CEE 483, Winter 2009, Exam 1 Solutions

1. (a) The mass balance for chloride in the CSTR is:

$$V\frac{dc}{dt} = Q\left(c_{\text{in}} - c_{\text{out}}\right) + \cancel{Vr}$$

where the Vr term is zero because chloride is a tracer. Since the reactor is a CSTR, $c = c_{\text{out}}$. Making that change and rearranging, we find:

$$V\frac{dc}{dt} = Q\left(c_{\text{in}} - c\right)$$

$$\int_{c(0)}^{c(t)} \frac{dc}{c_{\text{in}} - c} = \frac{Q}{V}\int_0^t dt$$

$$\int_{c(0)}^{c(t)} \frac{dc}{c - c_{\text{in}}} = -\frac{Q}{V}\int_0^t dt$$

Q/V is $1/\tau$. Therefore, the above equation can be integrated as follows:

$$\ln\frac{c(t) - c_{\text{in}}}{c(0) - c_{\text{in}}} = -\frac{t}{\tau_{\text{CSTR}}}$$

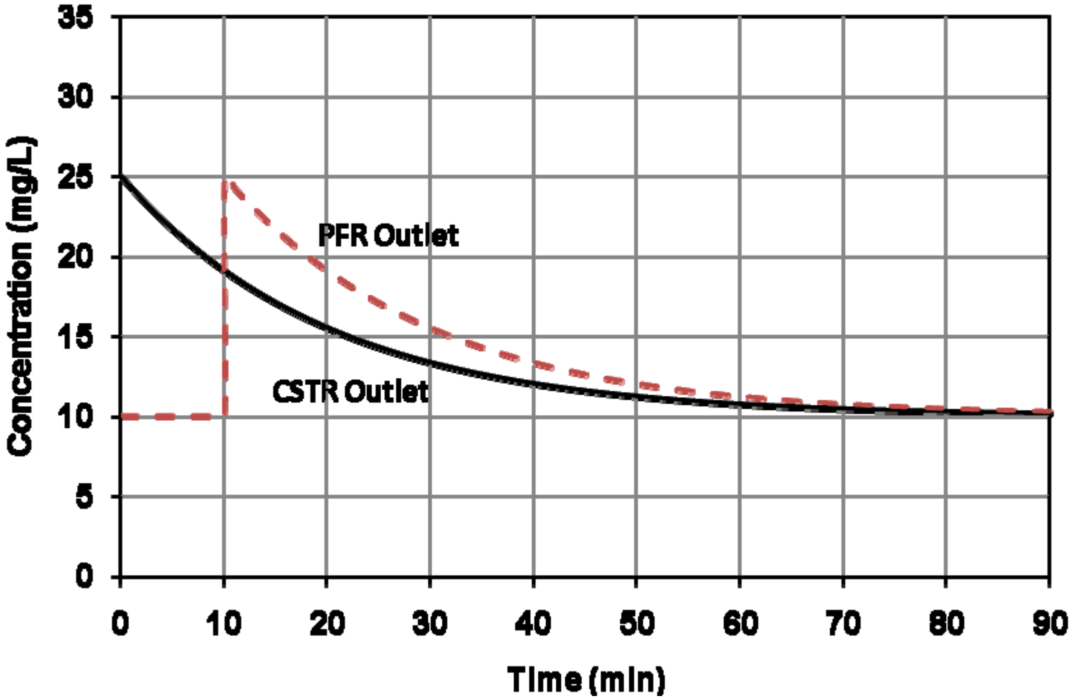
$$\frac{c(t) - c_{\text{in}}}{c(0) - c_{\text{in}}} = \exp\left(-\frac{t}{\tau_{\text{CSTR}}}\right)$$

$$c(t) = c_{in} + \left(c(0) - c_{in}\right)\exp\left(-\frac{t}{\tau_{\text{CSTR}}}\right)$$

Inserting the given values of $\tau_{\text{CSTR}} = 20$ min, $c(0) = 25$ mg/L, and $c_{in} = 10$ mg/L, we find:

$$\begin{aligned} c(t) &= 10\frac{\text{mg}}{\text{L}} + \left([25-10]\frac{\text{mg}}{\text{L}}\right)\exp\left(-\frac{t}{20\text{ min}}\right) \\ &= 10\frac{\text{mg}}{\text{L}} + \left(15\frac{\text{mg}}{\text{L}}\right)\exp\left(-\frac{t}{20\text{ min}}\right) \end{aligned}$$

This equation gives values of $c(0) = 25$ mg/L, $c(\infty) = 10$ mg/L, and an exponential decay between those two time limits. The function is shown as the solid curve on the graph below. A qualitative explanation that the concentration would have to decay smoothly from 25 to 10 mg/L, and that washout of a tracer from a CSTR typically follows an exponential decay curve with a decay constant of τ_{CSTR}, is also be an acceptable explanation. However, the equation for c as a function of t is needed for part c of the problem, so if it is not derived here, it has to be derived later.



(b) The chloride concentration that enters the PFR exits it 10 min later. Since there is no mixing and no reaction in the PFR, the chloride concentration curve exiting the PFR is identical to what entered the PFR (i.e., what exited the CSTR), delayed by 10 min; this curve is also shown on the graph (the broken line).

(c) The rate of change of the chloride concentration is *dc*/*dt*. According to the mass balance in part *a*, this rate can be computed as:

$$\frac{dc}{dt}=\frac{Q}{V}\left(c_{\text{in}}-c\right)=\frac{1}{\tau_{\text{CSTR}}}\left(c_{\text{in}}-c\right)$$

The value of c_{CSTR} at $t=20$ min can also be determined using the equation developed in part *a*:

$$c_{20\text{ min}}=10\frac{\text{mg}}{\text{L}}+\left(15\frac{\text{mg}}{\text{L}}\right)\exp\left(-\frac{20\text{ min}}{20\text{ min}}\right)=15.5\frac{\text{mg}}{\text{L}}$$

Then, substituting known values into the equation for *dc*/*dt*, we find:

$$\frac{dc}{dt}=\frac{1}{20\text{ min}}\left(10\frac{\text{mg}}{\text{L}}-15.5\frac{\text{mg}}{\text{L}}\right)=-0.275\frac{\text{mg/L}}{\text{min}}$$

2. (a) 100% of the particles settle at least 0.2 m in two hours. Therefore no particle in the suspension settles more slowly than 0.1 m/h.

(b) The overflow rate is the ratio of the flow rate to the surface area, so:

$$\text{O/F} = \frac{Q}{A} = \frac{150\ \text{m}^3/\text{h}}{\left(15\text{m}\right)\left(6\text{m}\right)} = 1.67\frac{\text{m}}{\text{h}}$$

(c) A settling velocity of 1.3 cm/min corresponds to 78 cm/h or 0.78 m/h. The overflow rate, determined in part *a*, is also the critical velocity, v_{crit}. Thus, the particles of interest have settling velocities less than the critical velocity, and they are therefore removed with an efficiency equal to v/v_{crit}. This ratio is:

$$\eta = \frac{v}{v_{crit}} = \frac{0.78\ \text{cm/min}}{1.67\ \text{cm/min}} = 0.47$$

3. The author of the article claimed that mixing the coagulant into the feed in a PFR rather than a CSTR reduced energy consumption and also reduced the required coagulant dose.

The author’s claim is that there is really no need to “backmix” the water (i.e., to mix water that has passed the coagulant addition point with water that has not yet reached that point. That is the kind of mixing that occurs in a CSTR. Rather, he points out that the only requirement is that the coagulant be mixed uniformly into all the water that passes the addition point. He says that this can be done efficiently with different types of mixing devices. In the conventional (CSTR) approach, water that has been dosed with coagulant can be transported upstream and dosed again, which is inefficient.

4. The larger dose leads to precipitation of $Al(OH)_3(s)$ or $Fe(OH)_3(s)$ and sweep flocculation of the particles. Although more sludge is generated, this approach is more "forgiving" in the sense that it is impossible to overdose the system. Thus, if the influent water quality fluctuates unpredictably, the overall process might work more reliably if the higher dose is used.

5. Three mechanisms of flocculation are usually considered: differential shear, differential sedimentation, and Brownian motion. Since the two particles of interest are of identical size and density, they will settle at the same rate, thereby eliminating differential settling as a potential flocculation mechanism. Brownian motion is significant only for particles that are very small – usually a few microns or smaller – so that is also unlikely to be an effective flocculation mechanism for these particles. Thus, we expect differential shear to be dominant.

6. The particles that have already settled out are likely to be larger and/or more dense than those in the influent. In addition, the concentration of particles in the influent might be so low that, even though they are sticky, they don't collide with each other often enough to generate large

flocs. When the settled flocs are mixed with the new particles entering the flocculation tank, more opportunities arise for collisions, and the collisions that occur generate bigger and heavier particles. These advantages sometimes outweigh the potential disadvantage of putting particles back in suspension that have already been removed.