The Basic Concepts of Set Theory

1. Sentence-form: “A *sentence-form* is an expression that is a sentence or is obtainable from a sentence by replacing some or all direct occurrences of names by variables” (Mates, p. 28).

2. Satisfying a sentence-form: “... a given object *satisfies* [a] sentence form if the result of interpreting the free occurrences of the variable as referring to that object is true” (p. 31).

3. Sets & set membership ($\in$): The crucial property of sets is that “sets having the same members are identical” (p. 33).

4. Objects: *Ox* iff there is a set of which *x* is a member.

5. The basic notation of sets: $\{x|Fx\}$ = the set of all *objects* that are *F*. For all *x*, $x \in \{x|Fx\}$ iff (*Ox* & *Fx*).

6. Universal set: $V =_{df} \{x|x = x\}$

7. Null set: $\Lambda =_{df} \{x|x \neq x\}$

8. Unit set: $\{a\} =_{df} \{x|x = a\}$

9. Unordered pair: $\{a,b\} =_{df} \{x|x = a \text{ or } x = b\}$

10. Subsets: $A \subset B$ iff every member of *A* is also a member of *B*.

11. Proper subsets: *A* is a proper subset of *B* iff $A \subset B$ & $A \neq B$.

12. Union: $A \cup B =_{df} \{x|x \in A \text{ or } x \in B\}$

13. Intersection: $A \cap B =_{df} \{x|x \in A \ \& \ x \in B\}$

14. Complement: $A' =_{df} \{x|x \notin A\}$

15. Difference: $A \sim B =_{df} A \cap B'$

16. Ordered pair: $\langle x,y\rangle =_{df} \{ \{x\}, \{x,y\} \}$

17. Ordered triple: $\langle x,y,z\rangle =_{df} \langle x,\langle y,z\rangle\rangle$

18. Binary relation: a set *R* is a binary relation iff every member of *R* is an ordered pair: $\{<x,y> \mid xRy\} =_{df} \{z \mid \text{there is an } x \text{ and a } y \text{ such that } z = <x,y> \text{ and } xRy\}$

19. Domain: "the *domain* of a binary relation *R* is the set of all objects *x* such that for some *y*, *xRy*" (p. 39).

20. Converse domain or range: "the *converse domain* [or *range*] of a binary relation *R* is the set of all objects *y* such that for some *x*, *xRy*" (p. 39).

21. Field: the field of a binary relation *R* is the union of its domain and range.

22. Function: a binary relation *R* is a function iff for all objects *x*,*y*,*z*, if *xRy* & *xRz*, then $y = z$.

23. Arguments and values: an object *x* is an *argument* of a function *R* iff *x* is a member of the domain of *R*. An object *y* is a *value* of a function *R* iff *y* is a member of the range of *R*.

24. The value of a function for a particular argument: If *R* is a function, then $R(x) = y$ iff $<x,y> \in R$.

25. Converse of a binary relation: $\check{R} =_{df} \{<y,x> \mid xRy\}$

26. 1-1 relation: a binary relation *R* is 1-1 iff *R* is a function & $\check{R}$ is a function.

27. Equipollence: $A \approx B$ iff there is a 1-1 relation *R* whose domain is *A* and whose range is *B*.