Foreign Aid: Where Your Money Could Be? An Optimal Two-Sector Growth Model Out Lining a Simple Answer

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Abstract

In this paper, we present an optimal two-sector growth model involving foreign aid as an input into the production function. We characterize the optimal resources allocation across sectors. Once calibrated, mainly, on Latin American countries, the model exhibits weak substitutability between aid and capital stock. Nonetheless, using numerical simulations, the model reproduces the main stylized facts outlined in the literature.

JEL classification: E13; O1; O11; O21; O4; O41

Key words: Foreign Aid; Economic Development; Investment; Economic Growth; Quantitative Approach

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“Unlike many African countries, we do not need aid in Latin America, but justice and equality. Economic policies introduced in the 90s are now obsolete because they did not contribute to alleviate poverty. A possible solution to overcome ‘asymmetries’ existing in the world would be to encourage a Global Third Way, which means undertaking broad reforms inside Public International Institutions like the IMF and the ONU, which are now out-of-date.”

Chilean President Ricardo Lagos addressing the Progressive Governance Conference held in London on July 12, 2003.

1. Introduction

Too much time has passed since foreign aid programs were launched massively in the 1960s and, in the words of Krugman (2002), “Latin America is too far from the rose garden that developed countries promised it. On the contrary, too many people there got nothing but thorns.” From this non-normative affirmation (and others) it seems clear that our knowledge about both the effectiveness of foreign aid and its credibility is far from being complete, notably from the macroeconomics perspective. Here are some issues related to this problem.

On the real world side, it is well known that for nearly forty years, many poor and developing countries (PDCs) that have been facing a combination of severe balance of payment problems, high and variable inflation, slow growth, and high unemployment, have steadily viewed foreign aid as a crucial ingredient in their development strategy. As a result, almost all PDCs have entered into agreement for multilateral financial support with some International Financial Institution (IFI) like the World Bank and the International Monetary Fund at least once since 1970. In the same way, bilateral financial agreements between most PDCs and different donors constitute a significant item in the agenda of the North-South relationship. As a consequence, PDCs have been undertaking heavy structural adjustment programs in their economies facing, in many cases, sharp crises. Moreover, aid can well be viewed as permanent income given its persistence.

On the academic side, most researchers using empirical models have found that, firstly, aid tends to be channeled rather to consumption than to investment. Secondly, positive changes in long run output have been channeled only via consumption. Thirdly, aid has not had an effect on investment in the long run and fourthly, aid has had, in some cases, a short run positive effect on investment. In sum, it is clear that the main goals that have been claimed for decades by the board of the IFIs and donors, namely to alleviate poverty and spur growth, still remain unachieved. Moreover, according to Easterly (1999), it is not less astonishing to realize that these international institutions have been applying models supposedly dead long ago to calculate short-run investment requirements for a target growth rate. Finally, surprisingly it turns out to be that the previous theoretical literature on the impact of foreign aid remains quite limited. While a great deal has been said about qualitative aspects of aid-effectiveness, little attention has been paid so far to some relevant quantitative issues. Indeed, the aid-conditionality-growth relationship and the role of ‘good’ policies and adequate political environments have dominated both the past and the recent debate, especially from the 1990s.

In light of all these considerations, the effectiveness of aid remains in question among many economists, especially in its conditional terms but in its non-conditional ones as well. This paper is intended to contribute to the foreign aid literature by proposing a new quantitative framework. Even though it involves simple assumptions and considers an optimal growth framework, its accordance with the stylized facts mentioned above make it worthy in itself as a stepping stone to more complete models involving decentralized markets.

Our main point is that a comprehensive analysis of the macroeconomic impact of aid should take into account the sectoral composition of the economy. Indeed, the classical way of formally tackling so far this kind of problem has been to use one-sector models in which aid is taken as an exogenous variable. Close to the spirit of Benhabib and Farmer (1996), we follow a different approach and propose an optimal two-sector growth model involving aid explicitly as one of the inputs into both consumption and investment sector. The fraction of aid

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1 See Barro and Lee (2002).
2 Corbo and Fischer (1995) offer a starting point for this theme.
4 In very recent Working Paper, Ranaweera (2003) adds some other aspects to this debate.
allocated in each sector is endogenized and the only remaining input in the economy is the stock of physical capital.

The model is then calibrated for two “representative” economies, being the average ratio capital to output the main variable behind this choice. This suitability of the model makes it very attractive to account for the aid-effectiveness problem faced by most PDCs. The model spells out, among other things, that the first best response is to consume aid rather than to invest it. On the other hand, it also predicts no effect of aid on the investment sector, either in the short or in the long run; only a permanent aid shock can affect the capital accumulation but it has only a small effect. The aggregate capital stock remains constant over time and when aid arrives it is distributed following an aid-consumption marginal productivity incentive. Thus if the positive relationship between investment and economic growth is accepted, the model offers a possible rationale of why most of PDCs have not exhibited major improvements in their growth performance since foreign aid programs were launched massively in the 1960s. Furthermore, the model predicts that changes in output are only through consumption. In sum, our model predicts that the main stylized facts mentioned above are indeed optimal!!.

The remainder of this paper is organized as follows. Turn to Section 2 and you will find the model. In Section 3 we spend some time explaining a calibration process. Next, move to Section 4 and you will find some simulations. Finally, in Section 5 we conclude.

2. The Base Model

2.1 The basic structure 6

We consider here a conventional neo-classical discrete-discounted-time two-sector economy having an infinitely-lived representative agent with single period linear utility function given by

\[ u(C) = C \]

and being run by a planner who has at her disposal two resources: an economy-wide stock of capital, ‘K’, and an economy-wide flow of foreign aid, ‘F’. There are two distinct commodities that we refer to as an investment good, ‘I’, and a consumption good ‘C’. Each commodity is assumed to be produced with a CES technology without sector-specific externalities. Letting \( KC_t \) and \( KI_t \) denote the fraction of the economy-wide stock of capital at time \( t \) in the consumption (investment) sector; \( F_t \), the economy-wide flow of foreign aid at time \( t \), and \( \mu_{F,t} \), and the fraction of this aid used in the consumption goods industry at time \( t \), I can write the output of the two industries at this time as follows:

\[
C_t = A_t \cdot \left[ a \cdot KC_t^\psi + (1 - a) \cdot \left( \mu_{F,t} \cdot F_t \right)^\psi \right]^{\frac{1}{\psi}} \\
I_t = B_t \cdot \left[ a \cdot KI_t^\psi + (1 - a) \cdot \left( 1 - \frac{\mu_{F,t}}{\mu_{F,t}} \cdot F_t \right)^\psi \right]^{\frac{1}{\psi}}
\]

where (1) \( A > 0 \) represents exogenous productivity index for the consumption (investment) sector; (2) \( 0 < a < 1 \) representing the share of capital stock assumed as being the same for both industries; (3) \( 0 \leq \mu_{F,t} \leq 1 \), and (4) \( \psi < 1 \).

Notice that (1) the two industries use identical technology with the exception of the two scaling factors \( A, B \); (2) the production functions exhibit both constant returns to scale for all values of \( \psi \), and (3) the elasticity of substitution between capital and foreign aid is \( 1/(1 - \psi) \).

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6 I thank Jess Benhabib for providing me with the right reference from which this structure was partially borrowed. A more “acceptable” utility function and its possible intertemporal effects will be exploited in subsequent papers.
Finally, we assume that the capital stock is depreciated at a constant level of $\delta$ in one period so that the capital accumulation of capital has the standard structure $K_{t+1} = (1 - \delta)K_t + I_t$.

Thus the planner’s optimization program is given by

$$
\max_{K_{t+1}, K_t, I_t, K_{t-1}} \sum_{t=0}^{\infty} \rho^t A_t \left[ a \cdot K_t^\psi + (1 - a) \cdot (1 - \mu_{F,t}) \cdot F_t^\psi \right] \frac{1}{\psi} \\
\text{s.t.} \quad I_t = B_t \left[ a \cdot K_t^\psi + (1 - a) \cdot (1 - \mu_{F,t}) \cdot F_t^\psi \right] \frac{1}{\psi} \\
K_t = K_t^C + K_t^I \\
K_{t+1} = (1 - \delta)K_t + I_t \\
K_0 > 0 \quad \text{given} \\
F_t > 0 \\
0 \leq \mu_{F,t} \leq 1 \\
0 < a < 1
$$

where $0 < \rho < 1$ is related to the constant rate of time preference. At first sight, the most striking aspect of this maximization program is, perhaps, the strictly positive condition for foreign aid. This condition means that aid is viewed as permanent income by aid-recipient countries. According to Easterly (1999), “…there is considerable justification for viewing it as such given its persistence…”. This kind of argument may become quite disputable depending on the definition of aid. In fact, if aid involves only multilateral agreements, the existing data shows that the argument can be wrong depending on the country under analysis. In contrast, if aid also involves bilateral agreements, the same data shows that the argument is quite right for most countries. Thus, based on this last empirical evidence, we pursue Easterly’s (1999) argument in order to keep the simplicity of the model as far as possible. Therefore, aid is considered here as a whole, involving its bilateral as well as multilateral flows. On the other hand, the above optimal program also shows that aid enters into the economy as a resource having the same price as consumption goods. Put differently, in order to keep the framework as realistic as possible we shall assume that aid is fungible and the planner can allocate the funds as needed.

### 2.2 Characterization of equilibrium paths

By using the standard tools to solve the above maximization problem (3), the following proposition summarizes the first-order optimality conditions and the necessary constraints to describe the solution of the optimization problem of a representative family.

**Proposition 1.** Given the initial condition $K_{-1}$, an optimal equilibrium is a path

$$\left\{ \frac{1}{\psi} \mu_{F,t}, K_t^C, K_t^I, K_{t+1}, C_t, I_t \right\}_{t \geq 0}$$

satisfying the following conditions:

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7 Notice that although this maximization problem involves $K_{t+1}$, the planner also decides at time $t$ this stock variable for the next period.

8 All previous empirical models assume this property of money. Nevertheless, a model involving aid as a ‘durable’ good seems to be an interesting extension of the model. The motivation behind this future research is the result found by Boone (1996) that aid can be effective in narrow cases where it is non-fungible.
\[
\frac{KC_t}{KI_t} = \frac{\mu F, \bar{J}}{1 - \mu F, \bar{J}}
\]

(4)

\[
(1 - \delta) \cdot \left( \frac{A_{t+1}}{B_{t+1}} \right)^{\psi} \cdot C_{t+1}^{1-\psi} \cdot I_{t+1}^{\psi - 1} \cdot \frac{\mu_{t+1}^{\psi - 1}}{1 - \mu_{t+1}^{\psi - 1}} - \frac{1}{\rho} \cdot \left( \frac{A_t}{B_t} \right)^{\psi} \cdot C_t^{1-\psi} \cdot I_t^{\psi - 1} \cdot \frac{\mu_t^{\psi - 1}}{1 - \mu_t^{\psi - 1}} = 0
\]

(5)

\[
K_t = KC_t + Kl_t
\]

(6)

\[
K_{t+1} = (1 - \delta)K_t + B_t[a \cdot Kl_t^\psi + (1 - a) \cdot (1 - \mu F, \bar{J} \cdot F_t)^\psi]^{1/\psi}
\]

(7)

\[
\lim_{t \to \infty} [\dot{K}_t \cdot K_t] = 0
\]

(8)

and \(C_t, I_t\) satisfying Equations (1) – (2).

Proof: See the appendix. ♦

In this set of equations, Equation (4) shows that the consumption (investment) industry exhibits bigger aid intensity for any value of \(\mu > (\leq) 0.5\). Equation (5) is just the Euler equation written in terms of the variables \(C\) and \(I\). Equation (7) gives an explicit law of motion of the aggregate stock of capital in this economy\(^9\) and Equation (8) represents the standard transversality condition.

At this point of the analysis something about the output in this economy should be mentioned. Like in Benhabib and Farmer (1996), it can be defined as

\[
C_t + (A / B)I_t = C_t + pI_t = Y_t,
\]

(9)

where \(p = A / B\) denotes the relative price of the investment goods. Taking a closer look at this definition, there is a relevant aspect related to aid hiding in its. Actually, what this equation implicitly exhibits is the assumption that aid enters in this economy as an intermediate good and not as a lump-sum transfer, which is unusual in the existing literature. Therefore, due to this new way of looking at aid there is no a term \(F_t\) adding to the output \(Y_t\) in Equation (9).

2.3 The steady state

To compute the steady state value we assume that the exogenous productivity indexes \(A_t, B_t\) and aid \(F_t\) as being constants. The above system of equations in Proposition 1 thus provides 6 equations for 6 unknowns \((\bar{\mu}, \bar{K}, K\bar{I}, \dot{K}, \bar{C}, \dot{I})\). Fortunately, it is possible to analytically reach the steady state value in terms of the parameters and the exogenous variables of the model. The steady state of our model is unique. Of course, we assume that technology is a sufficient producer to ensure a positive steady state value. The following proposition summarizes this estimate:

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\(^9\) Writing the Euler equation in this form instead of doing it explicitly in terms of the respective CES functions looks better for simulations in DYNARE.
Proposition 2. If \( \left(1 - \rho \cdot (1 - \delta)\right) \frac{\psi}{(A/p) \cdot \rho \cdot a} > a \), there is a positive steady state characterized by the following values of the endogenous variables\(^{10}\):

\[
\hat{\mu} = 1 - \delta \cdot \left(1 - \rho \cdot (1 - \delta)\right) \left(\frac{\psi}{(A/p) \cdot \rho \cdot a}\right)^{-1}
\]

(10)

\[
K\hat{C} = \hat{\mu} \cdot \left(1 - a\right) \cdot \left[\left(1 - \rho \cdot (1 - \delta)\right) \left(\frac{\psi}{(A/p) \cdot \rho \cdot a}\right)^{-1} - a\right]^{-1} \left(\frac{\psi}{(A/p) \cdot \rho \cdot a}\right)^{-1} \cdot F
\]

(11)

\[
K\hat{I} = (1 - \hat{\mu}) \cdot \left(1 - a\right) \cdot \left[\left(1 - \rho \cdot (1 - \delta)\right) \left(\frac{\psi}{(A/p) \cdot \rho \cdot a}\right)^{-1} - a\right]^{-1} \left(\frac{\psi}{(A/p) \cdot \rho \cdot a}\right)^{-1} \cdot F
\]

(12)

\[
\hat{K} = \left(1 - a\right) \cdot \left[\left(1 - \rho \cdot (1 - \delta)\right) \left(\frac{\psi}{(A/p) \cdot \rho \cdot a}\right)^{-1} - a\right]^{-1} \left(\frac{\psi}{(A/p) \cdot \rho \cdot a}\right)^{-1} \cdot F
\]

(13)

\[
\hat{C} = A \cdot \hat{\mu} \cdot \left(1 - a\right) \cdot \left[\left(1 - \rho \cdot (1 - \delta)\right) \left(\frac{\psi}{(A/p) \cdot \rho \cdot a}\right)^{-1} - a\right]^{-1} \left(1 - a\right) \cdot F
\]

(14)

\[
\hat{I} = B \cdot (1 - \hat{\mu}) \cdot \left(1 - a\right) \cdot \left[\left(1 - \rho \cdot (1 - \delta)\right) \left(\frac{\psi}{(A/p) \cdot \rho \cdot a}\right)^{-1} - a\right]^{-1} \left(1 - a\right) \cdot F
\]

(15)

Proof: See Appendix 2. ♦

Looking at these equations we realized, firstly, that the aid-consumption share \( \hat{\mu} \) and the level of the aggregate capital \( \hat{K} \) are both functions of the efficiency parameter \( B (A/p) \). This should not be seen as a surprise at all. In fact, the law of motion of the aggregate stock of capital explains in itself this dependence for the latter variable. On the other hand, this same law along with the issue that the aggregate capital stock is allocated between the two sectors of the economy explains the case for \( \hat{\mu} \). Secondly, all these endogenous variables exhibit a linear dependence with respect to aid, which is a direct consequence of the framework assumed in the paper.

\(^{10}\) Notice that we prefer to keep the dependence of the endogenous variables explicitly in terms of \( \hat{\mu} \). As it will be seen later, this option facilitates the interpretation of these equations. In any event, the reader can always considers the value of this aid-consumption share as given by Equation (10).
Finally, as a result of this proposition and the issue that \( \psi < 1 \) the following corollary establishes a bottom limit for the capital stock share \( a \):

**Corollary:** If \( \left( \frac{1 - \rho \cdot (1 - \delta)}{(A/p) \cdot \rho \cdot a} \right)^{\frac{\psi}{1-\psi}} > a \), any value of 'a' must satisfy the following inequality in order to hold \( \psi < 1 \):

\[
a^2 > \frac{1 - \rho \cdot (1 - \delta)}{(A/p) \cdot \rho} \tag{16}
\]

**Proof:** See Appendix 2 ♦

### 2.4 Some comparative statics and their economic interpretations

We will now derive some important theoretical properties of the steady state, some of which not only facilitate the economic interpretation of the results spelt out by the simulations but they also proportionate a rationale behind the planner’s decision-making process. To begin with, we introduce, firstly, the steady state value of the output \( \hat{Y} \) for this economy. Actually, Equations (9), (14) and (15) are sufficient to demonstrate that this variable takes the form

\[
\hat{Y} = A \cdot \left[ (1 - a) \cdot \left( \frac{1 - \rho \cdot (1 - \delta)}{(A/p) \cdot \rho \cdot a} \right)^{\frac{\psi}{1-\psi}} - a \right]^{-1} \cdot \frac{1}{\psi} \cdot F \tag{17}
\]

Then, Equations (14), (15) and (17) are sufficient to demonstrate that the following inequalities hold around the steady state value:

\[
\frac{\partial \hat{C}}{\partial F \text{ c.p.}} = A \cdot \frac{\partial \hat{Y}}{\partial F \text{ c.p.}} = A \cdot (1 - \hat{\mu}) \cdot \left[ (1 - a) \cdot \left( \frac{1 - \rho \cdot (1 - \delta)}{(A/p) \cdot \rho \cdot a} \right)^{\frac{\psi}{1-\psi}} - a \right]^{-1} \cdot \frac{1}{\psi} \tag{18}
\]

\[
p \cdot \frac{\partial \hat{Y}}{\partial F \text{ c.p.}} = A \cdot (1 - \hat{\mu}) \cdot \left[ (1 - a) \cdot \left( \frac{1 - \rho \cdot (1 - \delta)}{(A/p) \cdot \rho \cdot a} \right)^{\frac{\psi}{1-\psi}} - a \right]^{-1} \cdot \frac{1}{\psi} \tag{19}
\]

\[
\frac{\partial \hat{Y}}{\partial a \text{ c.p.}} > 0, \quad \frac{\partial \hat{Y}}{\partial \psi \text{ c.p.}} > 0 \quad \text{if} \quad \psi < 0 \tag{20}
\]

Two main facts are worth pointing out here:

Firstly, Equation (20) shows what it could be called the ‘internal’ technological mechanisms (possibilities) available to improving the long run output performance. As it will be shown in Section 4 the effect of
augmenting \( a \) or \( \psi \) is quantitatively very different, though. In addition, there is also a significant consequence of augmenting the value of \( \psi \) when it is negative. In fact, when the expression for the elasticity of substitution is shown - which is given by the expression \( \varepsilon = 1/(1 - \psi) \) - it is easy to realize that an increase in \( \psi \) also raises \( \varepsilon \). Then, an increase in \( \psi \) makes the economy more dependent on aid which, depending on the magnitude of the variation in the output, might not be a good strategy at all. Secondly, Equations (18)-(19) exhibit the consumption (investment) marginal productivity of aid CMPA (IMPA) at the steady state point.\(^{11}\) According to these equations, it is clear that the CMPA is bigger than the IMPA for all \( \bar{\mu} > 0.5 \). Put correctly,

\[
\text{CMPA}_{s,l} > \text{IMPA}_{s,l} \quad \text{if} \quad 1 - \delta \left[ \frac{1 - \rho - (1 - \delta)}{(A/\rho)\, \psi \cdot \rho \cdot a} \right] \frac{1}{\psi - 1} > 0.5
\] (21)

Then, if the parameters of the model satisfy the above inequality, the consumption sector will always benefit in the long run with more external resources in detriment of the investment sector.

### 3. A Calibration

**Data:** There are several sources for data on foreign aid. Nevertheless, it seems more appropriate to use the database built by Craig Burnside and David Dollar (2002)\(^ {12}\) (B&D) which is based on the last database on foreign aid developed by the World Bank (Chang et al., 1998). Even though the original data is in current U.S. dollars, B&D converted it into constant 1985 dollars using the unit-value of import price index from International Financial Statistics. Finally, they divided the aid figures by real GDP in constant 1985 prices from Robert Summers and Alan Heston (1991; Penn World Tables 5.6). Thus the advantages of using this database is that it provides, on the one hand, a more accurate measure of the real amount of aid going to the recipient. On the other hand, it also provides a measure of aid and of the ratio aid / GDP that are constants in terms of its purchasing power over a representative bundle of world imports, which makes the figures comparable to each other. Besides this, the way that the aid figures are presented in the data set supports the assumption that aid can be viewed as permanent income by aid-recipient countries.\(^ {13}\) The countries covered in this data set are listed in Table 2. The National Accounts figures are taken from Penn World Tables 5.6 and 6.0 and the \( K/Y \) ratios were estimated from Penn World Tables 5.6. The ratio consumption to output is equivalent here to the share only consumed by families (civil consumption), i.e. government expenditure and public inversion are contained in the ratio investment to output.\(^ {14}\)

**General criterion:** In order to speak in generalities we will not restrict the calibration to a special country. In contrast, a positive characteristic of the model is that of it can be calibrated for several receipts exhibiting a ratio \( K/Y \) bigger than 2.0.\(^ {15}\) The intuition behind this choice has to do with the issue that the aid-effectiveness problem seems to be the same for most PDCs when it is put in pure economic terms like in this paper. Therefore, trying to get a general model as far as possible is not an unreasonable project. Thus, we offer two calibrations each representing two ‘representative’ economies listed in Table 2. Hence, countries like Bolivia, Chile, Colombia,

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\(^{11}\) Notice that the investment marginal productivity of aid is multiplied by \( \rho \) in order to make the comparison with the consumption marginal productivity of aid correctly.

\(^{12}\) We thank Craig Burnside for providing me his database.

\(^{13}\) In fact, the data is presented in such a way that three kinds of aid are distinguished: Bilateral Aid; Multilateral Aid (IMF), and The World Bank Aid.

\(^{14}\) Even though there are all these advantages, there is also a disadvantage to using this database. It has to do with the issue that it imposes a hard constraint for the calibration process due to the aid figures are relatively smaller, compared with those involved in other data sets like, for example, Barro-Lee Data Set for a Panel of 138 Countries. Therefore, we think that the calibration process presented here constitutes a real added value to the paper.

\(^{15}\) Ratios less than this value imply rates of depreciation higher than 14% per year, which are not supported by any empirical evidence.
Ecuador, Honduras, Mexico, Peru, Venezuela, Syria, Turkey, Korea, Philippines, Sri Lanka and Zimbabwe enter into this categorization.16

**Step 1:** The set of parameters \( \{A, p, \rho, F\} \) is fixed a priori. The exogenous productivity index for the consumption sector is fixed at 1 and the corresponding index for the investment sector is fixed at 0.5 or 1.2. This choice allows to simulating the model for a high and a low value of \( p \), the relative price of the investment goods.17 On the other hand, the value of the time-discounted factor is calibrated at 0.98, which represents a typical value used in the real business cycle literature. Finally, the value of \( F \) is fixed at 1.

**Step 2:** The value of the ratio aid to output is ‘fixed’ around 1%. Although the planner sees this ratio as being exogenous in her decision-making process, the model spells out an endogenous value for this share. For this, the model offers a very approximate value of 1% for this ratio.

**Step 3:** The rate of depreciation of physical capital is calibrated in such a way that the economy exhibits a ratio \( pK / \bar{Y} \) equal to 2.5 or 3.29, which are “representative” values of those included in Table 2.

**Step 4:** The value of the endogenous variable \( \hat{\mu} \) can be easily obtained directly from the calibration process 18. Equations (9), (14) and (15) are sufficient to demonstrate that the value of the aid-consumption share is equivalent to the ratio

\[
\hat{\mu} = \hat{C} / \hat{Y}
\]

Then, identifying this endogenous ratio with the corresponding National Income Accounts ratio and considering the steady state values that the above steps imply for the endogenous variables \( \hat{pI} / \bar{Y}, \hat{\mu}, \delta \) I get:

<table>
<thead>
<tr>
<th>Parameter (low-( p ) case)</th>
<th>( A )</th>
<th>( p )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated value</td>
<td>1</td>
<td>0.60</td>
<td>0.98</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>( C/Y )</th>
<th>( pI/Y )</th>
<th>( pK/Y )</th>
<th>( \hat{\mu} )</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady state</td>
<td>0.73</td>
<td>0.27</td>
<td>2.50</td>
<td>0.729</td>
<td>0.108</td>
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</table>

<table>
<thead>
<tr>
<th>Parameter (high-( p ) case)</th>
<th>( A )</th>
<th>( p )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated value</td>
<td>1</td>
<td>1.5</td>
<td>0.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>( C/Y )</th>
<th>( pI/Y )</th>
<th>( pK/Y )</th>
<th>( \hat{\mu} )</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady state</td>
<td>0.73</td>
<td>0.27</td>
<td>3.29</td>
<td>0.73</td>
<td>0.082</td>
</tr>
</tbody>
</table>

**Step 5:** At this stage of the calibration we are now able to find the remaining parameters \( a \) and \( \psi \). They can be obtained by solving the system of equations given by Equation (10) and the ratio \( F/Y \) implicitly deduced from Equation (17) and fixed around 1%.

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16 Notice that if the ratio consumption to output is defined involving government expenditures also, the model can also be well calibrated for countries exhibiting ratios \( K/Y \) higher than 1.6. Then, countries like Argentina, Dominican Republic, Guatemala, Jamaica and Madagascar can be added to the above list.

17 We make this distinction due to the fact that we do not dispose of any empirical evidence about the value of the relative price of investment goods.

18 This does not mean, of course, that this variable had been ‘calibrated’.
Some comments can be brought out from the calibration process.

Firstly, the value of $\hat{\mu}$ obtained from the calibration process responds by itself to the question posed in the paper. Accordingly, the optimal response is to consume aid rather than to invest it. This result is in line with Boone’s (1996) empirical work, which found that foreign aid has mainly been channeled into consumption in most PDCs. Secondly, both low-$p$ and high-$p$ cases exhibit a negative value of $\psi$. In economic terms, as it was mentioned before, this finding means that the planner considers both the capital stock and foreign aid as having a low elasticity of substitution. There is significant insight hiding behind this. Actually, what the negative value of $\psi$ is reflecting is the issue that the planner does not view foreign aid as a strategic resource to be put into the economy. One unit of this input can be replaced for small units of the existing capital stock, or vice versa. The problem then is to know if the planner disposes of these small units of capital stock. If not, a higher grade of substitutability between factors is needed in order to supply this lack and aid then turns more strategic for the planner’s decision-making process. This condition might offer a rationale for understanding why the figures for foreign aid in African countries in Table 2 look so high in comparison with the rest of the globe. Therefore, the low figures for aid in Latin American countries and others might be viewed as an argument of plausibility to justify the negative value of the elasticity of substitution. In addition, many local political heads’ opinions would also support, in part, this finding. Finally, both low-$p$ and high-$p$ cases exhibit large values of $a$, which means that the capital stock is the main input in this economy and aid plays a small role in both cases.

### 4. Foreign aid shock

As anticipated before, the two different calibrated models (economies) are now shocked by introducing separately non-anticipated impulse and permanent shocks at time zero and having all of them a 0.5% intensity. Figures 1 and 2 display the estimated impulse and permanent responses for the low-$p$ economy ($p = 0.6$) and Figures 3 and 4 do the same for the high-$p$ case ($p = 1.5$). Of course, all the variables are expressed in levels.

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19 The same conclusion can be found in Easterly (1999). However, his work is focused uniquely on the short run evaluation of aid.

20 Unfortunately, because my model does not work for most African countries I cannot offer proof of this assumption.
Figure 1: Impulse responses to a positive shock in Aid (low – $p$)

- **Consumption 'C'**
- **Inversion 'pl'**
- **Consumption capital stock 'KC'**
- **Investment capital stock 'KI'**
- **Aid-consumption share 'u'**
- **Output 'Y'**

*Graphs showing the impulse responses over 33 periods for each indicator.*
Figure 2: Permanent responses to a positive shock in Aid (low-p)
Figure 3: Impulse responses to a positive shock in Aid (high –p)

- Consumption 'C'
- Inversion 'pl'
- Consumption capital stock 'KC'
- Investment capital stock 'KI'
- Aid-consumption share 'u'
- Output 'Y'
Figure 4: Permanent responses to a positive shock in Aid (high $\rho$)

- **Consumption 'C'**
- **Inversion 'pl'**
- **Consumption capital stock 'KC'**
- **Investment capital stock 'KI'**
- **Aid-consumption share 'u'**
- **Output 'Y'**
The following lessons can then be put forward:

(i) For both economies, impulse and permanent foreign aid shocks have no effect on the aid-consumption share in the long run. Only short run effects are observed and they have opposite directions depending on the kind of shock implemented. Accordingly, when an impulse (permanent) shock takes place, the short run response is to increase (decrease) the value of the aid-consumption share. However, the magnitudes of the short run variations in $\mu$ look a little bit different for each economy. On the one hand, under impulse shocks, a rise of around 0.125% is observed for the low-$p$ economy ($A<B$), while a rise of around 0.122% is detected for the high-$p$ one ($A>B$). On the other hand, under permanent shocks, a drop in $\mu$ of around 1.59% is observed for the low-$p$ economy, while a fall of around 2.13% is detected for the high-$p$ one. Thus the higher the relative price of investment goods the lower (higher) the short run increment (drop) of the aid-consumption share. Then, planners running this kind of economy would distribute, comparatively speaking, more fresh aid resources towards the investment sector than planners running low-$p$ economies.

(ii) For both economies, impulse aid shocks have no effect on output in the long run. Positive responses of this variable are observed only in the short run and they are channeled only via consumption. As in the previous case, there is a small difference between the short run variations in output for each economy: under impulse shocks a rise of around 0.339% is observed for the low-$p$ economy ($A<B$), while a rise of around 0.331% is detected for the high-$p$ one ($A>B$). On the other hand, permanent aid shocks increase the long run output but this rise is also via consumption. This last result is in line with the theoretical contribution of Gong and Zou (2001) who, based on an one-sector optimal growth model, predict that permanent rise in foreign aid increases long run consumption.21

(iii) Foreign aid shocks provoke exactly balanced opposite effects on the consumption and investment capital stocks. Looking at Equations (11) and (12) it is easy to understand this logic. Any change in $\mu$ will induce contrary balanced movements of these variables. As a consequence, aggregate capital stock will be constant over time when impulse shocks are implemented. In contrast, under permanent shocks a higher aggregate short run capital stock is achieved but aid has no effect on this variable in the long run. This last result is similar to those found in the theoretical work of Obstfeld (1999) but not with that of Gong and Zou’s (2001) contribution that predicts that foreign aid decreases the steady state capital stock.

(iv) For impulse shocks, foreign aid has no effect on inversion, either in the short or in the long run. See Figures 1 and 3. In contrast, under permanent shocks a higher short run value of inversion is achieved. To a certain degree, this last result could fit with the empirical work of McGillivray (2000) that finds that, for Pakistan, grants and loans are positively associated with investment.22 On the other hand, permanent aid-shocks keep the long run value of inversion unchangeable. This finding is consistent with Boone (1994a,b; 1996) who finds that foreign aid has hardly any effect on investment.

(v) Due to the linearity exhibited by all the endogenous variables with respect to aid, a change of 0.5% in this variable is entirely (positively or negatively) absorbed when permanent shocks take place. See Section 2.3 and Figures 2 and 4.

(vi) Consumption in both economies responds negatively in the short run under a permanent shock. However, the drop is more intensive for the low-$p$ economy (around 1.25%) than for the high-$p$ one (around 1.80%).

Thus, taking into account all these results the next natural step is to wonder: Why do these economies behave like this? Or, more exactly: What is the rationale beneath the planner’s decision-making process? And the answer is quite simple. It lies in Equation (21). According to this equation, the long run consumption marginal

---

21 The issue that aid increases the short run output either under impulse or permanent shock might seem not to fit with the empirical finding of Przeworski and Vreeland (2000) who find that aid reduces growth in the short run. However, the comparison of my results and theirs is not totally adequate due to the fact that they only consider Multilateral Aid (IMF programs) in their estimates. On the other hand, using the same data for aid used here, Svensson (1999) also finds that aid has a positive long run effect on growth, but the impact of aid is conditional on the degree of political and civil liberties in the recipient country. However, the main problem with this finding is that the question about what channels would drive this effect remains open. Then, the long run increase in output via consumption can perfectly be possible.

22 Unfortunately, I have no evidence about the ratio K/Y (see Table 2) for this country from which I would be able to calibrate the model for this economy. Therefore, this comment is nothing more than an “educated” guess.
productivity of aid (CMPA) is bigger than the long run *investment marginal productivity of aid* (IMPA) for all aid-consumption shares bigger than 0.5. This is, in fact, the case studied here.

We shall now see how this property can explain the above lessons. To make things easier, we start by examining the planner’s response to impulse shocks. If $\text{CMPA}_{s.1} > \text{IMPA}_{s.1}$, it is clear that when fresh aid arrives in the planner her first response will be to increment both the consumption capital stock ‘KC’ and the aid-consumption share ‘$u$’. Due to this same marginal consumption incentive, the planner decides to invest just what is needed to replace the depreciated capital stock. Then, a drop in the investment capital stock ‘KI’ compensates exactly for this rise in ‘KC’. Thus both the aggregate capital stock and inversion remain constant over time. There is no capital accumulation and short run output increases occurs only via consumption. Then, what looks different in a high-$p$ economy respect to a low-$p$ one? Well, even though both planners respond to shocks armed with their consumption marginal productivity incentive, the high-$p$ economy’s planner smoothes this incentive a bit in order not to misbalance the two sectors of the economy too much. In other words, when $A > B$ (high-$p$) it seems to be that the planner pays a little more attention to trying to pursue a strategy based on a more balanced distribution of inputs.

On the other hand, the planner’s behavior looks a little bit different under permanent shocks. When aid arrives at time 0, the short run response is to reallocate resources in favor of the investment sector. The law of motion of capital works during this period but it does not work afterwards. The level of inversion returns to its initial value and aid plays no role in the long run because of the consumption marginal productivity incentive embodied into the planner’s decision making process. Why is the planner then willing to break away from this incentive in the short run? Just because her knows very well that this is the only way of accumulating capital stock. In other words, she is willing to pay this short run cost because fresh aid is *steadily* arriving in, which allows her to modify things in the long run in line with the consumption marginal productivity incentive. Again, as viewed for impulse shocks, output goes up in the long run via consumption and the planner running a high-$p$ economy also smooths the consumption marginal productivity incentive by pursuing a more balanced distribution of resources. Table 1 summarizes all the above comments.

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>$a$</th>
<th>$\psi$</th>
</tr>
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<tr>
<td>Calibrated value</td>
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<td>0.982</td>
<td>-0.78</td>
</tr>
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<td>$\Delta$ Parameter (%)</td>
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<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\Delta$ Output (%)</td>
<td>50</td>
<td>2.17</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$p$</th>
<th>$a$</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated value</td>
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<td>-1.38</td>
</tr>
<tr>
<td>$\Delta$ Parameter (%)</td>
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<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$\Delta$ Output (%)</td>
<td>65</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

4.1 Some local dynamics

Let us now study the transition dynamics to the steady state growth paths. Our motivation in doing this is to go a little step further from Equation (20). This little step consists of studying what kind of ‘internal’ mechanism (possibilities) available to improving the long run output performance is more efficient. To do this, both already calibrated economies are shocked by making small independent variations in the parameters $a$ and $\psi$. The following figures summarize the results of these simulations:

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>$a$</th>
<th>$\psi$</th>
</tr>
</thead>
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<tr>
<td>Calibrated value</td>
<td>0.6</td>
<td>0.982</td>
<td>-0.78</td>
</tr>
<tr>
<td>$\Delta$ Parameter (%)</td>
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<td>$\Delta$ Output (%)</td>
<td>50</td>
<td>2.17</td>
<td></td>
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</tbody>
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>$p$</th>
<th>$a$</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated value</td>
<td>1.5</td>
<td>0.999</td>
<td>-1.38</td>
</tr>
<tr>
<td>$\Delta$ Parameter (%)</td>
<td>0.05</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$\Delta$ Output (%)</td>
<td>65</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>
From the above results, it is clear that an increase in $a$ about 0.5% (0.05%) has a strong positive effect on output, about 50% (65%). In contrast, an increase in $\psi$ about 0.5% (0.05%) rises the output approximately only by 2.17% (0.2%).

Accordingly, a possible interpretation of Equation (20) as a whole is that for a low substitutability of inputs ($\psi < 0$) the planner will always prefer to use technologies involving mainly the use of her own resources; in this case, capital stock.

5. Concluding remarks

Existing empirical one-sector models involving foreign aid as an exogenous variable have seldom found a positive relationship between this variable and both investment and growth. Furthermore, previous empirical works also show that positive changes in output are only channeled via consumption. Some of the very few existing theoretical contributions in this area reinforce these findings. Moreover, many authors behind these models have made general statements about whether foreign aid is or is not effective, even though there are many problems in making such an evaluation, most of all the endogeneity of aid.

In this paper we offer a new approach that, broadly speaking, can be viewed as a good starting point to work out better models for studying the effectiveness of aid. Two of the most striking virtues of this framework are its multi-sectoral character and the sector allocation endogeneity of aid; its applicability to many countries is another one. The use of similar technology in both the consumption and the investment sector is, perhaps, its most significant weakness. However, the strength of its results seems to be its most relevant output. The model predicts, on the one hand, that the optimal response of the planner is to consume aid rather than to invest it. On the other hand, it also predicts no effect of aid on the investment sector, either in the short or in the long run; only a permanent aid shock can affect the capital accumulation but it has only a small effect. Finally, the model also predicts that changes in output are only through consumption. In sum, even though the normative approach of the model, it reproduces the main stylized facts mentioned above quite well.

Besides this, the model also offers a clear formalization for both the empirical finding of Boone (1996) and Easterly’s (1999) statement, which says that a bigger aid-consumption marginal productivity incentive would drive the effectiveness of aid. However, planners running high-$p$ economies would tend to pursue strategies that imply more distribution of fresh aid towards the investment sector, in comparison with those planners running low-$p$ economies. To a certain degree, to pursue a more “balanced” strategy to allocate resources seems to be the right decision in order to start up the law of motion of capital and then investment and eventually growth. Nevertheless, the aid-consumption marginal incentive is a strong economic reason to do the opposite in the long run.

Finally, the model allows addressing a more sophisticated issue. From the low substitutability of factors exhibited by the model, it puts in evidence that recipient’s planners (excluding the African Continent) do not see aid as a strategic resource to be put in the economy. In addition, the high-calibrated values for the capital share ‘$a$’ reinforce this attitude. In short, the fundamental message at this point is that aid does not play an important role in the planner’s decision-making process. Unfortunately, only arguments of plausibility coming from many local political heads’ opinions would support, in part, this finding. If this issue could be empirically borne, a fundamental question would raise: What is the true role, if any, of aid in the process of development of PDCs? For the moment, answering this question is out of the scope of the paper.

In conclusion, the development effectiveness of aid is much more than a pure economic problem. In fact, when all the country-specific political and social aspects also underlying the planner’s decision-making process are taken into account, this problem becomes a hard dilemma for the planner and trying to solve it in pure economic terms constitutes a reduced approach. Moreover, due to the fact that there is a huge wedge in terms of

23 See Easterly (1999), page 429.
24 In this line of argument, I would not dare qualify the IFIs and donors’ performances with a passing grade and the PDCs’ performances with a failing grade; a bias conclusion that could rise from a fast reading of the paper. This caution is engaged, firstly, in a wide-ranging and comprehensive reexamination of the nature of assistance programs offered by IFIs like the World Bank and the IMF. See Caballero (2002), Fischer (2000), Stiglitz (2002). See also the IMF website http://www.imf.org for a number of papers on aspects of this debate. Secondly, it is also engaged in the current debate originated from the provocative contribution of Burnside and Dollar (2002) that conditions the effectiveness of aid in terms of growth to the application of “good” policies by recipients.
robustness between results spelled out by optimal models and the real economy, our model is far from representing reality. In any event, this wedge must be reduced and our ongoing agenda has plenty of room for further improvements.
## Table 2: Country-Specific Summary Statistics

<table>
<thead>
<tr>
<th>Country</th>
<th>C/Y (share of GDP)</th>
<th>I/Y (share of GDP)</th>
<th>K/Y (percent of GDP)</th>
<th>Aid (percent of GDP)</th>
</tr>
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<td>0.33</td>
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<td>0.02</td>
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</tbody>
</table>

*Notes:* The figures for C/Y were taken mostly from the Penn Tables 6.1 and some of them from Przeworski and Vreeland (2000)’s database. The figures for K/Y were estimated from the Penn Tables 5.6 and the figures for aid from Craig and Burnside (2000)’s database.
Appendix 2: Proofs

A Proof of proposition 1.

Following Dixit (1976), the intertemporal hamiltonian for the maximization problem (3) is defined as

\[ H_t = A_t \cdot [a \cdot KC_t^{\psi} + (1 - a) \cdot (\mu_t \cdot F_t)]^{1/\psi} + \rho \cdot \lambda_{t+1} \cdot [B_t \cdot [a \cdot KL_t^{\psi} + (1 - a) \cdot (1 - \mu_t) \cdot F_t]]^{1/\psi} - \delta K_t \]

\[ + \omega_t (K_t - KC_t - KL_t) \]

Then, the first-order conditions respect to \( \{KC_t, KL_t, \mu_t\} \) can be easily obtained from this expression. They imply, respectively, the following equalities:

\[ \omega_t = a \cdot A_t^{1-\psi} \cdot C_t^{1-\psi} \cdot KC_t^{\psi-1} \]

\[ \frac{\omega_t}{\rho \cdot \lambda_{t+1}} = a \cdot B_t^{1-\psi} \cdot I_t^{1-\psi} \cdot KL_t^{\psi-1} \]  \hspace{1cm} (A.1)

\[ \frac{A_t^{1-\psi} \cdot C_t^{1-\psi} \cdot \left( \frac{\mu_t}{1 - \mu_t} \right)^{\psi-1}}{B_t^{1-\psi} \cdot I_t^{1-\psi}} = \rho \cdot \lambda_{t+1} \]  \hspace{1cm} (A.2)

By combining adequately these three equations it is easy to get Equation (4). Now only the Euler equation must be obtained. Actually, it can be obtained from the canonic relationship evaluated in time \( t+1 \):

\[ \rho \cdot \lambda_{t+2} - \lambda_{t+1} = -\frac{\partial H_{t+1}}{\partial K_{t+1}} = \delta \cdot \rho \cdot \lambda_{t+2} - \omega_{t+1} \]  \hspace{1cm} (A.4)

Then, by substituting Equations (A.3) and (A.1) into this last condition, Equation (5) arises.\(^25\)

B Proof of proposition 2.

The Euler equation at the steady state point takes the form

\[ \frac{1 - \rho \cdot (1 - \delta)}{\rho \cdot a \cdot \hat{B}} = \hat{I}^{1-\psi} \cdot \hat{KC}^{\psi-1} \cdot \left( \frac{1 - \hat{\mu}}{\hat{\mu}} \right)^{\psi-1} \]  \hspace{1cm} (A.5)

On the other hand, Equations (6), (7) and the first-order condition (4) evaluated at the same point give

\[ \delta \cdot (\hat{KC} + \hat{KI}) = \frac{\delta \cdot \hat{KC}}{\hat{\mu}} = \hat{I} \]  \hspace{1cm} (A.6)

Then, by substituting the above value of the inversion into (A.5) and replacing the value of \( B \) for \( (A/p) \), Equation (10) arises.

\(^{25}\) Notice that the Euler Equation can be rewritten in a simpler way. In fact, taking into account that the discounted shadow price \( \rho \cdot \lambda_{t+1} \) equals the relative price of investment goods \( p \), Equation (3) takes the form

\[ p \cdot (1 - \delta) - p / \rho = -a \cdot A_t^{1-\psi} \cdot C_t^{1-\psi} \cdot KC_t^{\psi-1} \]
Now, looking at Equation (4), it is easy to see that Equation (A.6) can also be written in terms of the investment capital stock as follows:

\[
\delta \cdot (K^C + K^I) = \frac{\delta \cdot K^I}{(1 - \hat{\mu})} = \hat{i}
\]

(A.7)

By putting this last expression into Equation (2) and replacing the value of \( B \) for \( (A/p) \), it is trivial to get Equation (12) and then, by using Equation (4), Equation (11). Hence, Equation (13) is simply the sum of Equation (11) and (12). Finally, known \( \{\hat{\mu}, K^C, K^I\} \), the equations for consumption and inversion are easily carried out.

Thus the condition

\[
\left(1 - \rho \cdot (1 - \delta)\right) \frac{\psi}{(A/p) \cdot \rho \cdot a} > a
\]

is required to guarantee positive values in all capital stocks. ♦

**Corollary:** If \( \left(1 - \rho \cdot (1 - \delta)\right) \frac{\psi}{(A/p) \cdot \rho \cdot a} > a \), any value of ‘\( a \)’ must satisfy the following inequality in order to hold \( \psi < 1 \):

\[
a^2 > \frac{1 - \rho \cdot (1 - \delta)}{(A/p) \cdot \rho}
\]

(A16)

**Proof:** It is trivial. ♦
References


