AMATH 568
Winter Quarter 2023
Professor J. Nathan Kutz

HOMEWORK #4
Due: February 3, 2023

1. Consider the weakly nonlinear oscillator:

$$\frac{d^2y}{dt^2} + y + \epsilon y^5 = 0$$

with $y(0) = 0$ and $y'(0) = A > 0$ and with $0 < \epsilon \ll 1$.

(a) Use a regular perturbation expansion and calculate the first two terms.
(b) Determine at what time the approximation of part (a) fails to hold.
(c) Use a Poincare-Lindstedt expansion and determine the first two terms and frequency corrections.
(d) For $\epsilon = 0.1$, plot the numerical solution (from MATLAB), the regular expansion solution, and the Poincare-Lindstedt solution for $0 \leq t \leq 20$.

2. Consider Rayleigh's equation:

$$\frac{d^2y}{dt^2} + y + \epsilon \left[-\frac{dy}{dt} + \frac{1}{3}\left(\frac{dy}{dt}\right)^3 \right] = 0$$

which has only one periodic solution called a "limit cycle" ($0 < \epsilon \ll 1$). Given

$$y(0) = 0$$

and

$$\frac{dy(0)}{dt} = A\,.$$

(a) Use a multiple scale expansion to calculate the leading order behavior.
(b) Use a Poincare-Lindsted expansion and an expansion of $A = A_0 + \epsilon A_1 + \cdots$ to calculate the leading-order solution and the first non-trivial frequency shift for the limit cycle.
(c) For $\epsilon = 0.01, 0.1, 0.2$ and 0.3, plot the numerical solution and the multiple scale expansion for $0 \leq t \leq 40$ and for various values of A for your multiple scale solution. Also plot the limit cycle solution calculated from part (b)
(d) Calculate the error

$$E(t) = |y_{numerical}(t) - y_{approximation}(t)|$$

as a function of time ($0 \leq t \leq 40$) using $\epsilon = 0.01, 0.1, 0.2$ and 0.3.