Modeling Microbial Growth Dynamics: Two Key Equations

Michaelis-Menten equation characterizes food ('substrate') consumption rate:

$$-\frac{r_S}{X} = \frac{k_{max} S}{K_S + S}$$

$-r_S$: rate of substrate utilization (mg BOD_5/L-d)

X: biomass concentration (mg VSS/L)

$-r_S/X$: specific rate of substrate utilization (F/M) (mg BOD_5/ mg VSS-d)

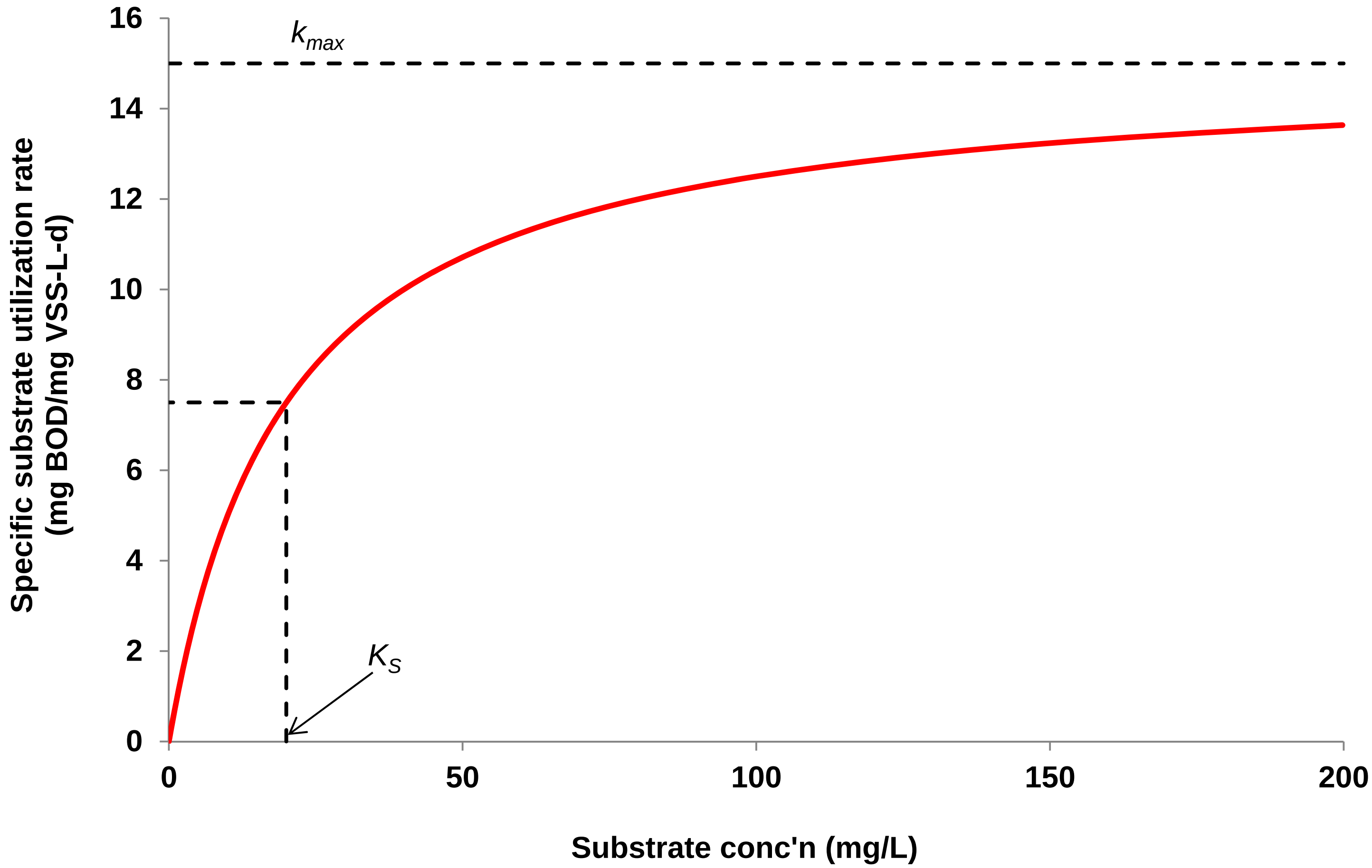
k_{max}: maximum specific substrate utilization rate (mg BOD_5/ mg VSS-d)

K_S: 'half-saturation' constant (mg BOD_5/L)

VSS: Volatile suspended solids, a measure of biomass

$$-\frac{r_S}{X} = \frac{k_{max}S}{K_S + S}$$

Michaelis-Menten equation indicates that the substrate consumption rate per gram of biomass is proportional to substrate concentration when S is low (<<K_S) and independent of S when S is high (>>K_S)



Modeling Microbial Growth Dynamics: Two Key Equations

Monod equation relates substrate consumption to microbial growth:

$$r_X = -Y r_S - k_d X$$

r_X: rate of biomass formation (mg VSS/L-d)
Y : yield coefficient (mg VSS formed/ mg BOD_5 consumed)
k_d: biomass decay rate (d^{-1})

Monod equation indicates that the biomass grows in proportion to the substrate (food) consumption, but also decays at a rate proportional to the biomass concentration.

$$r_X = -Y r_S - k_d X$$

$$r_X = +Y \frac{k_{max} S X}{K_S + S} - k_d X$$

$$\frac{r_X}{X} \equiv \mu = Y \frac{k_{max} S}{K_S + S} - k_d = \frac{\mu_{max} S}{K_S + S} - k_d$$

r_X/X or μ: specific growth rate (d^{-1})

μ_{max}: maximum specific growth rate, Yk_{max} (d^{-1});

$\mu = \mu_{max}$ when $S \gg K_S$

Batch Microbial Growth Dynamics

Mass balances on S and X:

$$V\frac{dS}{dt} = \cancel{Q}\left(S_{in} - S_{out}\right) + r_S V$$

$$V\frac{dX}{dt} = \cancel{Q}\left(X_{in} - X_{out}\right) + r_X V$$

$$\frac{dS}{dt} = -r_S = \frac{k_{max} S X}{K_S + S}$$

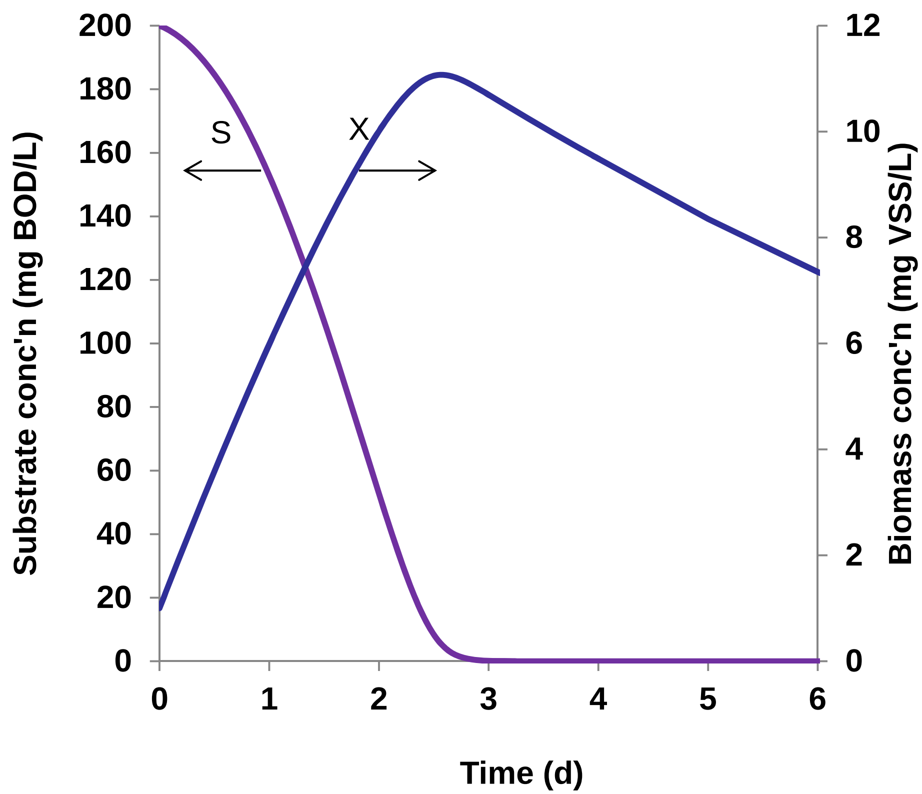
$$\frac{dX}{dt} = r_X = -Y r_S - k_d X$$

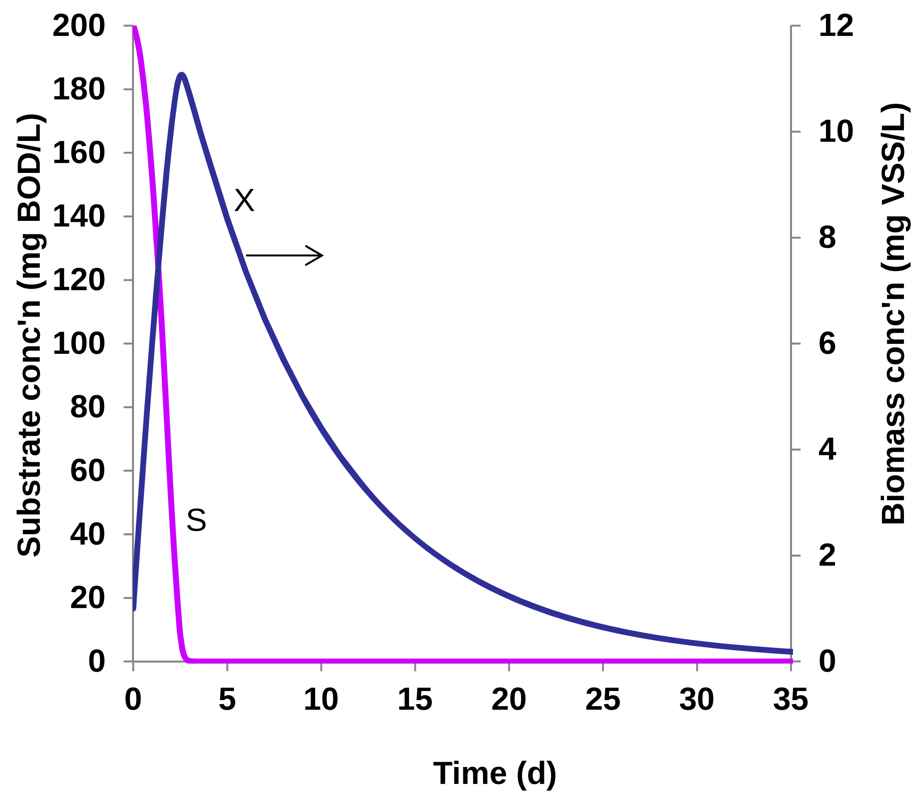
Batch Microbial Growth Dynamics

Example: $S(0) = 200$ mg BOD/L $\quad X(0) = 1$ mg VSS/L

$k_{max} = 15$ mg BOD/mg VSS-d $\quad K_S = 20$ mg BOD/L

$Y = 0.4$ mg VSS/mg BOD $\quad k_d = 0.12\ d^{-1}$





Microbial Growth in a Steady-State CSTR

Mass balance on S:

$$V\frac{dS}{dt} = 0 = Q\left(S_{in} - S_{out}\right) + r_S V$$

$$0 = Q\left(S_{in} - S\right) - \frac{k_{max} S X}{K_S + S} V$$

Rearrange, write Q/V as 1/τ :

$$\frac{1}{\tau}\frac{S_{in} - S}{X} = \frac{k_{max} S}{K_S + S}$$

Mass balance on X:

$$V\frac{dX}{dt} = 0 = Q\left(X_{in} - X_{out}\right) + r_X V$$

$$0 = Q\left(X_{in} - X\right) + QY\left(S_{in} - S\right) - k_d XV$$

$$\frac{Q}{V}\frac{S_{in} - S}{X} = \frac{1}{Y}\left[\frac{Q}{V}\left(1 - \frac{X_{in}}{X}\right) + k_d\right]$$

Assume $X \gg X_{in}$, write Q/V as 1/τ :

$$\frac{1}{\tau}\frac{S_{in} - S}{X} = \frac{1}{Y}\left[\frac{1}{\tau} + k_d\right]$$

Equate final expressions from the two mass balances and solve for S. Then substitute back into mass balance on S to solve for X:

$$\frac{k_{max} S}{K_S + S} = \frac{1}{Y}\left[\frac{1}{\tau} + k_d\right]$$

$$S = \frac{K_S\left(1 + k_d \tau\right)}{\left(Yk_{max} - k_d\right)\tau - 1}$$

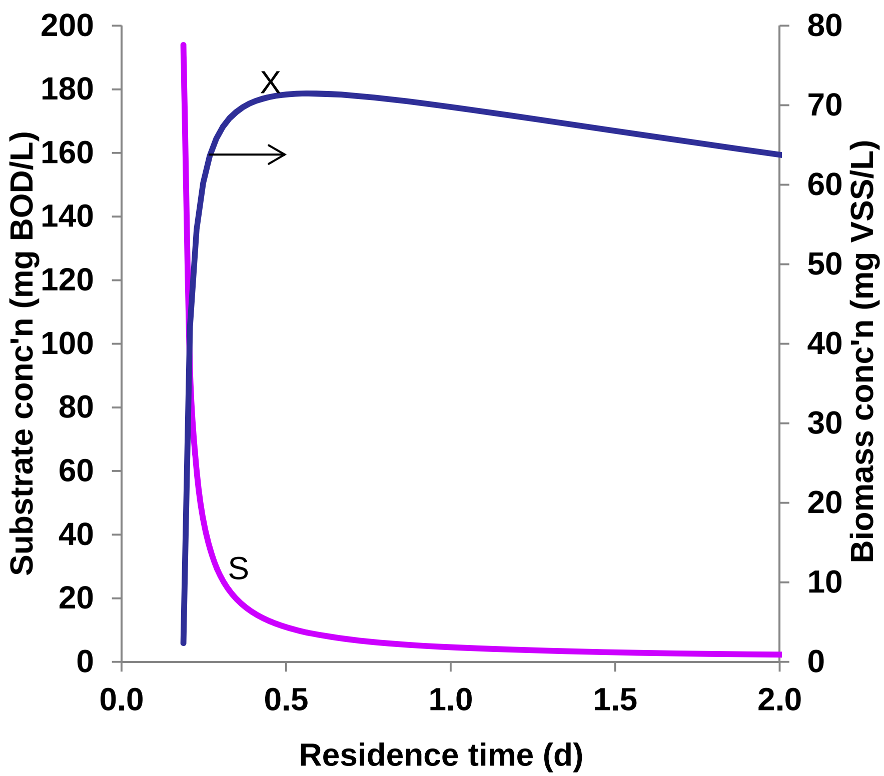
$$X = \frac{Y\left(S_{in} - S\right)}{1 + k_d \tau}$$

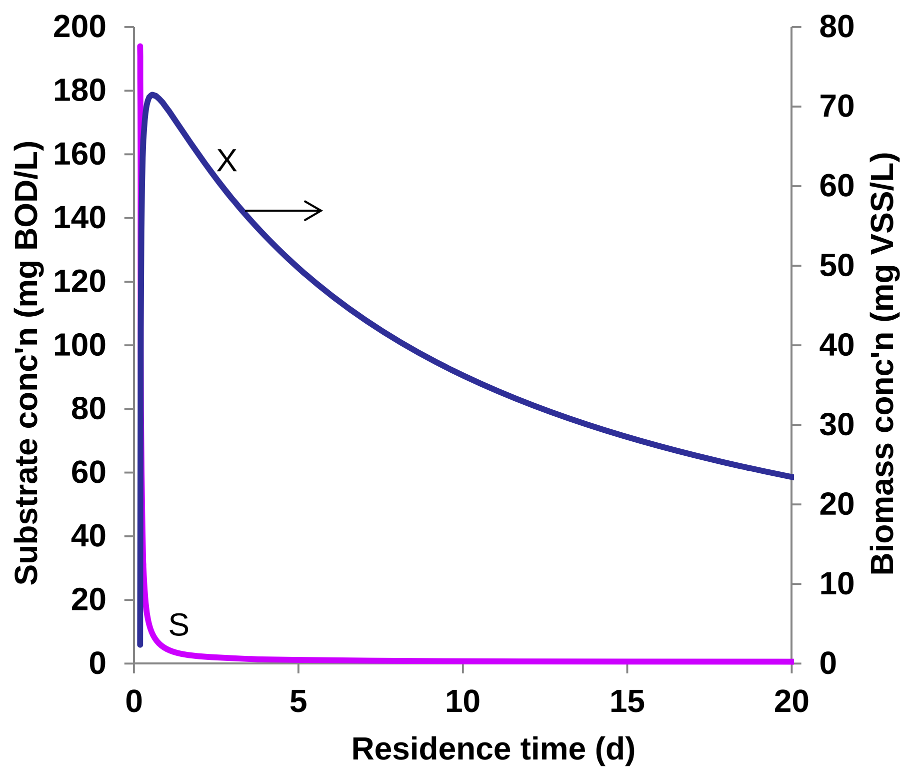
Microbial Growth in a CSTR

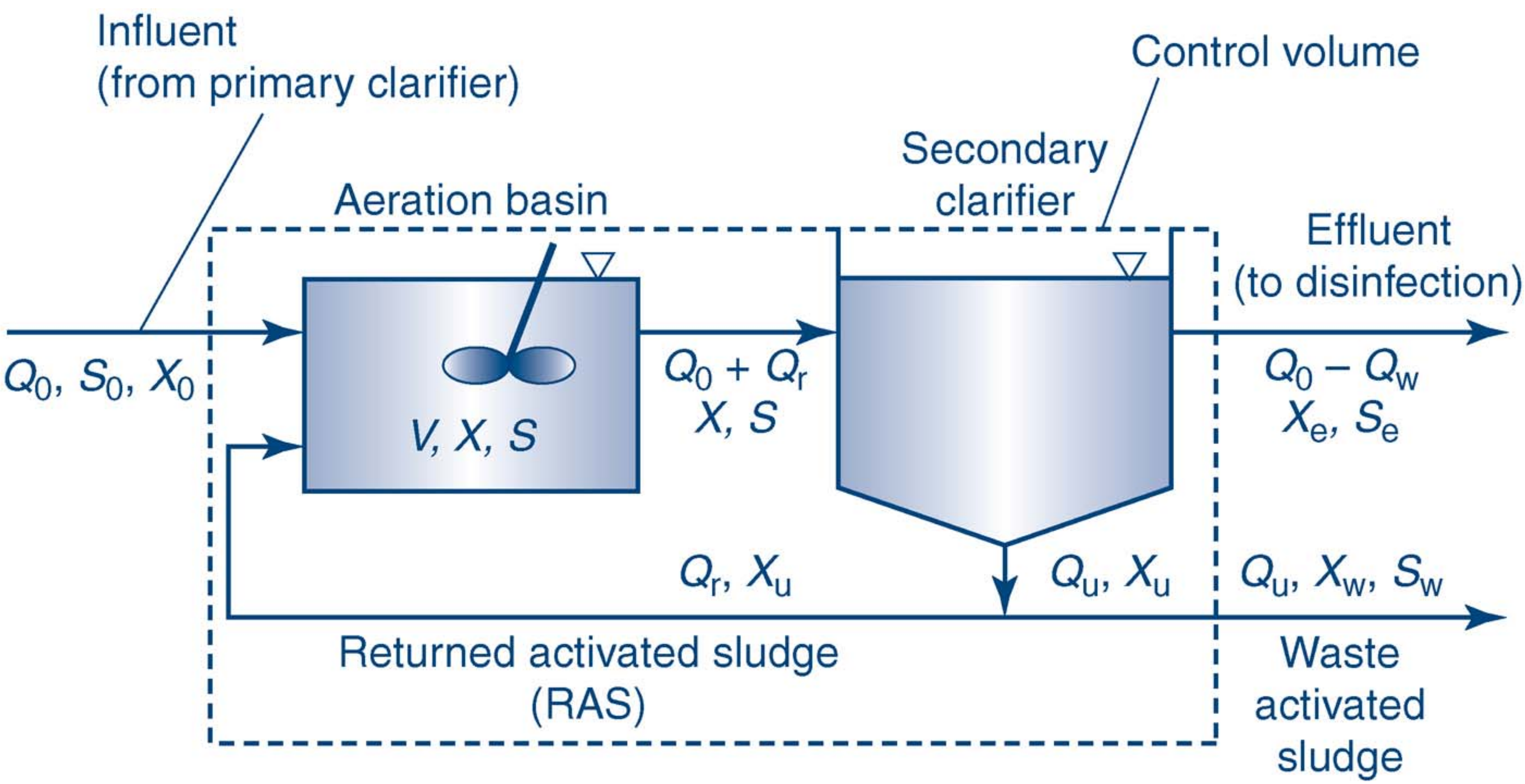
Example: $S(0) = 200$ mg BOD/L $X(0) = 1$ mg VSS/L

$k_{max} = 15$ mg BOD/mg VSS-d $K_S = 20$ mg BOD/L

$Y = 0.4$ mg VSS/mg BOD $k_d = 0.12\ d^{-1}$






Influent
(from primary clarifier)
Control volume
Secondary
clarifier
Aeration basin
Effluent
(to disinfection)
Q_0, S_0, X_0
V, X, S
$Q_0 + Q_r$
X, S
$Q_0 - Q_w$
X_e, S_e
Q_r, X_u
Q_u, X_u
Q_u, X_w, S_w
Returned activated sludge
(RAS)
Waste
activated
sludge

Microbial Growth in a Steady-State CSTR with Sludge Recycle

Mass balance on S:

$$V\frac{dS}{dt}=0=Q\left(S_{in}-S_{out}\right)+r_S V$$

$$0=Q\left(S_{in}-S\right)-\frac{k_{max}SX}{K_S+S}V$$

Rearrange, write Q/V as 1/τ :

$$\frac{1}{\tau}\frac{S_{in}-S}{X}=\frac{k_{max}S}{K_S+S}$$

Mass balance on X:

$$V\frac{dX}{dt} = 0 = QX_{in} - Q_{out}X_{out} - Q_u X_w + r_X V$$

Assume $Q_w X_w$ >> $Q_{in}X_{in}$, $Q_{out}X_{out}$, write Q/V as 1/τ :

$$Q_w X_w = QY\left(S_{in} - S\right) - k_d XV$$

$$\frac{Q_u X_w}{VX} = \frac{Q}{V}\frac{Y\left(S_{in} - S\right)}{X} - k_d$$

$$\frac{Q_u X_w}{VX} = \frac{1}{\tau}\frac{Y\left(S_{in} - S\right)}{X} - k_d$$

$$\frac{Q_u X_w}{VX} = \frac{Q}{V} \frac{Y\left(S_{in} - S\right)}{X} - k_d$$

$$\frac{VX}{Q_u X_w} = \frac{\text{biomass in reactor}}{\text{rate of biomass removal from reactor}}$$

$$= \begin{pmatrix} \text{average residence time} \\ \text{of biomass in reactor} \end{pmatrix} = \text{SRT}$$

SRT is Solids Retention Time

$$\frac{Q_u X_w}{VX} = \frac{Q}{V}\frac{Y\left(S_{in} - S\right)}{X} - k_d$$

$$\frac{1}{\mathrm{SRT}} = \frac{1}{\tau}\frac{Y\left(S_{in} - S\right)}{X} - k_d$$

$$\frac{1}{\tau}\frac{\left(S_{in} - S\right)}{X} = \frac{1}{Y}\left(\frac{1}{\mathrm{SRT}} + k_d\right)$$

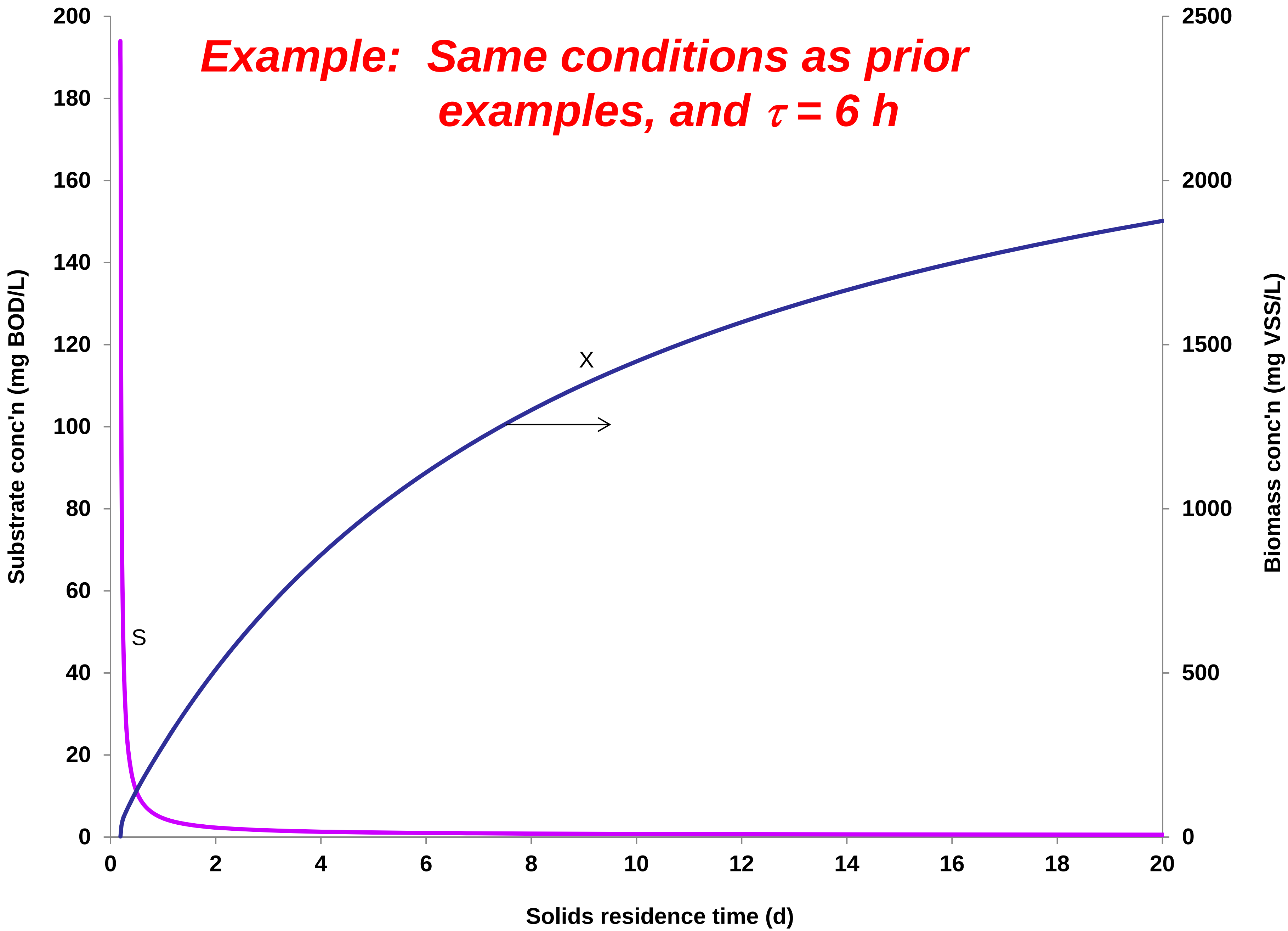
Equate final expressions from the two mass balances and solve for S. Then substitute back into mass balance on S to solve for X:

$$\frac{k_{max} S}{K_S + S} = \frac{1}{Y}\left(\frac{1}{\mathrm{SRT}} + k_d\right)$$

$$S = \frac{K_S\left(1 + k_d[\mathrm{SRT}]\right)}{\left(Yk_{max} - k_d\right)[\mathrm{SRT}] - 1}$$

$$X = \frac{Y\left(S_{in} - S\right)}{1 + k_d[\mathrm{SRT}]}\frac{\mathrm{SRT}}{\tau}$$

Example: Same conditions as prior examples, and τ = 6 h



- *Key Advantages and Disadvantages of Sludge Recycle*
  - *Increases biomass concentration in aeration tank, increasing r_S and decreasing required hydraulic residence time to achieve given level of contaminant removal*
  - *In theory, allows adjustable control over treatment efficiency*
  - *Requires extra tank and maintenance of environmental conditions favoring sludge flocculation and good settling*
- *Potential of MBRs*
  - *Provide benefits of recycle without risk of failure due to poor solid/liquid separation*
  - *Potential problem with fouling*