CEE 483, Win 2009. Solutions to HW#1

Note: In these answers, the mean hydraulic detention time is represented as t_d.

1. (a) When a first-order reaction is occurring in a PFR at steady state, the extent of reaction is given by:

$$c_{out} = c_{in} \exp\left(-k_1 t_d\right)$$

$$\ln \frac{c_{out}}{c_{in}} = -k_1 t_d$$

We are interested in finding the time required to achieve 90% destruction of the contaminant, corresponding to $c_{out} = 0.1\, c_{in}$. Substituting in this value:

$$\ln \frac{0.10\ c_{in}}{c_{in}} = \ln\left(0.10\right) = -2.30$$

The rate constant for the decay reaction is given as $0.05\,\text{d}^{-1}$, so:

$$t_d = \frac{-2.30}{-0.05\ \text{d}^{-1}} = 46.1\ \text{d}$$

The flow rate is given as 395 m^3/d, so the required volume of the reactor is:

$$V = Qt_d = \left(395\ \text{m}^3/\text{d}\right)46.1\ \text{d} = 18{,}190\ \text{m}^3$$

(b) When a first-order reaction is occurring in a CMR at steady state, the extent of reaction is given by:

$$c_{out} = \frac{c_{in}}{1 + k_1 t_d}$$

$$t_d = \frac{\frac{c_{in}}{c_{in}} - 1}{k_1}$$

For the specified conditions of 90% destruction of the contaminant, c_{in}/c_{out} equals 10, so:

$$t_d = \frac{10 - 1}{0.05\ \text{d}^{-1}} = 180\ \text{d}$$

$$V = Qt_d = \left(395\ \text{m}^3/\text{d}\right)\left(180\ \text{d}\right) = 71{,}100\ \text{m}^3$$

(c) Calling the concentration in the first of the two CMRs in series c_1, the mass balances on these reactors yield:

$$\frac{c_1}{c_{in}} = \frac{1}{1+k_1 t_d} \quad \text{and} \quad \frac{c_{out}}{c_1} = \frac{1}{1+k_1 t_d}$$

so: $$\frac{c_{out}}{c_{in}} = \left(\frac{c_1}{c_{in}}\right)\left(\frac{c_{out}}{c_1}\right) = \frac{1}{\left(1+k_1 t_d\right)^2}$$

$$\frac{c_{in}}{c_{out}} = \left(1+k_1 t_d\right)^2$$

Once again setting c_{in}/c_{out} equal to 10, we find:

$$10 = \left(1+k_1 t_d\right)^2$$

$$\sqrt{10} = 3.16 = 1+k_1 t_d$$

$$t_d = \frac{3.16-1}{0.05\ \text{d}^{-1}} = 43.2\ \text{d}$$

$$V = Qt_d = \left(395\ \text{m}^3/\text{d}\right)\left(43.2\ \text{d}\right) = 17{,}064\ \text{m}^3$$

The volume computed above is for each of the reactors, so the total detention time is 86.2 d, and the total volume required is 34,128 m^3. Note that the total required volume decreases in the order 1 CMR > 2 CMRs > PFR, corresponding to our conceptual understanding that dividing a CMR into two equal portions in series makes it more PFR-like.

2. The mass balance for a steady-state CMR in which a second-order reaction is occurring is:

$$V\cancel{\frac{dc}{dt}} = Q\left(c_{in} - c\right) - k_2 c^2 V$$

In the above mass balance, dc/dt has been set to zero because the system is at steady-state, and c has been equated with c_{out}, because the reactor is a CMR. The equation can be converted to a standard polynomial, which can then be solved using the quadratic equation:

$$c = c_{out} = \frac{-1+\sqrt{1^2 + 4\left(k_2 t_d\right)c_{in}}}{2k_2 t_d}$$

For each reactor in the series, the residence time is:

$$t_d = \frac{V}{Q} = \frac{80 \text{ m}^3}{150 \text{ m}^3\text{/h}} = 0.533 \text{ h}$$

The value of k_2 is given as 0.063 L/mg-h, and c_{in} into the first reactor is 560 mg/L, so the concentration in the first reactor (and the concentration exiting that reactor) is:

$$c_1 = \frac{-1 + \sqrt{1^2 + 4\left(0.063 \text{L/mg-h}\right)\left(0.533 \text{ h}\right)\left(560 \text{ mg/L}\right)}}{2\left(0.063 \text{L/mg-h}\right)\left(0.533 \text{ h}\right)} = 115 \text{ mg/L}$$

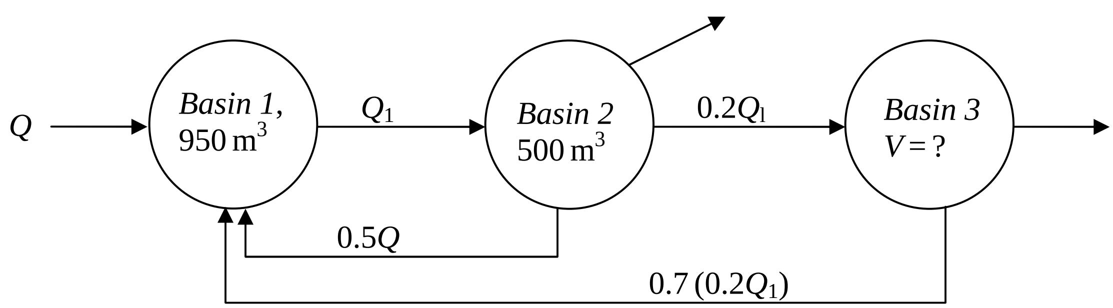
The effluent from the first reactor is the influent (c_{in}) to the second. All other values in the mass balance are the same, so:

$$c_2 = \frac{-1 + \sqrt{1^2 + 4\left(0.063 \text{L/mg-h}\right)\left(0.533 \text{ h}\right)\left(115 \text{ mg/L}\right)}}{2\left(0.063 \text{L/mg-h}\right)\left(0.533 \text{ h}\right)} = 45.5 \text{ mg/L}$$

Similarly, for the third reactor:

$$c_3 = \frac{-1 + \sqrt{1^2 + 4\left(0.063 \text{L/mg-h}\right)\left(0.533 \text{ h}\right)\left(45.5 \text{ mg/L}\right)}}{2\left(0.063 \text{L/mg-h}\right)\left(0.533 \text{ h}\right)} = 24.8 \text{ mg/L}$$

3. A schematic of the sequence of basins and the given information about the flow rates is shown below. In addition, we are told that $Q = 3800 \text{ m}^3\text{/d}$.



We can write a mass balance on water using Basin 1 as the control volume. The "concentration" of water in mass/volume is just its density (ρ_w), which is the same throughout the system. In addition, water is non-reactive ($r_w = 0$), and the system is at steady state, so the mass balance really just turns out to be a balance on the flow rates, with the following form:

$$\cancel{\frac{d\left(V\rho_w\right)}{dt}} = \sum_{inflows} Q_i \rho_w - \sum_{outflows} Q_i \rho_w + \cancel{r_w V}$$

$$0 = Q\rho_w + 0.5Q\rho_w + 0.7\left(0.2Q_1\right)\rho_w - Q_1\rho_w$$

$$1.5Q = \left(1.0 - 0.14\right)Q_1$$

$$Q_1 = \frac{1.5}{0.86}Q = 1.74\left(3800 \text{ m}^3/\text{d}\right) = 6628 \text{ m}^3/\text{d}$$

$$t_{d,1} = \frac{V_1}{Q_{tot,1}} = \frac{950 \text{ m}^3}{6628 \text{ m}^3/\text{d}} = 0.143 \text{ d} = 3.44 \text{ h}$$

The total flow into and out of the second basin is Q_1, so:

$$t_{d,2} = \frac{V_2}{Q_1} = \frac{500 \text{ m}^3}{6628 \text{ m}^3/\text{d}} = 0.075 \text{ d} = 1.81 \text{ h}$$

The total flow into and out of the third basin is $0.2\,Q_1$, so for the detention time in the basin to be 15 d, the volume must be:

$$V_3 = 0.2Q_1 t_{d,2} = 0.2\left(6628 \text{ m}^3/\text{d}\right)\left(15 \text{ d}\right) = 19{,}880 \text{ m}^3$$