HOMEWORK #5
Due: February 15, 2023

1. Consider the singular equation:

$$\epsilon y'' + (1+x)^2 y' + y = 0$$

with $y(0) = y(1) = 1$ and with $0 < \epsilon \ll 1$.
(a) Obtain a uniform approximation which is valid to $O(\epsilon)$, i.e. determine the leading order behavior and first correction.
(b) Show that assuming the boundary layer to be at $x = 1$ is inconsistent. (hint: use the stretched inner variable $\xi = (1-x)/\epsilon$).
(c) Plot the uniform solution for $\epsilon = 0.01, 0.05, 0.1, 0.2$.

2. Consider the singular equation:
$$\epsilon y'' - x^2 y' - y = 0$$

with $y(0) = y(1) = 1$ and with $0 < \epsilon \ll 1$.
(a) With the method of dominant balance, show that there are three distinguished limits: $\delta = \epsilon^{1/2}$, $\delta = \epsilon$, and $\delta = 1$ (the outer problem). Write down each of the problems in the various distinguished limits.
(b) Obtain the leading order uniform approximation (hint: there are boundary layers at $x = 0$ and $x = 1$).
(c) Plot the uniform solution for $\epsilon = 0.01, 0.05, 0.1, 0.2$.