AMATH 481/581 Autumn Quarter 2024

Homework 2: Quantum Harmonic Oscillator
DUE: Friday, October 18 at midnight

The probability density evolution in a one-dimensional harmonic trapping potential is governed by the partial differential equation:

$$i\hbar\psi_t + \frac{\hbar^2}{2m}\psi_{xx} - V(x)\psi = 0\,, \tag{1}$$

where ψ is the probability density and $V(x) = kx^2/2$ is the harmonic confining potential. A typical solution technique for this problem is to assume a solution of the form

$$\psi = \sum_1^N a_n \phi_n(x) \exp\left(-i\frac{E_n}{2\hbar}t\right) \tag{2}$$

which is called an eigenfunction expansion solution (ϕ_n=eigenfunction, E_n=eigenvalue). Plugging in this solution ansatz to Eq. (1) gives the boundary value problem:

$$\frac{d^2\phi_n}{dx^2} - \left[Kx^2 - \varepsilon_n\right]\phi_n = 0 \tag{3}$$

where we expect the solution $\phi_n(x) \to 0$ as $x \to \pm\infty$ and ε_n is the quantum energy. Note here that $K = km/\hbar^2$ and $\varepsilon_n = E_n m/\hbar^2$. In what follows, take $K = 1$ and always normalize so that $\int_\infty^\infty |\phi_n|^2 dx = 1$.

(a) Calculate the first five *normalized* eigenfunctions (ϕ_n) and eigenvalues (ε_n) using a shooting scheme. For this calculation, use $x \in [-L, L]$ with $L = 4$ and choose $xspan = -L : 0.1 : L$. Save the absolute value of the eigenfunctions in a 5-column matrix (column 1 is ϕ_1, column 2 is ϕ_2 etc.) and the eigenvalues in a 1x5 vector.

ANSWERS: Should be written out as A1 (eigenfunctions) and A2 (eigenvalues)