Physics 564 Assignment #3 October 21, 2016

Due Thursday, November 3, 2016

1. In D-dimensional spacetime, consider a metric of the form

$$ds^2 = \tilde{g}_{\mu\nu}(x, r)\, dx^\mu\, dx^\nu + 2dx^0 dr\,,$$

where $\mu, \nu = 0, \cdots, D{-}2$, and $\tilde{g}$ (for every fixed value of r) is a Lorentzian signature $D{-}1$ dimensional metric. Notice the absence of any dr^2 term in the line element. Explain why this implies that curves along which r varies, with all other coordinates held fixed, are null curves (curves whose tangent vectors are always null). Then show that these null curves are actually geodesics. Are surfaces on which $x^0 =$ const. spacelike, timelike, or null surfaces? (What, in curved space, does it mean for a surface to be timelike, spacelike, or null?)

2. In D-dimensional spacetime, consider a metric of the form

$$ds^2 = -A(r)\, dt^2 + \Sigma(r)^2\, dS^2_{D-2} + 2\, dt\, dr\,,$$

where dS^2_{D-2} is a metric on a unit $D{-}2$ dimensional sphere. This line element is a specialization of the form in the previous problem in which, by assumption, the geometry is static and posesses $D{-}2$ dimensional rotational symmetry. (Static means that $t \to t +$ const. is a symmetry.) What is the residual diffeomorphism freedom of this metric ansatz? That is, what changes of coordinates can be made which preserve the form of the metric? What is the area (or “length” or “volume”) of the surface defined by $t =$ const. and $r =$ const.? What conditions must $A(r)$ and $\Sigma(r)$ satisfy for this metric to be a vacuum solution of Einstein's equations? Explain why one may set $\Sigma(r) = r$ without loss of generality. Solve the resulting equation for $A(r)$ and find the most general static, spherically symmetric solutions to Einstein's equations. Are these geometries asymptotically flat? What radial curves (i.e., curves with fixed values of all angular coordinates) are infalling null geodesics? What curves are outgoing radial null geodesics? Discuss the meaning of any integration constants appearing in the solution. (Hint: consider $D = 3$ separately from $D > 3$.)