CEE 483 Winter 2009 HW#2 Solutions

1. We can solve this problem by imagining that the stormwater contains a tracer at a concentration of, say, 10 mg/L. We can then determine how much tracer enters during the storm and how much remains in the pond at the end of the storm. (Note that this question differs somewhat from the calculation in the Mass Balances handout. The calculation in the notes addresses the question of what fraction of the water that was *originally* in the control volume [the retention basin, in this case] is still there after some amount of time, whereas this question asks about the fraction of the *incoming* water that is still in the basin.)

The mass of (hypothetical) tracer entering the basin during the storm is 10,000 m^3 x 10 mg/L, which equals 100 kg. The mass balance on the tracer in the basin is as follows:

$$V\frac{dc}{dt} = Qc_{in} - Qc_{out} + \cancel{rV} \qquad (1)$$

Because the basin is well mixed, the concentration exiting the pond (c_{out}) is the same as the concentration in it (c), so the mass balance can be rearranged and integrated as follows:

$$\int_{c(0)}^{c(t)} \frac{dc}{c_{in} - c} = \frac{1}{t_d}\int_0^t dt = \frac{t}{t_d} \qquad (2)$$

In this case, the initial tracer concentration in the pond is zero, and tracer is assumed to enter continuously thereafter at $c_{in} = 10$ mg/L. Inserting that information, we find:

$$\int_0^{c(t)} \frac{dc}{10\frac{\text{mg}}{\text{L}} - c} = \frac{t}{t_d} \qquad (3)$$

$$-\ln \frac{10\frac{\text{mg}}{\text{L}} - c(t)}{10\frac{\text{mg}}{\text{L}} - \cancel{c(0)}} = \frac{t}{t_d}$$

$$c(t) = \left(10\frac{\text{mg}}{\text{L}}\right)\left[1 - \exp\left(-\frac{t}{t_d}\right)\right] \qquad (4)$$

The hydraulic residence time in the pond is $V/Q = 15000$ m^3/2500 m^3/h, or 6.0 h. At $t = 4$ h, we find $c(t) = 4.87$ mg/L. The total mass of tracer in the pond is thus (4.87 mg/L)(1500 m^3), or 73.0 kg. Thus, 73% of the tracer, and therefore 73% of the water, that entered during the storm is still in the pond at the end of the storm. Correspondingly, 27% of the water has exited.

2. (a) A mass balance on the toxicant in the absence of reaction is:

$$V\frac{dc}{dt} = Qc_{in} - Qc_{out}$$

Since the flocculation basin is being treated as a CMR, we can equate c inside the reactor (the term on the left of the mass balance equation) with c_{out}, the concentration exiting the reactor. Doing this, dividing through by Q, and rearranging, we obtain:

$$\frac{V}{Q}\int \frac{dc}{c_{in} - c} = \int dt$$

Noting that V/Q is the hydraulic residence time t_d (= 2 h), and inserting the relevant limits of integration and the known values of c_{in} (= 0.1 mg/L) and $c_{t=0}$ (= 0 mg/L), we find:

$$\int_{c(0)}^{c(3)} \frac{dc}{c_{in} - c} = \frac{1}{t_d}\int_0^3 dt$$

$$-\ln\frac{c_{in} - c_{t=3\text{h}}}{c_{in} - c_{t=0\text{h}}} = \frac{1}{t_d}(3\text{h} - 0\text{h})$$

$$\frac{0.1\frac{\text{mg}}{\text{L}} - c_{t=3\text{h}}}{(0.1-0)\frac{\text{mg}}{\text{L}}} = \exp\left(-\frac{3\text{h}}{2\text{h}}\right)$$

$$c_{t=3\text{h}} = \left(0.1\frac{\text{mg}}{\text{L}}\right)\left(1 - \exp\left(-\frac{3}{2}\right)\right) = 0.0777\frac{\text{mg}}{\text{L}}$$

(b) This problem is slightly different from the ones we have considered previously, in that it involves a reaction in a reactor with flow that is not at steady state. Nevertheless, we can solve it using the same approach we have used for other systems where the steady-state simplification applied. Writing a mass balance on the toxicant in this system, we have:

$$V\frac{dc}{dt} = Qc_{in} - Qc_{out} + rV$$

where r is the rate at which the reaction is *forming* the toxicant in the reactor. Again, since we are treating the flocculation basin as a CMR, the concentration inside the basin is the same as that exiting, and that also is the concentration that controls the rate of reaction at all locations in the reactor (i.e., it is the concentration that is used in the rate expression). Substituting c for c_{out} and $-kc$ for r, and dividing through by Q, we obtain:

$$V\frac{dc}{dt} = Qc_{in} - Qc - kcV$$

$$t_d \frac{dc}{dt} = c_{in} - c - kct_d$$

$$\int_{c_{init}}^{c_t} \frac{dc}{c_{in} - \left(1 + kt_d\right)c} = \frac{1}{t_d}\int_0^t dt$$

$$-\frac{1}{k}\ln\frac{c_{in} \ - \left(1 + kt_d\right)c_t}{c_{in} \ - \left(1 + kt_d\right)c_{init}} = \frac{t}{t_d}$$

The sum $1 + kt_d$ equals 1.7. Substituting this and the other known values into the above equation yields:

$$-\frac{1}{1.7}\ln\frac{0.1 \text{ mg/L} \ - 1.7c_{t=3\text{h}}}{0.1 \text{ mg/L} \ - \left(1.7\right)\left(0 \text{ mg/L}\right)} = \frac{3\text{h}}{2\text{h}}$$

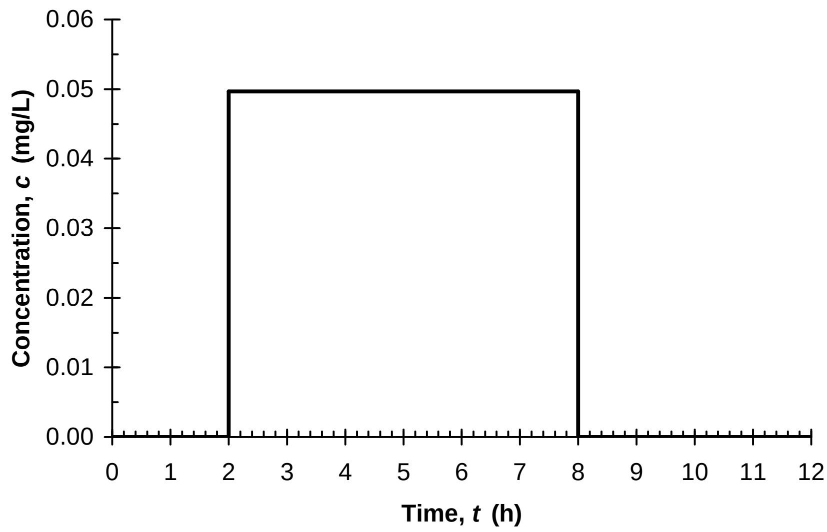
$$c_{t=3\text{h}} = \frac{0.1 \text{ mg/L}}{1.7}\left(1 - \exp\left(-\left[1.7\right]\frac{3}{2}\right)\right) = 0.0542 \text{ mg/L}$$

(c) If the flocculation basin operated as a PFR, none of the toxicant would arrive at the effluent for 2 hours, so c_{tox} would be zero until $t = 2$ h. Immediately thereafter, the toxicant concentration would jump up to the value that corresponded to two hours of reaction in a batch reactor, for the given influent concentration and reaction rate. Substituting the known values into the expression for the extent of reaction of a substance undergoing a first-order reaction in a PFR or batch reactor, we obtain:

$$c = c_{in} \exp\left(-kt_d\right)$$

$$= 0.1\exp\left(-0.35 * 2\right) = 0.050 \text{ mg/L}$$

This concentration would persist in the reactor for six hours (until $t = 8$ h), at which time the concentration would drop to zero again. The c vs. t plot is shown below.



3. (a) The reaction for the formation of $Al(OH)_3(s)$ from alum is:

$$Al_2(SO_4)_3 \cdot 14\ H_2O + 6\ OH^- \rightarrow 2\ Al(OH)_3(s) + 3\ SO_4^{2-} + 18\ H_2O$$

The reaction could be written in a few different ways, but the important relationship is that two moles of $Al(OH)_3(s)$ are created per mole of alum added. The daily alum dose is:

$$(30\ \text{mg/L})(7500\ \text{m}^3/\text{d})(10^3\ \text{L/m}^3)(1\text{kg}/10^6\ \text{mg}) = 225\ \text{kg alum/d}$$

The molecular weight of alum is 594, and that of $Al(OH)_3(s)$ is 78, so if all the Al precipitates as $Al(OH)_3(s)$, the amount of $Al(OH)_3(s)$ formed is:

$$\left(2\frac{\text{mol Al(OH)}_3}{\text{mol Alum}}\right)\left(30\frac{\text{mg Alum}}{\text{L}}\right)\left(\frac{1\text{ mol Alum}}{594,000\text{ mg Alum}}\right)\left(\frac{78,000\text{ mg Al(OH)}_3}{\text{mole Al(OH)}_3}\right)$$

$$= 7.9\ \text{mg Al(OH)}_3(s)/\text{L}$$

The total concentration of solids in the water is 7.9 mg $Al(OH)_3$/L + 9 mg/L solids in water initially, or 16.9 mg/L total suspended solids. The daily sludge production is:

$$\left(16.9\frac{\text{mg solids}}{\text{L}}\right)\left(10^3\frac{\text{L}}{\text{m}^3}\right)\left(7500\frac{\text{m}^3}{\text{d}}\right)\left(\frac{1\text{ kg}}{10^6\text{ mg}}\right) = 126.7\frac{\text{kg}}{\text{d}}$$