AMATH 568
Winter Quarter 2023
Professor J. Nathan Kutz

HOMEWORK #3
Due: January 25, 2023

1. *Particle in a box*: Consider the time-dependent Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi$$

which is the underlying equation of quantum mechanics where $V(x)$ is a given potential.

(a) Let $\psi = u(x)\exp(-iEt/\hbar)$ and derive the time-independent Schrödinger equation (Note that E here corresponds to energy).
(b) Show that the resulting eigenvalue problem is of Sturm-Liouville type.
(c) Consider the potential

$$V(x) = \begin{cases} 0 & |x| < L \\ \infty & \text{elsewhere} \end{cases}$$

which implies $u(L) = u(-L) = 0$. Calculate the normalized eigenfunctions and eigenvalues.
(d) What is the energy of the ground state (the lowest energy state $\neq 0$)
(e) If an electron jumps from the third state to the ground state, what is the frequency of the emitted photon. Recall that $E = \hbar\omega$.
(f) If the box is cut in half, then $u(0) = u(L) = 0$. What are the resulting eigenfunctions and eigenvalues (Think!)

2. Find the Green's function (fundamental solution) for each of the following problems, and express the solution u in terms of the Green's function.

(a) $u'' + c^2u = f(x)$ with $u(0) = u(L) = 0$.
(b) $u'' - c^2u = f(x)$ with $u(0) = u(L) = 0$.

3. Calculate the solution of the Sturm-Liouville problem using the Green's function approach (See the notes as I already showed you what the answer should be)

$$Lu = -[p(x)u_x]_x + q(x)u = f(x) \quad 0 \leq x \leq L$$

with

$$\alpha_1 u(0) + \beta_1 u'(0) = 0 \quad \text{and} \quad \alpha_2 u(L) + \beta_2 u'(L) = 0$$