Large $N$ Volume Independence in Confining and Conformal Theories

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Outline:

- Large $N$ equivalences
- Large $N$ volume independence
- Example 1: Yang-Mills
- Example 2: QCD(adj)
- Example 3: $N=4$ SYM
Large $N$ equivalences

Differing finite $N$ gauge theories can have identical* large $N$ limits:

Gauge group independence

$U(N) \sim O(N) \sim Sp(N)$

Volume independence

$\mathbb{R}^d \sim \mathbb{R}^{d-1} \times \mathbb{S}^1 \sim \mathbb{R}^{d-2} \times (\mathbb{S}^1)^2 \sim \ldots$

Orbifold projections

$U(2N) \sim U(N) \times U(N)$, etc.

Orientifold projections

antisymmetric $\sim$ adjoint matter

Proof: Comparisons of loop equations or large $N$ coherent state dynamics

*With important caveats...
Large $N$ volume independence

$SU(N)$ gauge theory on toroidal compactifications of $\mathbb{R}^d$:

no volume dependence in leading large $N$ behavior of topologically trivial single-trace observables (or their connected correlators)

provided

no spontaneous breaking of center symmetry or translation invariance

Proof: comparison of large $N$ loop equations (EK) or $N=\infty$ classical dynamics (LY)
Example 1: $SU(N)$ YM on $\mathbb{R}^3 \times S^1$

- $Z_N$ center symmetry, order parameter = Wilson line $\Omega$
- $L > L_c$: unbroken center symmetry
  \[ \langle \text{tr} \, \Omega^n \rangle = 0 \]
  confined phase
- $L < L_c$: broken center symmetry
  \[ \langle \text{tr} \, \Omega^n \rangle \neq 0 \]
  deconfined plasma phase
  failure of EK reduction
Center-symmetry stabilization

- Unwanted symmetry breaking? Fix it!
  - quenched EK: doesn’t work
  - twisted EK: doesn’t work
  - adding massless adjoint fermions: works
  - adding explicit stabilizing terms: works

\[
S_{YM} \longrightarrow S_{YM} + \Delta S
\]

\[
\Delta S = \int_{\mathbb{R}^3 \times S^1} \frac{1}{L^4} \sum_{n=1}^{[N/2]} c_n \left| \text{tr} \Omega^n \right|^2
\]

\{c_n\} sufficiently positive, \(O(1)\) as \(N \rightarrow \infty\)

deformation prevents symmetry breaking but has
no effect on \(N=\infty\) center symmetric dynamics

Bringoltz, Sharpe 2008
Teper, Vairinhos 2007
Azeyanagi, Hanada, Hirata, Ishikawa 2008
Kovtun, Unsal, Yaffe 2007
Unsal, Yaffe 2008
Dimensional Reduction?

• small $L$, asymptotic freedom $\Rightarrow$ heavy, weakly coupled KK modes

• usual case: broken center symmetry
  \[ \langle \text{tr } \Omega^n \rangle \neq 0 \Leftrightarrow \text{eigenvalues clump} \]
  \[ m_{KK} = 1/L, 2/L, \ldots, \text{perturbative control when } L \Lambda \ll 1 \]
  integrate out $\Rightarrow 3d$ effective theory, $L$-dependent

• center-symmetric case:
  \[ \langle \text{tr } \Omega^n \rangle = 0 \Leftrightarrow \text{eigenvalues repel} \]
  \[ m_{KK} = 1/NL, 2/NL, \ldots, \text{perturbative control when } NL \Lambda \ll 1 \]
  $SU(N) \Rightarrow U(1)^{N-1}$ Higgsing
  \[ m_W = \frac{2\pi}{\sqrt{N}} \quad m_\gamma \sim m_W e^{-8\pi^2/N g^2(m_W)} \Rightarrow \frac{m_\gamma}{m_W} \sim (NL \Lambda)^{11/6} \]
  topological defects (monopoles) $\Rightarrow$ mass gap, confinement

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Large $N$ vs. small $L$

- unbroken center symmetry $\Rightarrow$ relevant scale is $NL$, not $L$.

- $L \to 0$, $N$ fixed $\Rightarrow NL \Lambda \ll 1$
  - KK modes: decouple
  - IR physics = Abelian 3$d$ dynamics, semiclassical confinement
  - volume dependence

- $N \to \infty$, $L$ fixed $\Rightarrow NL \Lambda \gg 1$
  - KK spectrum $\Rightarrow$ continuum
  - IR physics = non-Abelian 4$d$ dynamics
Example 2: massless QCD(adj), $\mathbb{R}^3 \times S^1$

- $N_f \geq 1$ massless adjoint rep. fermions: periodic boundary conditions $\Rightarrow$ stabilized center symmetry

$$V_{1\text{-loop}}(\Omega) = \frac{2}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{1}{n^4} (-1 + N_f) |\text{tr} \, \Omega^n|^2$$

- $n_f > n_f^* = \text{conformal window lower limit: IR CFT on } \mathbb{R}^4$
  compactify on $\mathbb{R}^3 \times S^1 \Rightarrow \text{correlation length } \sim \text{NL, not L.}$
Massive QCD(adj), $\mathbb{R}^3 \times S^1$

- Suppose $1/L \gg m \gg \Lambda$: negligible change to $V_{1\text{-loop}}$? No:
  \[
  \frac{1}{n^4}(-1 + N_f) \implies \frac{1}{n^4}(-1 + N_f f(nLm)) ,
  \]
  \[
  f(z) = \frac{1}{2} z^2 K_2(z) \sim \begin{cases} 1, & z \ll 1; \\ e^{-z}, & z \gg 1. \end{cases}
  \]
  small $m \neq 0 \implies$ high winding loops become unstable

- $2 \leq n_f < n_f^\star$: non-uniform $m \to \infty$ and $L \to 0$ limits:

see also: Hollowood & Myers
Massive QCD(adj), $\mathbb{R}^3 \times S^1$

- $n_f = 1$: non-uniform $m \to 0$ and $L \to 0$ limits.
- $n_f^* \leq n_f < n_f^{AF}$: non-uniform $m \to 0$ and $L \to \infty$ limits.
- Require: $NLm < O(1)$ for center stabilization,
  
  $NL\Lambda \gg 1$ for volume independence

- $m/\Lambda$ must vanish as $N \to \infty$ for volume independence with fermion induced center stabilization
Massive QCD(adj), $T^4$

- Eigenvalues of commuting $\Omega_1,...,\Omega_4 \Rightarrow N$ particles on dual 4-torus
  - small mass $\Rightarrow$ particles repel
  - large mass $\Rightarrow$ particles attract
- $O(1)$ eigenvalue fluctuations for all $L$
  - $N$ finite: unbroken center symmetry
  - $N = \infty$: unbroken center symmetry for even when $m \gg \Lambda$!
- One site lattice theory:

Bringoltz & Sharpe: arXiv:0906.3538
Example 3: $N=4$ SYM on $\mathbb{R}^3 \times S^1$

- Moduli space = $\mathbb{R}^{6N}/S_N$ on $\mathbb{R}^4$,
  
  $= [\mathbb{R}^{6N} \times (\mathbb{S}^1)^N]/S_N$ on $\mathbb{R}^3 \times S^1$

- “Usual” $\Omega = 1$ compactification:
  
  spontaneously broken center symmetry
  
  $1/L$ = relevant scale, no volume independence
  
  IR physics = $N=8$ superconformal 3d SYM theory, enhanced $SO(8)_R$ symmetry

- $\text{tr} \, \Omega^n = 0$ compactification:
  
  unbroken center symmetry
  
  $1/NL$ = relevant scale
  
  $N<\infty$: IR physics = $U(1)^{N-1}$ massless 3d Abelian, no superpotential
  
  $N=\infty$: IR physics = 4d non-Abelian, $L$ independent
Concluding remarks

• Volume independence is a remarkable consequence of the large $N$ limit in an interesting class of non-Abelian gauge theories:
  * Does not require confining phase or continuity of phases.
  * Allows one to trade $V \to \infty$ limit for $N \to \infty$ limit.

• Compactification produces rich phase structure in QCD(adj).

• Promising practical utility for studying large $N$ limit of Yang-Mills, QCD(adj), and real QCD $\to$ QCD(adj) $\sim$ QCD(adj).
  * Numerical work underway: Bringoltz, Sharpe; Catterall, Galvez, Ünsal; Azeyanagi, Hanada, Yacoby, ...

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