 \frac{L}{h}$$

The concentrations in the permeate and the feed are related by the rejection, so:

$$\text{Rejection} = 1 - \frac{c_p}{c_f}$$

$$c_p = c_f (1 - \text{Rejection}) = \left(0.2 \frac{\text{mol}}{L}\right) (1 - 0.9) = 0.02 \frac{\text{mol}}{L}$$

Finally, the concentration in the concentrate stream can be found by a mass balance:

$$Q_f c_f = Q_p c_p + Q_c c_c$$

$$c_c = \frac{Q_f c_f - Q_p c_p}{Q_c} = \frac{\left(15,290 \frac{L}{h}\right) \left(0.2 \frac{\text{mol}}{L}\right) - \left(2,290 \frac{L}{h}\right) \left(0.02 \frac{\text{mol}}{L}\right)}{13,000 \frac{L}{h}} = 1.22 \frac{\text{mol}}{L}$$

6. The effective concentration of water on the feed side of the membrane is:

$$c_{eff,w} = c_w \exp\left(\frac{\bar{V}_w \Delta P}{RT}\right) = \left(55.1 \frac{\text{mol}}{\text{L}}\right) \exp\left(\frac{[0.018 \text{ L/mol}][10 \text{ atm}]}{24.47 \text{ L-atm/mol}}\right) = 55.4 \frac{\text{mol}}{\text{L}}$$

The effective concentration of water on the permeate side is the same as its actual concentration (i.e., 55.5 mol/L), since the pressure on that side is atmospheric. Therefore, the effective concentration on the permeate side is greater than that on the feed side, and water will diffuse from the permeate toward the feed, i.e., in the direction of normal (not reverse) osmosis.