AMATH 301
Homework 2: Autumn 2021

DUE: Midnight on Thursday, October 14

I Consider the circuit depicted in the figure. By using the two following facts: (i) the voltage drop across a resistor is $V = IR$ and (ii) the sum of all voltage drops in a closed loop sum to zero, The currents I_1, I_2, and I_3 are determined from the 3×3 system.

$$R_6I_1 + R_1(I_1 - I_2) + R_2(I_1 - I_3) = V_1$$
$$R_3I_2 + R_4(I_2 - I_3) + R_1(I_2 - I_1) = V_2$$
$$R_5I_3 + R_4(I_3 - I_2) + R_2(I_3 - I_1) = V_3$$

where $R_1 = 20$, $R_2 = 10$, $R_3 = 25$, $R_4 = 10$, $R_5 = 30$, $R_6 = 40$, $V_2 = 0$, $V_3 = 200$, and V_1 will be variable. In this form, the associated matrix is strictly diagonal dominant.

(a) Vary V_1 from 0 to 100 in steps of 2 (i.e. $V_1 = 0, 2, 4, ..., 100$) and calculate I_1, I_2 and I_3 as a function of increasing V_1 by solving the system with the standard backslash (matlab) or solve (python) command. Save your results in a matrix of 3 columns and 51 rows where the first, second and third column are I_1, I_2 and I_3 respectively.

ANSWER: Should be written out as A1

(b) Repeat part (a), but now solve it with three additional methods: **LU** decomposition, Jacobi Iterations, and Gauss-Seidel Iterations. For the iteration methods, begin with the guess $(I_1, I_2, I_3) = (0, 0, 0)$. This will give you three additional matrices of size 3 columns by 51 rows for the LU, Jacobi and Gauss-Seidel respectively.

ANSWERS: Should be written out as A2–A4

(c) For the two iteration methods, what is the **average** number of iterations required to solve the given equation with accuracy 10^{-6}. The accuracy constraint should be based upon looking at the norm of the difference between successive iterations. Thus it is required that $\|\vec{x}_{n+1} - \vec{x}_n\| < 10^{-6}$. Save the two answers (for Jacobi first and Gauss-Seidel second) as a row vector with two components.

ANSWER: Should be written out as A5