CEE 483, Winter 2009, Homework #4 Solutions

1. The filter is to be operated at a loading rate of 120 m^3/m^2-d, meaning that 120 m^3 of water is applied to each m^2 of surface area atop the filter each day. The depth of the filter is 0.60 m, so 120 m^3 of water is applied to each 0.60 m^3 of filter bed per day, corresponding to a ratio of 200 m^3 of water per m^3 of bed per day. In 20 hours, the total water application is therefore (20/24)*200, or 167 m^3 per m^3 of bed.

According to the problem statement, the influent contains 1.0 mg/L of solids, with a density of $\rho_{\text{solids}} = 1.2\ \text{g/cm}^3 = 1200\ \text{mg/cm}^3$. The volume of solids in each liter of influent water is therefore:

$$\frac{\text{Volume Solids}}{\text{Volume Water}} = \frac{c_{\text{solids}}}{\rho_{\text{solids}}} = \frac{1.0\ \text{mg/L}}{1200\ \text{mg/cm}^3} = 8.33\text{x}10^{-4}\ \frac{\text{cm}^3\ \text{solids}}{\text{L}}$$

The volume of solids in 167 m^3 of water is therefore:

$$\frac{\text{Volume Solids}}{\text{m}^3\ \text{bed - cycle}} = \left(8.33\text{x}10^{-4}\ \frac{\text{cm}^3\ \text{solids}}{\text{L water}}\right)\left(\frac{167\ \text{m}^3\ \text{water}}{\text{m}^3\ \text{bed - cycle}}\right)\left(\frac{1000\ \text{L water}}{\text{m}^3\ \text{water}}\right)$$

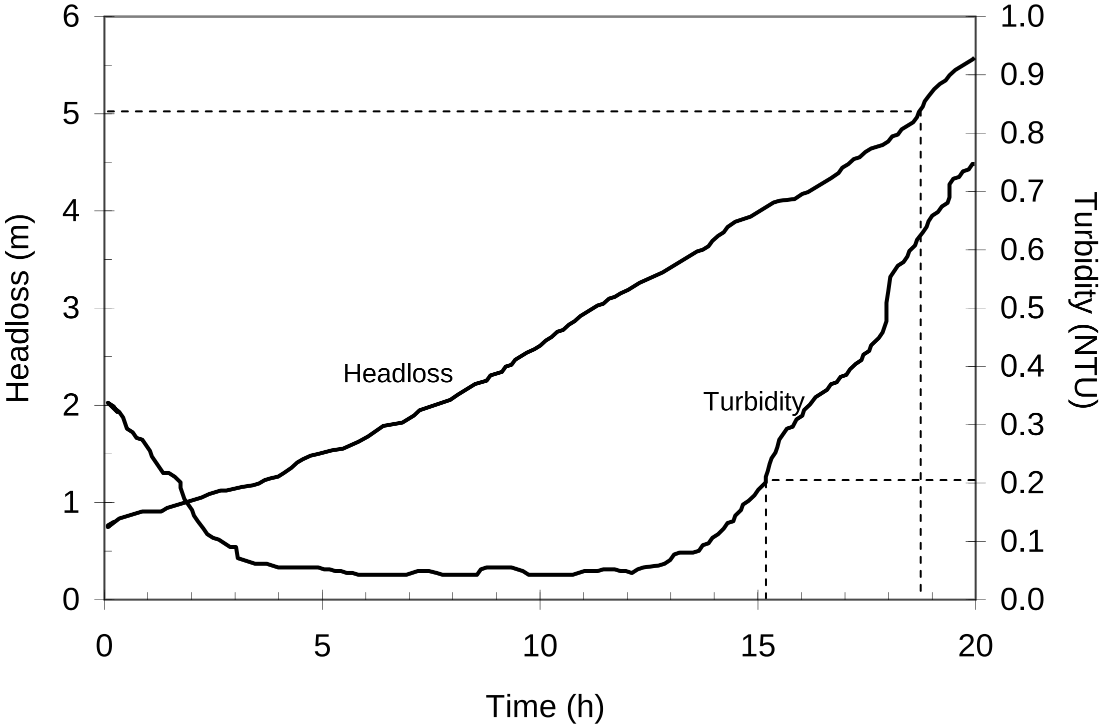
$$= 139\ \frac{\text{cm}^3\ \text{solids}}{\text{m}^3\ \text{bed - cycle}}$$

One cubic meter of bed has a void volume of 0.39 m^3, so the fraction of that volume filled by the particles in one cycle is:

$$\frac{\text{Volume solids}}{\text{Volume voids}} = \left(139\ \frac{\text{cm}^3\ \text{solids}}{\text{m}^3\ \text{bed}}\right)\frac{1.0\ \text{m}^3\ \text{bed}}{0.39\ \text{m}^3\ \text{void}} = 357\ \frac{\text{cm}^3\ \text{solids}}{\text{m}^3\ \text{void space}}$$

Since 1 $m^3 = 10^6$ cm^3, the fraction of the void space occupied by the solids is only 357×10^{-6}, or 0.0357%. Even if all of the solids accumulated in the top 1% of the bed (i.e., the top 6 mm), the fraction of the void space that was filled would be only 3.57%. Clearly, the idea that the filter gets “clogged” by the removed material is misleading; the buildup of headloss is due to increased friction associated with flow around the accumulated particles, not narrowing of the spaces between adjacent grains.

2. (a) The filter run will have to be stopped after ~15.2 hours, when the turbidity criterion is met. At this time, the headloss is below its allowable value; headloss does not interfere with the ability to continue the run until approximately 18.8 hours into the run.

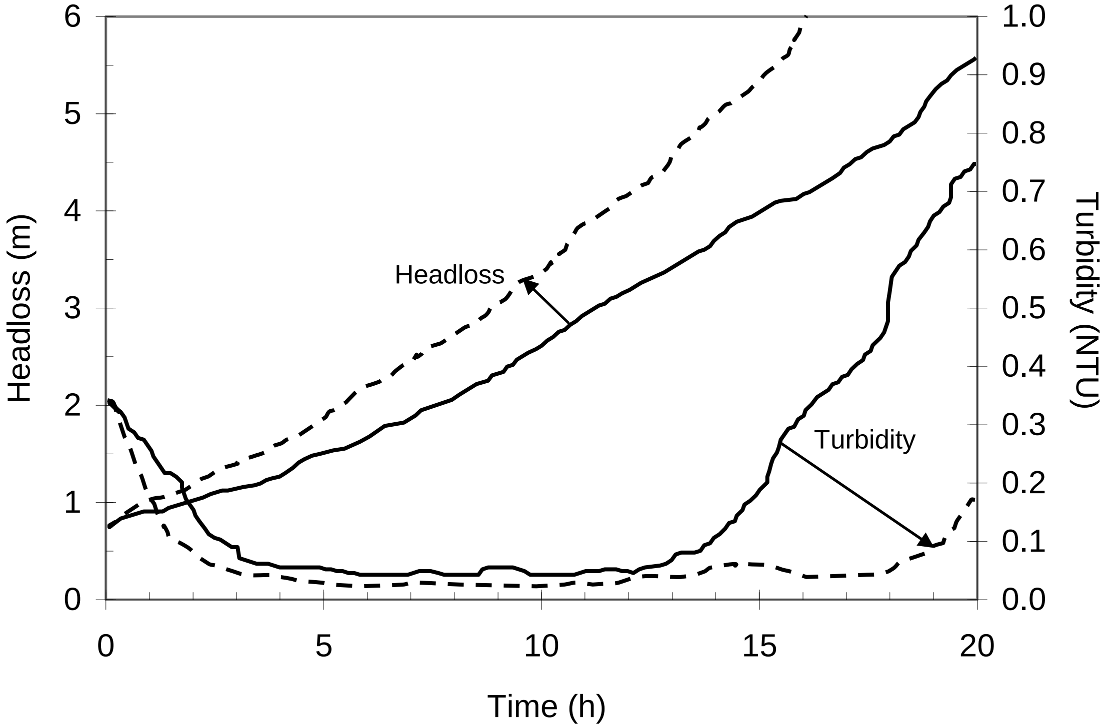


(b) (i) Adding coagulant would improve the quality of the effluent, while causing more removal in the upper portion of the filter and increased headloss. Since the run length is currently being limited by breakthrough of turbidity, a small addition of coagulant would increase the run length compared to the current operating conditions. Therefore, coagulant addition would be a good idea.

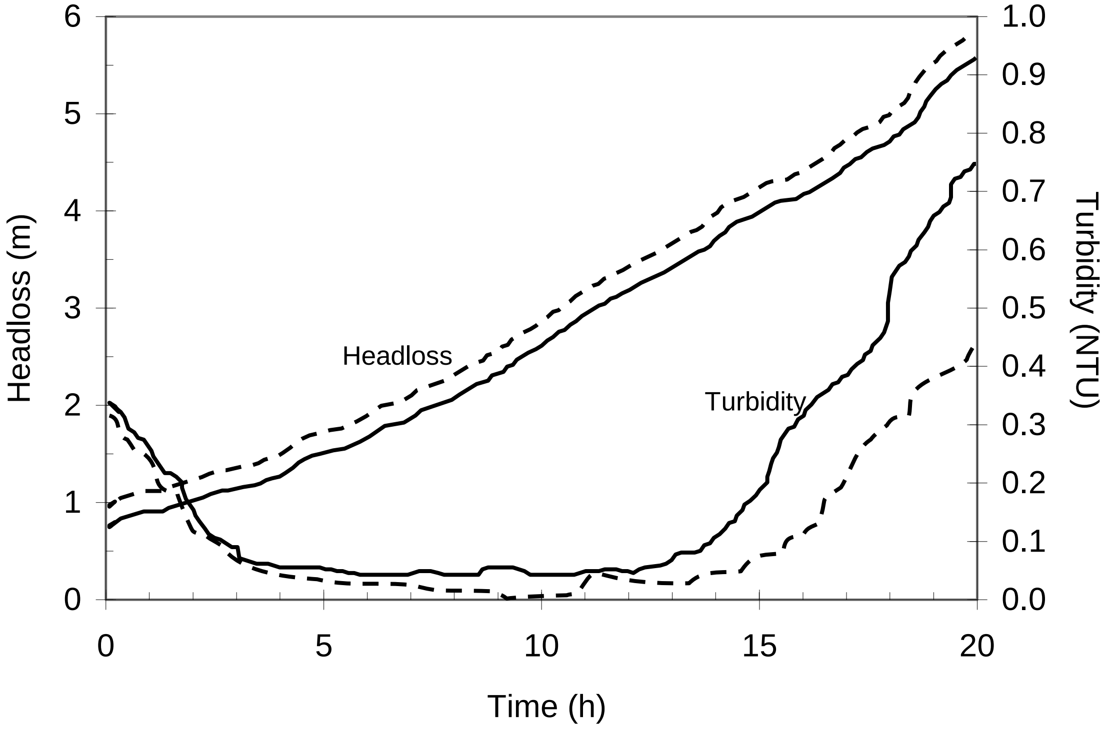
(ii) Like adding coagulant, increasing the depth of sand in the filter will improve the quality of the effluent at the expense of more headloss. As in part *i*, it would lengthen the filter run and be a good decision.

(iii) Using larger grain sand would increase the pore size and therefore trap less material near the top of the filter. This would decrease the headloss, and would probably increase the effluent turbidity. This would be a poor operating decision, since it would decrease the length of the filter run.

(c) Coagulant addition would increase the rate of headloss buildup (but not affect the initial headloss) and would delay particle breakthrough. Plausible changes in the development of these two parameters are shown in the figure below.



(d) If 15% more sand were added to the bed, the initial headloss would increase by approximately 15%. Thereafter, the rate of headloss buildup would be about the same as under the current operating conditions. Thus, the headloss curve would be approximately parallel to the current one, but higher by a fixed amount equal to 15% of the initial headloss, *i.e.*, by approximately 0.12 m. The added capacity of the bed to hold particles would also increase by approximately 15%. Therefore, the time at which the effluent turbidity started increasing would shift from $t \approx 14$ h to $t \approx 16$ h. Although the increased bed length would lead to slightly greater particle removal, during most of the run the vast majority of the turbidity is already being removed, so chances are that the incremental increase in removal would have a negligible effect on headloss. The predicted behavior is shown in the graph below.



3. The single collector efficiencies due to gravity (sedimentation), interception, and Brownian motion can be computed using the following expressions:

$$\eta_{\mathrm{G}} = \frac{\left(\rho_{\mathrm{p}} - \rho_{\mathrm{w}}\right) g d_{\mathrm{p}}^{\,2}}{18\mu v}; \quad \eta_{\mathrm{I}} = \frac{3}{2}\left(\frac{d_{\mathrm{p}}}{d_{\mathrm{c}}}\right)^2; \quad \eta_{\mathrm{Br}} = 0.9\left(\frac{k_B T}{\mu d_{\mathrm{p}} d_{\mathrm{c}} v}\right)^{2/3}$$

Once the single collector efficiency by each removal mechanism is found, we treat them as additive, and then can compute the overall “filter coefficient” (λ) as follows:

$$\eta_{\mathrm{tot}} = \eta_{\mathrm{G}} + \eta_{\mathrm{I}} + \eta_{\mathrm{Br}}; \qquad \lambda = \frac{3}{2}\frac{\left(1-\varepsilon\right)\alpha\eta_{\mathrm{tot}}}{d_{\mathrm{c}}}$$

Finally, the fraction of the particles of a particular type that remain in the filter at a depth *x* is computed as:

$$N\left(x\right)/N_{\mathrm{o}} = \exp\left(-\lambda x\right)$$

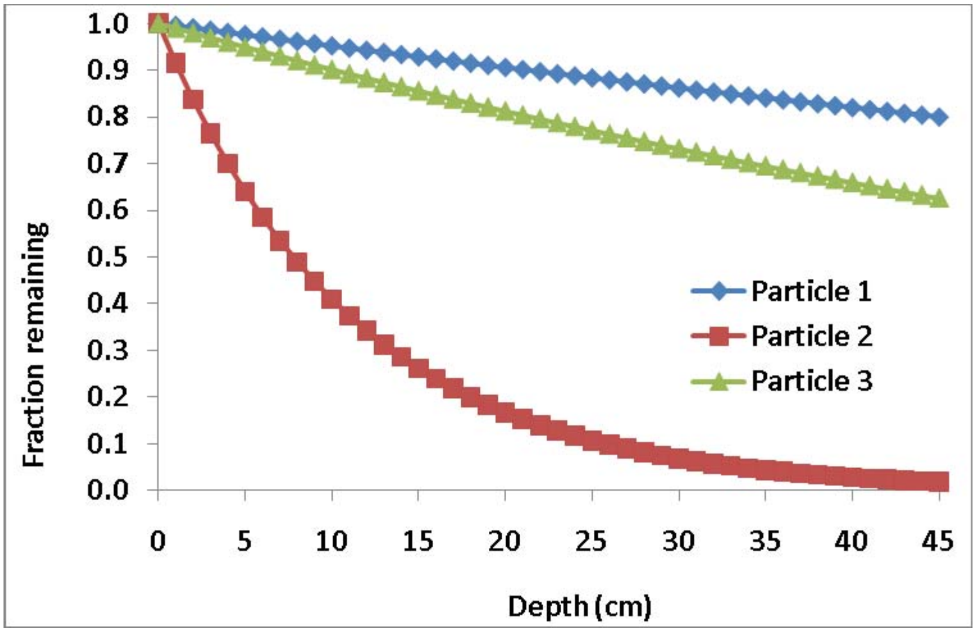
The values of all the relevant parameters for each type of particle are summarized in the following table, and the plot of fraction remaining as a function of depth is shown below.

Parameters applicable to all the particles

k_B:	1.38×10^{-16} g-cm^2/(s^2-K)	ε:	0.4	v:	0.25 cm/s
T:	293 K	d_c:	0.05 cm	g:	981 cm/s^2
L:	45 cm	α:	1.0		
μ:	1.00×10^{-2} g/cm-s	ρ_w:	1.0 g/cm^3		

Values applicable to individual particles

	Particle 1	***Particle 2***	***Particle 3***
ρ_p	1.0 g/cm^3	1.6 g/cm^3	1.6 g/cm^3
d_p	6.0×10^{-4} cm	6.0×10^{-4} cm	2.0×10^{-5} cm
η_G	0.00	4.70×10^{-3}	5.22×10^{-6}
η_I	2.16×10^{-4}	2.16×10^{-4}	2.40×10^{-7}
η_{Br}	5.93×10^{-5}	5.93×10^{-5}	5.73×10^{-4}
η_{tot}	2.75×10^{-4}	4.97×10^{-3}	5.78×10^{-4}
λ	4.95×10^{-3} cm^{-1}	8.95×10^{-2} cm^{-1}	1.04×10^{-2} cm^{-1}



Type 1 particles are not removed by sedimentation at all, since they are neutrally buoyant. They are moderately large, so their removal by Brownian motion is also not very substantial, but they are not so large that their removal by interception is significant, so overall their removal is not great.

Type 2 particles are the same size as type 1 particles, so their removal by interception and by Brownian motion is the same as for type 1 particles. However, type 2 particles are substantially more dense than type 1 particles, so the type 2 particles are removed significantly by sedimentation. In fact, that mechanism turns out to be the dominant one accounting for their removal, and <2% of the type 2 particles in the influent remain in the effluent.

Type 3 particles are just as dense as type 2 particles, but the type 3 particles are only 1/30 as large, so their removal by Brownian motion is more substantial, and that by sedimentation is less so. The net result is that the removal efficiency for type 3 particles is intermediate between that for types 1 and 3 – approximately 37% of the type 3 particles are removed in the filter.